Vehicle-following Model based Mechanical Model of Traffic Flow

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ABSTRACT: *Based on vehicle – following model, we propose a new traffic flow dynamics model from both micro and macro aspects. The model can overcome some of the problems that exist in the existing kinetic models, such as avoiding the phenomenon that the traffic jam occurs in the vicinity of the disturbance density, thus exceeding the blockage density. Through design of examples, the authors also simulated traffic flow under different conditions, which shows the model has good numerical simulation capabilities, thus proving its theoretical and practical value.*

Keywords: Vehicle – Following, Traffic Flow, Dynamical Model

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1. Introduction

With the needs of economic construction and social development, traffic problem has increasingly become the focus of people's attention which leads to the following theoretical study of traffic flow. Meanwhile, because of its cross-disciplinary, Traffic flow theory has also attracted attentions of many experts and scholars in fields like mechanics, applied mathematics, traffic engineering and control theory and other areas.

Currently, the research of Traffic flow theory includes vehicle -following model, dynamic model (also known as continuum mechanics model), molecular dynamics method and cellular automata methods[1] etc, among which dynamic model and vehicle-following model are the two most influential and representative categories. The former develops more quickly in applications because of its macroscopic properties. The latter can be used as a supplement to the former because of its microscopic nature. In addition, because they study the evolution regularity of traffic flow from different perspectives, the foundation on which they establish is the same. based on this knowledge, In this paper, starting with the microscopic vehicle-following model, Taking into account the effect of speed difference and the distance difference on dynamic regularity of the traffic flow, the existed vehicle-following model was improved and perfected and the new Microscopic vehicle-following model was established. Then, based on this, Macro-mechanical model of traffic flow was established and was numerically simulated using the link between micro and macro.

Vehicle-following mode was first proposed by Pipes. L.A in 1953. It mainly adopts stimulate response m odel to study the driving state of two cars with one following the other in the single queue of vehicles traveling on the lane. The specific form of it is:

Tracking response = sensitivity × stimulus

In the process of vehicle-following, the movement state of the latter car is affected and restricted by many elements. Such as the velocity difference and distance between the two vehicles, personalities of the drivers, weather and the road condition etc. However, in the actual modeling process, the different motives and interests of the researchers resulted in the various vehicle-following models.

Here are some representative models:

OVM vehicle-following model proposed by Bando in 1995:

$$\frac{dv_n(t)}{dt} = k\left[v(\Delta x) - v_n(t)\right] \tag{1}$$

$v(\Delta x)$ is optimized speed which is decided by the distance between the two cars; k is Sensitivity coefficient. Model (1) considered the distance between the two cars and the speed of target car, but it did not take the speed difference of the two cars into account. Practical application of the model can simulate many of the qualitative characteristics of traffic flow, such as Traffic instability, blocking ,*f* stopping here and there. But compared with the actual observed data, OVM model has excessive acceleration or unrealistic deceleration.

On the basis of OVM model, Helbing proposed a generalized mechanical model (GFM) [5], which is in form:

$$\frac{dv_n(t)}{dt} = k\left[v(\Delta x) - v_n(t)\right] + \lambda \Delta v_n H(-\Delta x) \tag{2}$$

When $x \geq 0$ 时, $H(x)=1$ when $x<0$ 时 $H(x)=0$。Model (2) overcome the problem of excessive acceleration in OVM model。But the simulated result shows: compared with the actual research data.

The result shows that the time to start the vehicle is delayed for too long and when the vehicle starts the propagation speed of the small disturbance is too slow.

Based on this, Chinese scholars Jiang Rei and Wu Qingsong put forward FVD vehicle-following model in year 2001. Its form is:

$$\frac{dv_n(t)}{dt} = k\left[v(\Delta x) - v_n(t)\right] + \lambda \Delta v_n \tag{3}$$

The model better reflects the characteristics of the actual traffic condition. It can solve the problem of excessive acceleration of OVM model.

In [7], the following vehicle-following model is given based on the fact that the speed of the following vehicle is decided by the distance between the two adjacent cars.

$$x_n(t+T) = G(x_{n-1}(t) - x_n(t)) \tag{4}$$

Wherein $x_n(t)$ is the location of the vehicle n at time t. T is the time lag for the driver to respond to the stimulus in front. $G(x_{n-1}(t) - x_n(t))$ is the function of the distance between the two adjacent vehicles.

The following vehicle -following model is given based on the condition that the speed of the following car is only affected by the distance between the two cars:

$$\dot{x}_n(t+T) = G(x_{n-1}(t) - x_n(t)) \tag{5}$$

Based on observation of vehicle -following phenomenon, we concluded that the speed of the following car is decided not only by the distance between the two cars but also related to the speed of the two cars. Based on this, we came to the following vehicle following model:

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$$\dot{x}_n(t+T) = G(x_{n-1}(t) - x_n(t)) + \lambda(\dot{x}_{n-1}(t) - \dot{x}_n(t))$$

$\lambda > 0$ is dimensionless constant.

Model (5) gives full consideration to Vehicle spacing, the effect of speed difference to the acceleration of the following vehicle. This model avoids the phenomenon that the following vehicle cannot accelerate no matter how long the distance between the two cars is, under the condition that when only speed difference is taken into consideration and the speed of the two vehicles is the same, Meanwhile, it also avoids the condition that if we only take vehicle spacing into consideration, the following cannot respond when the preceding car accelerate suddenly. In fact, for the two cars in the following state, the speed of the following car will always fluctuate around the speed the preceding car. The following car will not overtake the preceding car for too long a time so as to crush into the preceding car nor will it be slower than the preceding for too long a time so as to be lagged behind. When the preceding car accelerate or decelerate suddenly, the following car will respond immediately. When $\Delta v_n > 0$, it is when vehicle spacing is less than the safe distance, the following car can still accelerate. Therefore, it is very practical to take into account the impact of distance difference and speed difference on the acceleration at the same time.

Let's export a new dynamic based on the relationship between micro and macro.

2. The Establish of Model

In the formula (5), if we let:

$$v_n(t) = \dot{x}_n(t),\ x_{n-1}(t) - x_n(t) = h_n(t) \tag{6}$$

Then (5) equals to the following two functions:

$$\begin{cases} v_n(t+T) = G(h_n) + \lambda(v_{n-1}(t) - v_n(t)) \\ \dfrac{dh_n(t)}{dt} = v_{n-1}(t) - v_n(t) \end{cases}$$

Here, in order to transform them into Continuity equation, we Introduce the continuity variables $v(x, t)$, $h(x, t)$ to meet

$$v(x_n(t), t) = v_n(t)\ h(\frac{x_{n-1} + x_n}{2}, t) = h_n(t) \tag{7}$$

Then equation (7) can be

$$v(x_n(t+T), t+T) = G[h(x_n + \frac{1}{2}h_n, t)] + \lambda[v(x_{n-1}, t) - v(x_n, t)]$$

Then solve partial differential equations with t and h_n Expanding it by Taylor expansion and only retaining the linear term, we get:

$$\begin{aligned} v(x_n, t) + [\frac{\partial v(x_n, t)}{\partial t} + v(x_n, t)\frac{\partial v(x_n, t)}{\partial x}]T \\ = G[h(x_n, t)] + \frac{1}{2}h(x_n, t)\frac{\partial G[h(x_n, t)]}{\partial h} \cdot \frac{\partial h(x_n, t)}{\partial x} \\ + \lambda\frac{\partial v(x_n, t)}{\partial x}h(x_n, t) \end{aligned} \tag{8}$$

Because (7) can be

$$\frac{d}{dt}h(\frac{x_{n-1} + x_n}{2}, t) = v(x_{n-1}, t) - v(x_n, t),$$

In addition, when $x = \frac{x_{n-1} + x_n}{2}$, we can employ as an approximation.

$$h_t + vh_x = hv_x \tag{9}$$

Now, based on $\rho = 1/h, V(\rho) = G(h)$, (8), (9) can be transformed into

$$\begin{cases} v + (v_t + vv_x)T = V(\rho) + \frac{1}{2}\frac{V'(\rho)}{\rho}\frac{\partial \rho}{\partial x} + \frac{\lambda}{\rho}\cdot\frac{\partial v}{\partial x} \\ \rho_t + (\rho v)_x = 0 \end{cases}$$

$V'(\rho)$ is the decreasing function of ρ. It satisfies $V'(\rho) < 0$ so let $\gamma = -\frac{1}{2}V'(\rho) > 0$.

We get the following model:

$$\begin{cases} \rho_t + q_x = 0 \\ \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{v - V(\rho)}{T} - \frac{\gamma}{\rho T}\cdot\frac{\partial \rho}{\partial x} + \frac{\lambda}{\rho T}\cdot\frac{\partial v}{\partial x} \\ q = \rho v \end{cases} \tag{10}$$

In the model, $\gamma > 0$. It is because the density of the preceding vehicles is continuously growing, so the driver has to decelerate, otherwise accelerate. Meanwhile, the existence of the density difference shows that the traffic speed of a place will not be decided by the local density but be decided by the density of a certain place in the front. In addition, the vehicles in the low-density areas can be prevented from moving into high-density areas. Otherwise, the existence of it will accelerate the vehicles in high-density areas to flow to the low-density areas. The existence of the velocity gradient shows that when the preceding vehicle accelerates the following vehicle will accelerate correspondingly. Otherwise, the following vehicle will decelerate. The acceleration and deceleration of vehicles was particularly obvious in the state of linear vehicle-following. That is to say, the drivers will adjust the speed of their vehicles according to distribution of the density and the speed of the vehicle flows in front to make the front-body spacing of the two cars tend to be normal.

Note 1: in formula 10, if we let $\lambda = 0$ we get Payne model. If we let $\lambda = 0$ $k = 0, V(\rho) = u_f$,we get Ross model. u_f is free flow speed.

Note 2: if let $p = \frac{1}{T}(\gamma T - \lambda\rho)$, then (10) can be:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{v - V(\rho)}{T} - \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{11}$$

The model is consistent with the model in literature in [8], ρ is pressure.

3. The construction of differed the construction of differential format and numerical simulation entail format and numerical simulation

Both sides of formula (11) is multiplied by ρ, we get:

$$\rho\cdot\frac{\partial v}{\partial t} + \rho v\cdot\frac{\partial v}{\partial x} = -\frac{\rho v - \rho v(\rho)}{T} - \frac{\gamma}{T}\cdot\frac{\partial \rho}{\partial x} + \frac{\lambda}{T}\cdot\frac{\partial v}{\partial x}$$

After being simplified, we get

$$\frac{\partial q}{\partial t} + \frac{\partial(\frac{q^2}{\rho} + \frac{\gamma}{T}\cdot\rho - \frac{\lambda}{T}\frac{q}{\rho})}{\partial x} = \frac{\rho v - \rho v(\rho)}{T},$$

By this we can transform original equations (10) into

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial q}{\partial x} = 0 \\ \dfrac{\partial q}{\partial t} + \dfrac{\partial (\frac{q^2}{\rho} + \frac{\gamma}{T} \cdot \rho - \frac{\lambda}{T} \frac{q}{\rho})}{\partial x} = -\dfrac{\rho v - \rho v(\rho)}{T} \end{cases} \tag{11}$$

If let

$$U = \begin{pmatrix} p \\ q \end{pmatrix}, F = \begin{pmatrix} q \\ \frac{q^2}{\rho} + \frac{\gamma}{T} \cdot \rho - \frac{\lambda}{T} \cdot v \end{pmatrix}$$

$$S = \begin{Bmatrix} 0 \\ \frac{\rho v(\rho) - \rho v}{T} \end{Bmatrix}$$

the above equations can be turned into the conservation equations

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = S \tag{12}$$

Further, we can transform function (12) into the following quasilinear form:

$$\frac{\partial U}{\partial t} + J \frac{\partial U}{\partial x} = S$$

$$J = \frac{\partial F}{\partial U} = \begin{bmatrix} 0 & 1 \\ -\frac{q^2}{\rho^2} + \frac{\gamma}{T} + \frac{\lambda}{T} \cdot \frac{q}{\rho^2} & \frac{2q}{\rho} - \frac{\lambda}{\rho T} \end{bmatrix}$$

From which we can see matrix has two different real eigenvalues. Therefore, model (12) constitutes a conservative hyperbolic equation. Difference scheme can be constructed in accordance with the hyperbolic conservation.

In view of the fact that the high-end model is easy to produce unrealistic oscillations and nonlinear numerical instabilities, especially for the Larger gradient, and the format of an order of accuracy in the simulation of traffic flow shows good performance, so we choose Lax-Friedrichs format of an order of accuracy.

$$U_j^{n+1} = \frac{U_{j-1}^n + U_{j+1}^n}{2} - \frac{\Delta t}{2\Delta x}(F_{j+1}^n - F_{j-1}^n) + \Delta t S_j^n$$

for numerical simulation. Its stability condition (CFL condition) is: $\Delta t \leq \Delta x / v_f$

n is time number; *j* is space step number; *T* is Lag time; *Δt* is time step *Δx* is space step.

Let's study the validity of the model based on the following example:

Let maximum density $\rho_j = 0.25$ Units / m; maximum speed $v_f = 30 m/s$; length of road $l = 10km$;

Time of driver's response: $T = 3s$ Speed of balancing — relationship of density:

$$v = v_e(\rho) = v_f(1 - \frac{\rho}{\rho_j})$$

Initial condition:

$$\rho(x,0) = \begin{cases} 0.18, & 0 \le x < 5000 \\ 0.04, & 5000 \le x \le 10000 \end{cases}$$

$$v(x,0) = v_e(\rho(x,0))$$

Boundary condition:

$$\frac{\partial v(0,t)}{\partial x} = 0, \qquad \frac{\partial v(l,t)}{\partial x} = 0$$

The result of calculation is shown as figure 1 - 2.

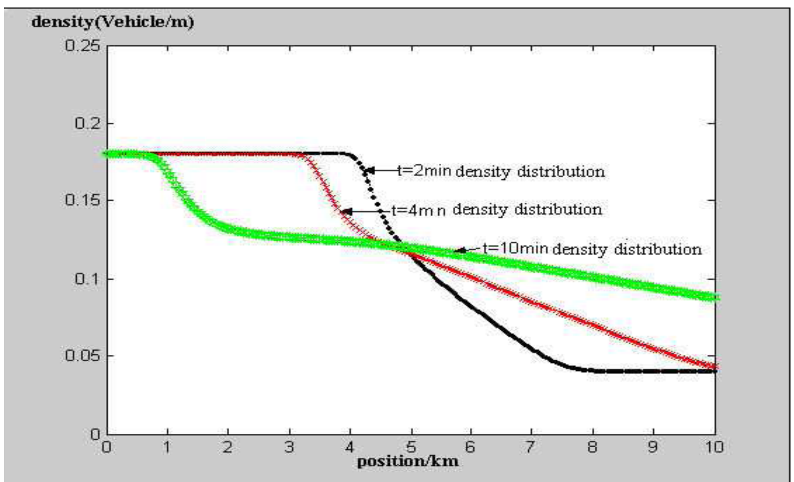

Figure 1. The distribution of density $\rho(x, t)$ for various value of t

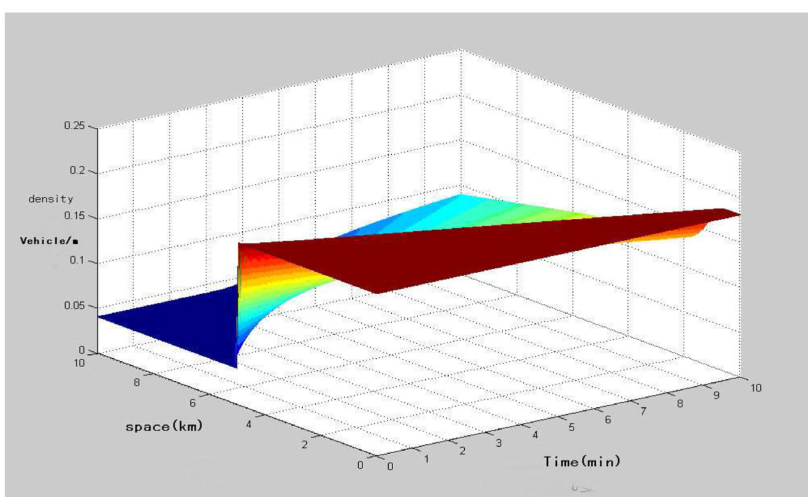

Figure 2. Temporal development of density $\rho(x, t)$

Example 2: Consider the process of dissipation in the traffic flow after the increase of the local density:

Let maximum density $\rho_j = 0.9$, maximum speed $v_f = 20m/s$, length of road l =10km;

Time of driver's response: T= 2.5s 。$\rho_a = 0.6$.

Speed of balancing — relationship of density:

$$V(\rho) = v_e(\rho) = v_f(1 - \frac{\rho}{\rho_j})$$

Initial condition:

$$\rho(x,0) = \begin{cases} \rho_a + \alpha\ \rho_a \cos\left(\frac{\pi(x - x_0)}{2\Delta x}\right) & 6000 \le x < 8000 \\ \rho_a, & x \in [0,6000) \cup [8000,10000] \end{cases}$$

$$v(x,0) = v_e(\rho(x,0))$$

Boundary condition:

$$\frac{\partial v(0,t)}{\partial x} = 0 \qquad \frac{\partial v(l,t)}{\partial x} = 0$$

The result of calculation is shown as figure 3-4.

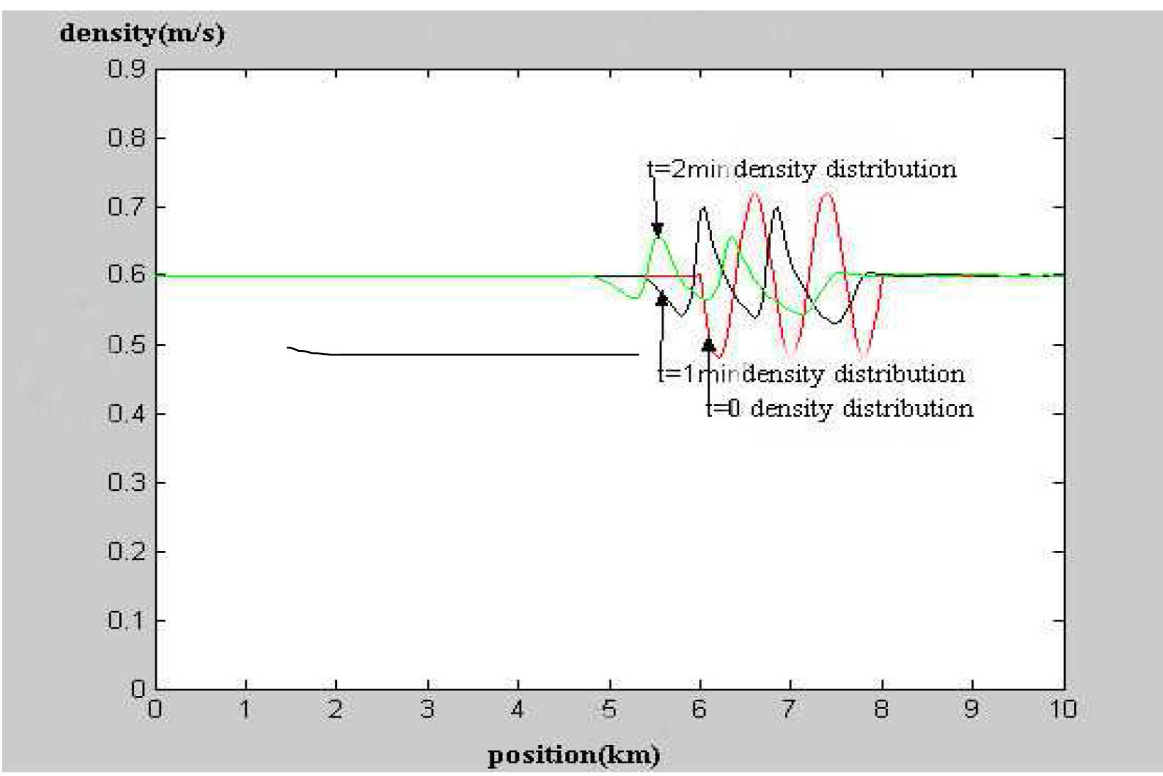

Figure 3. The distribution of density ρ (x, t) for various value of t

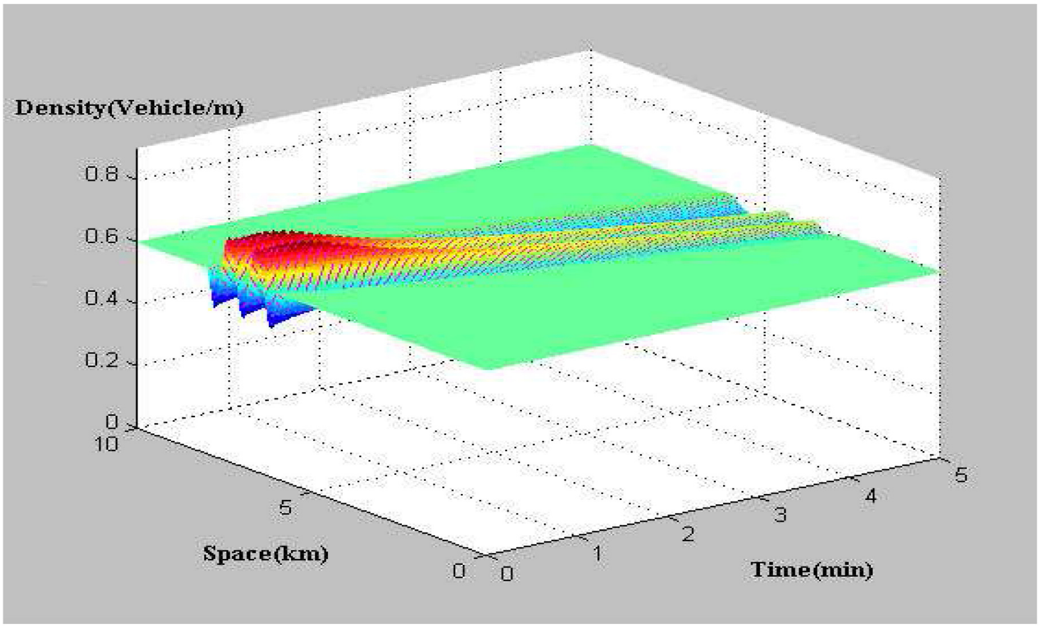

Figure 4. Temporal development of density $\rho(x, t)$

As can be seen from the simulation results, In Example 1, the high density of the traffic in upper reaches evacuate to the low-density areas of the downstream over time. It resulted in traffic rarefaction wave which is consistent with the nature of the traffic flow. In example 2, disturbance occurs in the middle sections of initial time, but when it is not so serious, it will gradually disappear with time passing by because the density resulted from disturbance increases. In addition, from figures we can see local density Increase and spread as figure 3 and form a shock wave following it. In fact, some mobiles move relatively fast, so drivers enter such an area of a local density increase from behind and rapidly decelerate when passing through the shock waves. However, he can only accelerate slowly when he left the traffic congestion. It is consistent with the actual situation. In addition, the further simulation shows that when $\alpha > 0.25$ density caused by disturbance increases and will continue until it forms blocks. This fully shows that small perturbations of stable traffic flow within a certain range will gradually disappear. With the increase of traffic density, perturbations will develop into a blockage and forms local clusters, it is the so-called stop-go phenomenon.

4. Conclusions

Based on the brief review of the research at the traffic flow model, taking the car- Following model as the starting point, this paper got the New traffic flow dynamics model with the method of converting microscopic into macroscopic and the method of Linear approximation. Meanwhile, it numerically simulated the different examples according to different boundary conditions. The results shows that the model can better capture the traffic shock wave formation, the formation and the ease of the blockings as well as the small disturbance instability of traffic and other traffic phenomena. These results not only agree with the evolution of the wave but also agree with the evolution of the traffic flow.

Of course, due to the limitations of the data acquisition, the validity of the model still requires inspection of the numerical simulation results. Meanwhile, as to the proposal of the model, We can be sure of the difference in distance and the speed difference between front and rear vehicles will surely affect the acceleration of the following vehicles. In addition, as we have put forward in [11], if we adjust the speed of the following car, the speed difference of the two cars will give the strongest response. but as to the proportion it accounts for, it is the size of the parameter λ we still need to further study.

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