

# Algorithms for the Spectrum Analysis of Digital Signals

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**ABSTRACT:** *In this current research we have introduced and discussed a method for analyzing the spectrum analysis of discrete signals with different shape forms. To do so we have used Discrete Fourier transformation and Fast Fourier transformation. We have also presented an algorithm and its implementation in MATLAB environment. We then have analysed the program code that influence the signals parameters on their spectrum. We do hope that this research results can serve as a study of the education activity.*

**Keywords:** Digital Signal Processing, Spectrum Analyses, Computer Simulation, MATLAB

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## 1. Introduction

The spectrum analysis is widely used in filtration, recognition of signals and noise, restrictions of bandwidth, etc. The development of digital electronics leads to the necessity of creating the equivalent of classical decomposition in Fourier series. In its simple form the Discrete Fourier Transform (DFT) is rarely used because of its computational complexity. A variety of methods to accelerate the computing process of DFT have been proposed, which generalized name is Fast Fourier Transform (FFT) [1].

Some of the commonly used methods for FFT are: time splitting, frequency splitting, Goertzel's algorithm (by linear filtration), Chirp-z transformation (by using cross correlation), real fast conversion etc. [1].

In the time splitting algorithm the input sequence is divided into one (or two) odd sequences and one (or two) even sequences. This process continues recursively until a plurality of sequences of 2 (or 4) samples each is achieved. Subsequently a 2-point (or a 4-point) operation butterfly have been repeatedly applied. After completing of the calculations at the output the harmonics are obtained. The two-point basic operation is applied when the length of the input string is up to 512 samples. Since the input sequences is longer the performance and the efficiency decreases.

Unlike time-splitting algorithm, in which the harmonics are obtained sequentially, at the frequency-splitting algorithm, the obtained harmonics at the output are divided into a set of even and odd sequences. After completing of the calculations the

obtained sequences need to be reordered. In this algorithm the input signal is divided into 2 (or 4) consecutive parts, each of which is subjected to a basic operation of two (or four) point butterfly. The subsequent separation of the two (or four) sequences to another two (or four) sequences and applying the basic operations continues to be applied recursively, until reaching a sequences of 2 (or 4) samples each. A base operation is applied on these sequences, after which they are fed to the exit as reordered results. The selection of two-point or four-point base operation is analogous as in the case with the time-division algorithm.

The Goertzel's algorithm is used when only parts of the signal's harmonics will be analyzed. The principle of operation of this algorithm is the separation of the individual harmonics by linear filtering of the input signal. The main advantage of the algorithm is that the number of input samples does not affect the computational complexity [2].

By Chirp-z transformation method an existing methods for fast calculation of cross correlation functions are used. For this purpose, the input sequence will be preliminary modified, and then by fast cross-correlation algorithm will be processed. The result is subjected to further processing in order to obtain a DFT [3].

In cases when the input signals are real, it is possible to obtain a further acceleration of the calculation in the time splitting and frequency-splitting algorithms. This is done by splitting the actual signal on two signals containing odd and even samples of the original signal. The obtained sub-sequences are presented as a real and imaginary part of a new signal, which is twice shorter than the original.

The DFT and FFT are studied in the course "Signals and Systems" at the undergraduate course in specialties "Telecommunications", "Computer and Software Engineering" and "Electronics" in Technical University of Sofia. In the labs the students will have the possibility to analyze the magnitude and phase spectrum of discrete signals. They will be able to change the signal parameters and analyze their influence on the signals in time and frequency domain. The students will be able to create the source code, which can generate signals with different form and different parameters, based on their mathematical models. The MATLAB environment has a definite advantage over other software products and other programming languages, because of builtin functions [4], [5]. This would considerably facilitate students in their work.

## 2. Problem Formulation

The main tasks to be solved by spectrum analysis of the signals are the variety of signals as types (periodic and aperiodic) and shapes (sinusoidal, rectangular, triangular, sawtooth, Gaussian, etc.). This paper presents an approach for studying the spectrum analysis of discrete signals with different shape forms, by using DFT and FFT. The DFT is defined by Eq.(1) [6], [7]:

$$S(k\Omega) = \frac{1}{N} \sum_{n=0}^{N-1} S(nT) e^{-j \frac{2\pi kn}{N}} \quad (1)$$

The inverse DFT is given by Eq. (2):

$$S(nT) = \sum_{k=0}^{N-1} S(k\Omega) e^{j \frac{2\pi kn}{N}} \quad (2)$$

The main tasks, which can be solved by computer simulations in MATLAB environment, are following:

1. To create an appropriate model for a presentation of the signals.
2. To create an algorithm for calculation of the DFT and FFT.
3. To investigate the influence of the waveforms and signal parameters on the signal spectrum.
4. To represent graphically the results of the investigations.

### 3. Basic Algorithm for Spectrum Analysis

The block diagram of the basic algorithm for spectrum analysis of discrete signals is shown in Figure 1.

The algorithm begins with setting up of global parameters and variables (number of periods for visualization, number of samples per period, number of coefficients in Fourier transformation, etc.). Then it is necessary to select signal shape and set parameters of the signal (basic parameters such as period, amplitude, etc. as well as specific parameters for the particular signal). Taking into account the signal shape and its parameters students must create mathematical description of the signal. Mathematical description of the following discrete signals will be created: single delta pulse, sinusoidal, rectangular, triangular, sawtooth, Gaussian.

It's envisaged that the students will create a source code that implements discrete Fourier transformation (in both directions), based on its mathematical model. They will also use the MATLAB built-in fast Fourier transformation. In order to see the

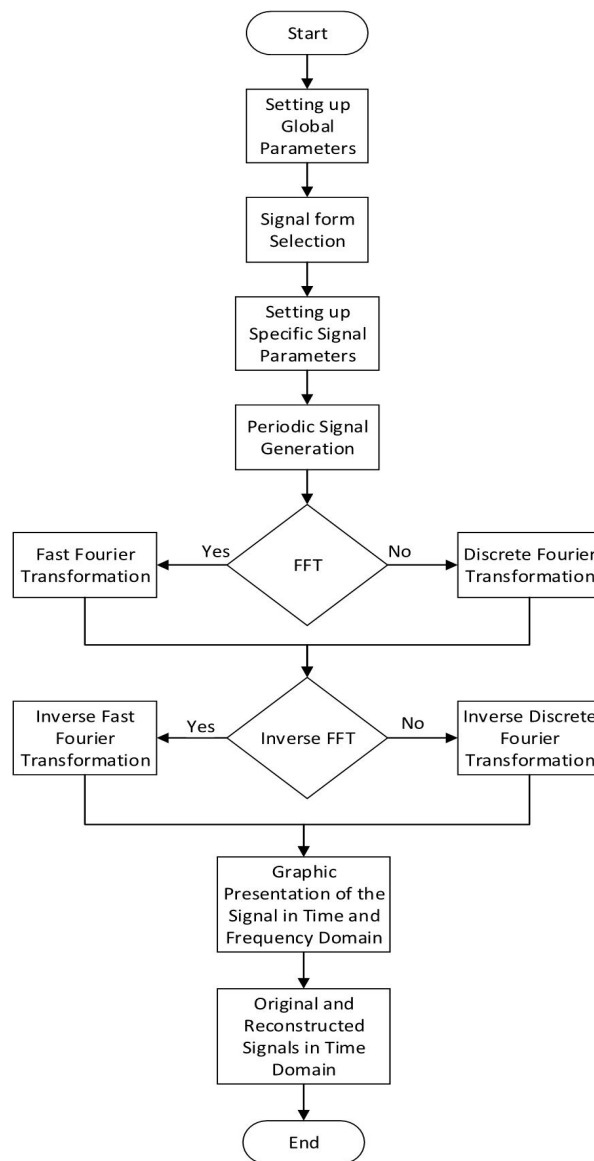


Figure 1. Block diagram of basic algorithm for spectrum analysis of digital signals

compatibility of different algorithms (discrete and fast Fourier transformations) the possibility to apply the conversion of one type and inverse conversion of the other type is provided. Created software modules for determining the spectrum of discrete signals allows creating a graphical representation of the signal, its magnitude and phase spectrum. This gives possibilities for investigation and visualization of the impact of certain signal parameters of the signal in time and frequency domain.

In order to observe how the reconstructed signal becomes closer to the original with the increase of the number of harmonics in the Fourier transform, it is provided an opportunity to plot/draw the original signal and the obtained reconstructed signal on the same graph. The graphic presentation is updated every 0.1 seconds. The obtained graphic presentation shows how really work the Fourier transform and can give deep knowledge of the process of spectrum analysis.

#### 4. Experimental Part

The formulated problems are solved by computer simulation in MATLAB environment by implementation of the proposed algorithm. Some results, which are obtained by computer simulations of discrete signals such as: rectangular pulse train, triangular pulse train, sawtooth pulse train and Gaussian pulse train, are shown in Figure 2, Figure 3, Figure 4 and Figure 5 respectively. Each figure shows a graphic presentation of the signal in the time domain (original and the reconstructed after inverse Fourier transform), magnitude and phase spectrum.

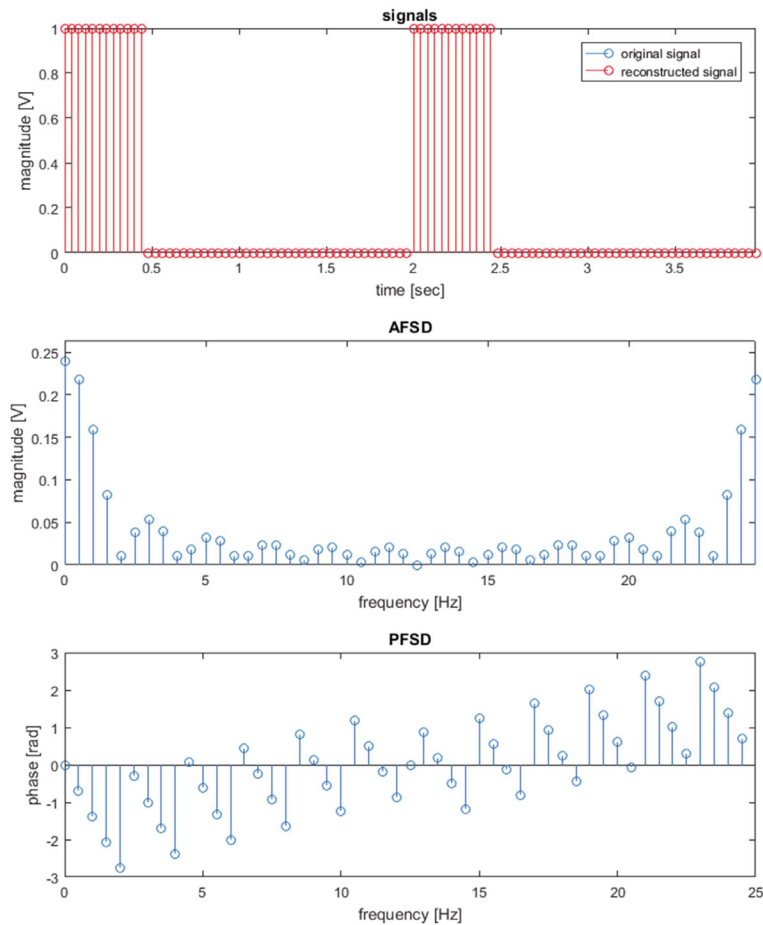


Figure 2. Rectangular pulse train for two periods and its spectrum

In Figure 6-8 are shown the original and the reconstructed signal in three different stages of reconstruction (represented as a sum of sinusoids with the appropriate magnitude, frequency and initial phase offset). On each subsequent step, the next harmonic is added to the current state of the reconstructed signal. This continually changing graphic presentation shows how with the increase of the number of harmonics that are summed, the reconstructed signal becomes closer to the original.

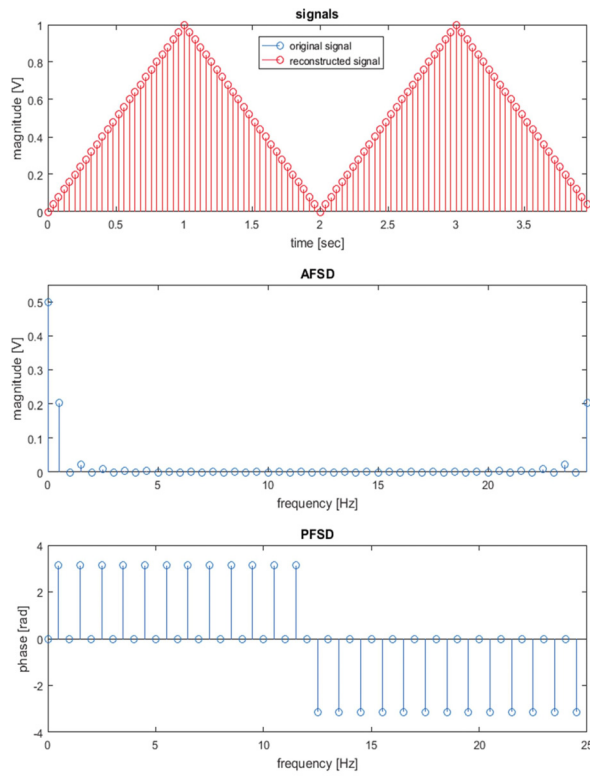


Figure 3. Triangle pulse train for two periods and its spectrum

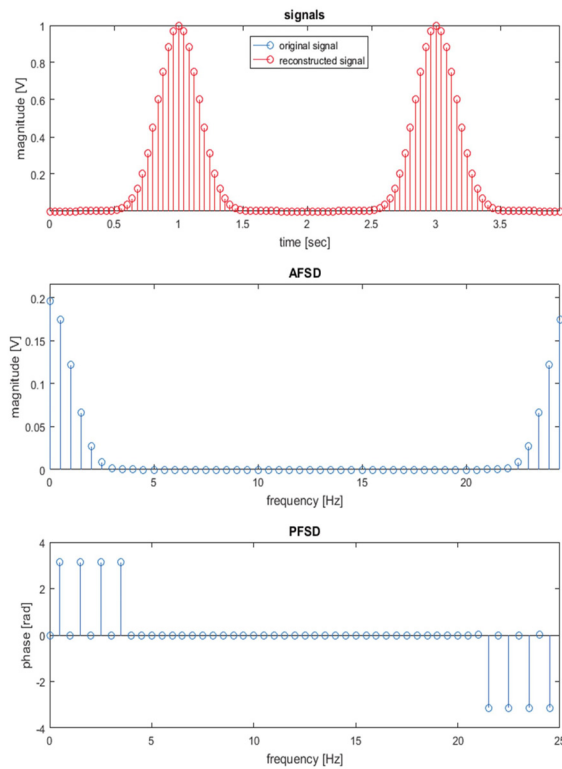


Figure 4. Gaussian pulse train for two periods and its spectrum

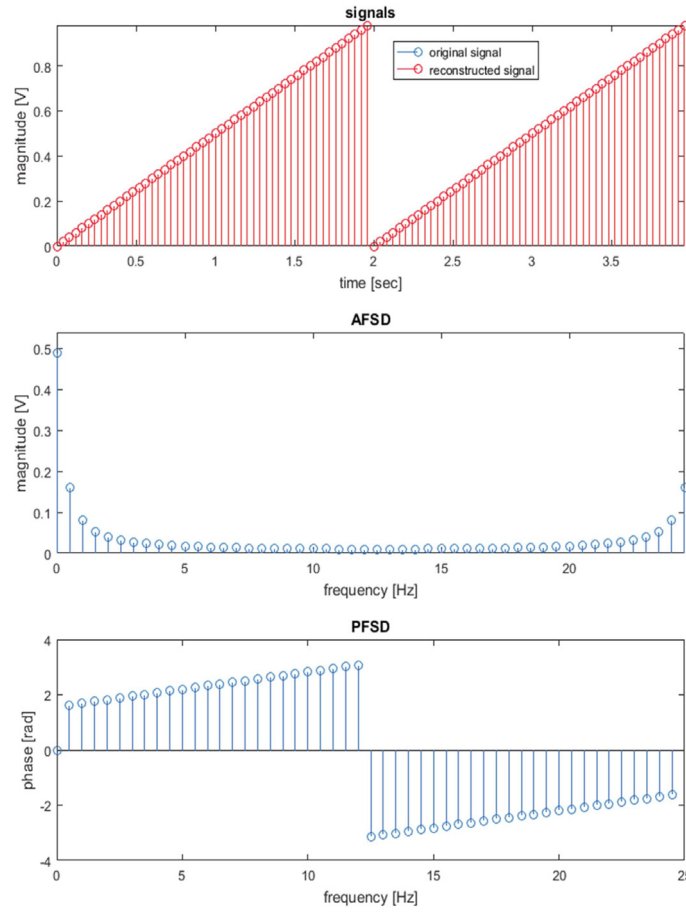


Figure 5. Sawtooth pulse train for two periods and its spectrum

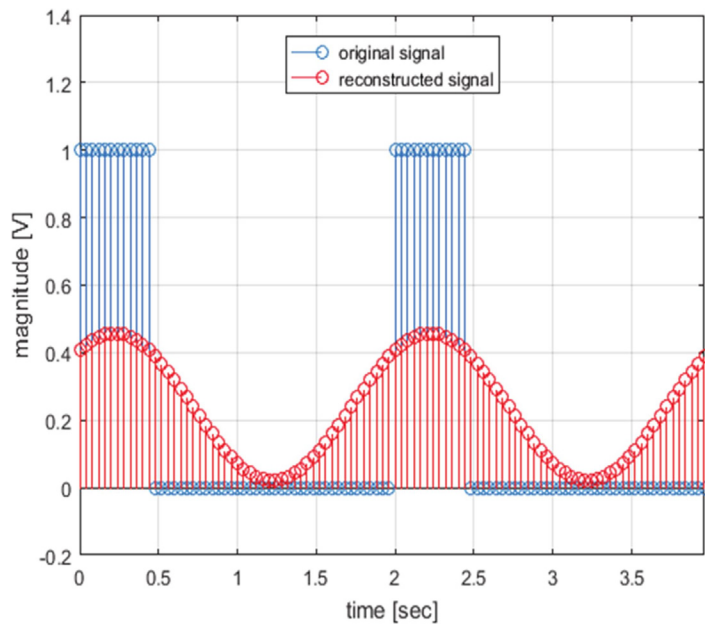


Figure 6. The original and reconstructed signals by 1 harmonic

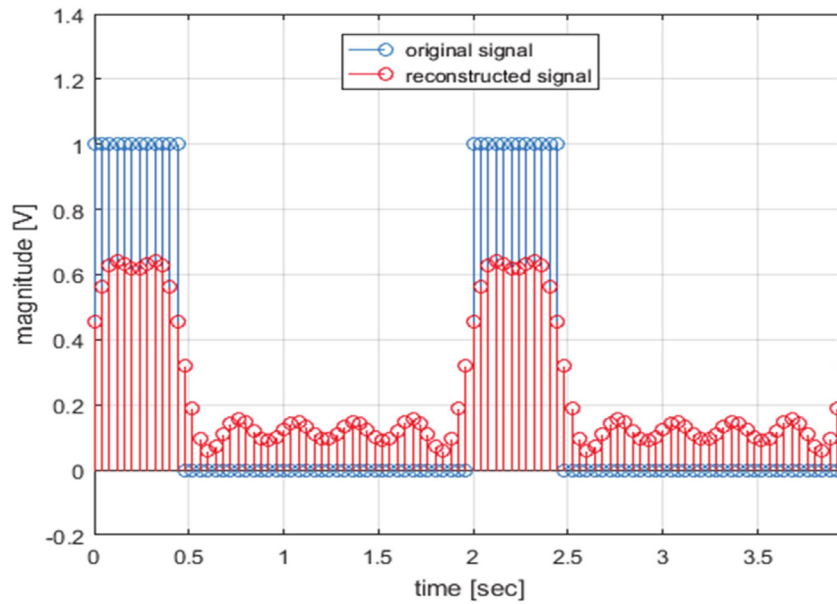


Figure 7. The original and reconstructed signals by 6 harmonics

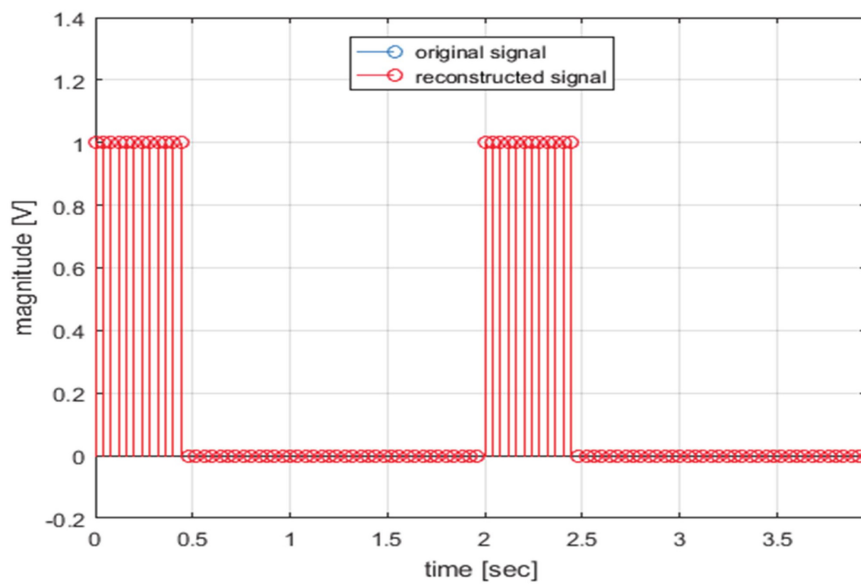


Figure 8. The original and reconstructed signals by 50 harmonics

## 5. Conclusion

In this paper an approach for in-depth studying on DFT and FFT has been proposed. An algorithm and corresponding program code in MATLAB environment have been developed. The students will have ability to create software models for discrete signals with different shape forms. They will be able to change the signal parameters and analyze their influence on the signal in time and frequency domain. The proposed algorithm provides the solution of the inverse task, associated with reconstruction of the discrete signal and its graphic presentation.

The proposed approach can be developed for making exercises, which will be used for web based distance education.

## References

- [1] Ingle, V., Proakis, J. (1997). *Digital Signal Processing Using MATLAB v4*, PWS Publishing Company, 1997.
- [ 2 ] Н. А. Галанина, Д. Д. Дмитриев, Д. И. Ахметзянов, *Алгоритм Герцеля для Спектрального Анализа Сигналов, Программные системы и вычислительные методы*, No. 4(5), 2013.
- [3] Alkishriwo, O. (2013). *The Discrete Linear Chirp Transform Algorithm and Its Applications*, Ph.D. Thesis, University of Pittsburgh, 2013.
- [4] Chaparro, L. (2011). *Signal and Systems using MATLAB*, Academic Press (Elsevier), 2011.
- [5] Karris, S. (2012). *Signal and Systems with MATLAB; Computing and Simulink; Modeling*, Fifth edition, Orchard Publications, 2012.
- [6] Donevska, S., Donevsky, B. (2014). *Advanced Engineering Mathematics*, TU-Sofia, Sofia, 2014.
- [7] Sadiku, M., Ali, W. (2016). *Signal and Systems: A Primer with MATLAB*, CRC Press, 2016.