A Model for Measuring the Outage Conditions in the Presence of Atmospheric Turbulence

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ABSTRACT: *We propose for assessing the outage conditions of FSO system using optimal parameters. We deployed the laser beam propagation with the presence of normal atmospheric turbulence and random jitter. We further derived the closed from expressions for measuring the outage conditions.*

Keywords: Random Jitter, Gamma-gamma Distribution, Atmospheric Turbulence, Log-normal Distribution, Outage Probability

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1. Introduction

The more demanding telecommunication services have become; the more bandwidth they need. This constant need of increasing the channel capacity poses the question of finding new means to transfer data at high bit rates. One fairly new alternative technology is the Free-Space Optical (FSO) communication systems. It is a wireless optical technology, that can transfer information at very high speeds, close to the channel capacity of optical fiber links.

Having such high-speed communication systems, however, raises the question for their availability and respectively their outage probability. There are a lot of research papers regarding the performance of high-speed FSO systems under various conditions such as: strong atmospheric turbulence [1- 4], different atmospheric visibility [5] and pointing errors [7- 9].

Our previous work [10-11] presents a method for optimizing the laser beam divergence angle θ_t, so that the FSO system can be reliable under various atmospheric conditions. Using optimal beam divergence also compensates for the fluctuations in the laser beam direction. In this paper we will show a method for evaluation of such FSO system in the presence of atmospheric turbulence.

From here on the paper is structured as follows: 2. Calculation of the optimal divergence of a gaussian beam, 3. Methodology for calculating the outage probability of FSO in the presence of atmospheric turbulence and 4 Conclusion.

2. Calculation of the Optimal Divergence of a Gaussian Beam

Figure 1 shows the basic parameters of a free-space optical system:

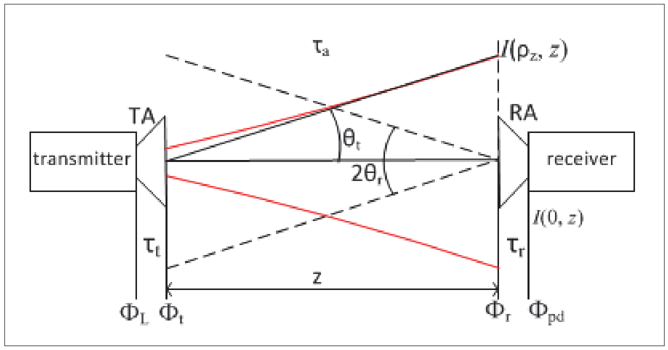

Figure 1. FSO link and system parameters

The transmitter (TA) and the receiver (RA) antennas are aligned. The distribution of the optical radiation intensity $I(\rho_z, z)$ in the plane z = *const* depends mainly on the phase and amplitude distribution of the field in the transmitting antenna. We accept equiphase and Gaussian amplitude distribution. The optical power radiated by the laser source is Φ_L, whereas Φ_t and Φ_r are the optical fluxes at TA and RA. Optical power at the entrance of the photodetector is represented with Φ_{pd}. $I(0, z)$ is the optical radiation intensity along the axis of the antenna. Diagram width of the receiving antenna is $2\theta_r$. The radius of the Gaussian laser beam is represented by ρ_z and is defined by:

$$I(\rho_z, z) = \frac{I(0, z)}{e^2} \qquad (1)$$

Equation (1) is also used to define the divergence θ_t of the radiation in the far field region. Losses in the transmitter and receiver antenna are denoted with τ_t and τ_r, while τ_a is the transparency of the atmospheric channel.

Radial distribution of the optical radiation intensity in the plane z = *const*, in which receiving aperture is situated, is shown in Figure 2.

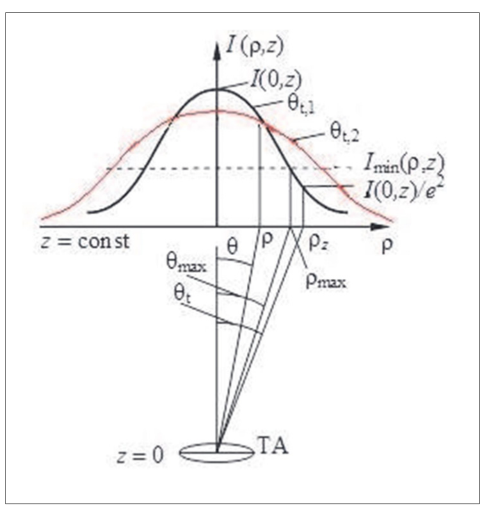

Figure 2. Radial distribution of the intensity in a plane z = const for two values of the divergence of the optical radiation θ_t

The intensity I_{min}, corresponds to the minimal optical power through RA, respectively the minimal Φ_{pd}, for which the FSO system works reliably, with bearable bit error ratio (BER). It also defines the optimal magnitudes of the optical beam divergence angle $\theta_{t, opt}$ and the corresponding beam radius, $\rho_{z, opt}$, which allows for maximal angular deviation θ_{max} (and the corresponding linear shifts ρ_{max}) of the optical beam axis from its original direction. These misalignments can be consequence of unstable foundations on which the antennas are mounted, building sway, pointing errors or large scale turbulent eddies.

In order to calculate optimal beam divergence angle and radius, we first need to find the minimal power in the entrance of the photodetector, that allows the FSO system to work reliably (corresponding to reasonable BER). We can calculate Φ_{pd} using the expression

$$\Phi_{\mathrm{pd}} = \frac{1}{2}\left[\frac{SNR^2.C_{\mathrm{I}}.e^-}{R_{\mathrm{I}}} + \left(\left(-\frac{SNR^2.C_{\mathrm{I}}.e^-}{R_{\mathrm{I}}}\right)^2 + \right.\right. \\ \left.\left. + \frac{4SNR^2.C_{\mathrm{I}}}{R_{\mathrm{I}}}\left(\frac{2k_{\mathrm{B}}.T.A}{R_{\mathrm{I}}.R_{\mathrm{Fb}}} + e^-.\Phi_{\mathrm{B}}\right)\right)^{\frac{1}{2}}\right] \tag{2}$$

In (2) R_I and η(λ0) are the integral sensitivity and the quantum efficiency of the photodetector, C_I is the channel capacity. A is a constant of the receiver; R_{FB} is the value of the resistor in the feedback of the preamplifier. The background optical flux Φ_B is defined by the brightness of the background radiation $L_{\lambda B}$, the transmission wavelength of the interference filter before the photodetector $\Delta_{\lambda F}$, and the parameters of the receiver; radius of the aperture R_r and its angular width θ_r.

$$\Phi_{\mathrm{B}} = \pi^2.\tau_{\mathrm{r}}.L_{\lambda,\mathrm{B}}.R_{\mathrm{r}}^2.\theta_{\mathrm{r}}^2.\Delta\lambda_{\mathrm{F}} \tag{3}$$

Signal to noise ratio (*SNR*) is defined by the BER needed for the particular use case [14]

$$BER = \frac{1}{2}\mathrm{erfc}\left(\frac{SNR}{2\sqrt{2}}\right) \tag{4}$$

According to Gaussian beam theory the intensity distribution in the far-field region, in which the receiver is placed is defined by

$$I(\rho, z) = I(0, z)\exp\left(-2\frac{\rho^2}{\rho_z^2(z)}\right) \tag{5}$$

The optical radiation along the axis of the laser beam is

$$I(0, z) = \frac{2.\tau_{\mathrm{t}}.\tau_a(\lambda_0, S_{\mathrm{M}}, z).\Phi_{\mathrm{L}}}{\pi.\rho_{\mathrm{z}}^2(z)} \tag{6}$$

When $\rho = \rho_{max}$ the optical radiation intensity $I = I_{min}$, i.e.

$$I_{\min} = I(0, z)\exp\left(-2\frac{\rho_{\max}^2}{\rho_z^2(z)}\right) \tag{7}$$

From (6) and (7) the equation for ρ_{max} can be derived:

$$\rho_{\max} = \frac{1}{\sqrt{2}}\rho_z\sqrt{\ln\frac{2.\tau_{\mathrm{t}}.\tau_{\mathrm{a}}.\Phi_{\mathrm{L}}}{\pi.\rho_{\mathrm{z}}^2.(1-e^{-2})I_{\min}}} \tag{8}$$

The value of the laser beam radius ρ_z for which FSO can bare the extreme magnitudes of the linear shifts is

$$I_{\min} = \frac{\Phi_{\text{pd}}\big|_{SNR=\text{const}}}{\pi.\tau_{\text{r}}.R_{\text{r}}^2} \tag{9}$$

I_{min} is calculated by the condition

$$\rho_z \equiv \rho_{z,\text{opt}} = \sqrt{\frac{2.\tau_{\text{t}}.\tau_{\text{a}}.\Phi_{\text{L}}}{\pi.\text{e}.I_{\min}}}, \quad \text{e} = 2{,}7183 \tag{10}$$

Having the expressions (2), (8) and (9), and knowing, that, $\theta_t = \rho_z/z$, we can easily derive the expression for optimal beam divergence angle, $\theta_{t,\,opt}$.

3. Methodology for Calculating the Outage Probability of FSO in the Presence of Atmospheric Turbulence

3.1. Channel Model

It is well known that under turbulent conditions the optical intensity is redistributed. The recent studies in this field widely accept that the optical intensity is modelled with lognormal distribution in the case of weak turbulence [6, 12] and with gamma-gamma distribution, when we want to simulate moderate to strong turbulence in the atmospheric channel.

Having this in mind the intensity distribution under weak turbulence is modelled with:

$$f(I) = \frac{1}{I\sqrt{2\pi\sigma_I}} \exp\left\{-\frac{\left[\ln\left(I/\langle I\rangle\right) + \sigma_I^2/2\right]^2}{2\sigma_I^2}\right\} \tag{11}$$

Where σ_I is the scintillation index described in the Raytov theory [12] eqn (12) through eqn (18).

Under moderate to strong turbulence, the optical intensity is modelled with the gamma-gamma distribution:

$$f(I) = \frac{2(\alpha\beta)^{\alpha+\beta/2}}{\Gamma(\alpha)\Gamma(\beta)\langle I\rangle}\left(\frac{I}{\langle I\rangle}\right)^{\alpha+\beta/2-1} \times \mathrm{K}_{\alpha-\beta}\left(2\sqrt{\frac{\alpha\beta I}{\langle I\rangle}}\right) \tag{12}$$

Where K is the Bessel function of the second kind and n^{th} order. α and β are positive parameters

$$\begin{aligned}
\alpha &= \left[\exp\left(\frac{0.49\chi^2}{\left(1 + 0.18d^2 + 0.56\chi^{\frac{12}{5}}\right)^{\frac{7}{6}}}\right) - 1\right]^{-1}, \\
\beta &= \left[\exp\left(\frac{0.51\chi^2\left(1 + 0.69\chi^2\right)^{-\frac{5}{6}}}{\left(1 + 0.9d^2 + 0.62d^2\chi^{\frac{12}{5}}\right)^{\frac{7}{6}}}\right) - 1\right]^{-1},
\end{aligned} \tag{13}$$

Where $\chi^2 = 0.56C_n^2k^{7/6}Z^{11/6}$ and $d = \sqrt{\left(kR^2/4Z\right)}$. Here, k is the optical wave number, λ is the wavelength and C^2_n stands for the altitude-dependent index of the refractive structure parameter and varies from 10^{-13} $m^{-2/3}$ for strong turbulence to 10^{-17} $m^{-2/3}$ for weak turbulence.

3.2. Outage Probability

Having the laser beam intensity distribution, we can define the outage probability as:

$$P_{\text{out}} = P(I \le I_{th}). \tag{14}$$

In other words, this is the probability that the instantaneous optical intensity falls below a specified threshold. Having in mind that the FSO system is set to work with optimal beam divergence angle, this means that the intensity should not fall below $I_{\min}$.. The outage probability can be calculated from:

$$P_{out} = \int_0^{I_{\min}} \frac{2(\alpha\beta)^{\alpha+\beta/2}}{\Gamma(\alpha)\Gamma(\beta)\langle I\rangle}\left(\frac{I}{\langle I\rangle}\right)^{\alpha+\beta/2-1} \times \mathrm{K}_{\alpha-\beta}\left(2\sqrt{\frac{\alpha\beta I}{\langle I\rangle}}\right)dI \tag{15}$$

By writing the modified Bessel function in terms of the Meijer's G-function [13:15], we get

$$\begin{aligned} P_{out} &= \frac{2(\alpha\beta)^{\alpha+\beta/2}}{\Gamma(\alpha)\Gamma(\beta)\langle I\rangle^{\alpha+\beta/2}} \times \\ &\times \int_0^{I_{\min}} \left(\frac{I}{\langle I\rangle}\right)^{\alpha+\beta/2-1} \times G_{0,2}^{2,0}\left[\frac{\alpha\beta}{\langle \mathrm{I}\rangle}\left(\frac{I}{\langle I\rangle}\right)\Bigg|_{\frac{\alpha-\beta}{2},\frac{\beta-\alpha}{2}}\right]dI \end{aligned} \tag{16}$$

We derive the closed form expression for (16) using [14] eqn (26):

$$\begin{aligned} P_{out} &= \frac{2(\alpha\beta)^{\alpha+\beta/2}}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{I}{\langle I\rangle}\right)^{\alpha+\beta/2} \times \\ &\times G_{0,2}^{2,0}\left[\alpha\beta\left(\frac{I}{\langle I\rangle}\right)\Bigg|_{\frac{\alpha-\beta}{2},\frac{\beta-\alpha}{2},-\frac{\alpha+\beta}{2}}^{1-\frac{\alpha+\beta}{2}}\right] \end{aligned} \tag{17}$$

Equation (17) is the final solution to (15) and can be used to calculate the outage probability of a free-space optical communication system.

4. Conclusion

In this paper we presented a methodology for calculating the outage probability of FSO. The system is using optimal beam divergence angle, which compensates any random jitter presented in the initial laser beam direction. We've considered the case where the wireless optical system is working in the presence of moderate to strong atmospheric turbulence. The atmospheric channel was modelled with gamma-gamma distribution. The result of this paper is a closed form expression for calculating the outage probability of free-space optical communication system in the presence of strong turbulence.

As a future work we consider performing numerical simulations of the outage probability depending on various parameters like: optical power of the transmitter, distance between transmitter and receiver, channel capacity (CI) and different radius of the receiving aperture. Such study would benefit the evaluation and design of reliable free space optical systems.

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