Noise Fluctuation in Lidar Signals

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ABSTRACT: *We studied in this paper, the non-lidar data that can be used when the lidas has good response. It is a fact that the noise fluctuations happen due to the noise fluctuations of the normal range lidar signal leads to relative errors. This issue has been addressed in this work.*

Keywords: Lidar, Atmosphere, Atmospheric Aerosol, Solving Lidar Equation, Lidar Sounding of the Atmosphere

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1. Introduction

There are various methods for remote sensing of the Earth's surface and its atmosphere. An important place among them occupy laser locational methods [1]. Light detection and ranging (lidar) systems are being successfully applied for the analysis of parameters of the atmosphere. Often lidar is used in combination with other methods to investigate atmospheric aerosol properties [2, 3]. This is because atmospheric aerosol seriously affects Earth's climate. The basis of the use of lidar is the processing of lidar data. Improving old methods for processing lidar data and development of new methods is an ongoing process.

In [4] the main methods for solving single-scattering elastic lidar equation are presented. Algorithms for retrieving atmospheric parameters and constituents from elastic lidar signals are shown in [5, 6]. They allow direct retrieval of the extinction coefficient profile from the lidar signals. In [7] Kovalev presents algorithms for extraction of the extinctioncoefficient profile from the elastic-lidar signal. The author discusses specific scenarios for profiling vertical aerosol loading.

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This work is a continuation of our work [8]. We propose a new approach to determining $\alpha(z_m)$ (the point at which we preknown the extinction coefficient value and from which we start solving the lidar equation) with the aid of lidar data. The basic idea consists in using the entire convex portion of the *S*-function around one of its maximum (or, alternatively, its concave portion around a minimum). This ensures a sufficient stability of the solution for $\alpha(z_m)$ with respect to $\Delta S(z_i)$, i.e. the S-function of raw lidar data. Once $\alpha(z_m)$ is found, we employ the Klett's algorithm to recover the entire $\alpha(z_i)$ profile without imposing any constraints on its shape.

2. Mathematical Modeling and Solution of the Problem

The study of the atmosphere by lidar sounding requires knowledge of the physics of the interaction of the optical radiation with the material content of the atmosphere. Depending on the size of the particles basically two types of scattering are defined. Both of them are known as elastic interaction of light with scattering particles. When the particles are very small compared to the wavelength of the light we have Rayleigh scattering. To describe interaction of light with particles whose sizes are similar to or larger than the wavelength of the light Mie scattering is used.

Elastic scattering is the most common interaction used in lidar systems. For most wavelengths of the laser radiation used in lidars, molecular scattering is negligible in comparison to the aerosol. In these cases, the range-dependent backscatter and extinction coefficients can be considered as functions only of the aerosols. In this case, in the absence of multiple scattering and monochromatic (laser) radiation, output single scattering elastic lidar equation is brought out. It plays a major role in the study of atmospheric aerosol content (natural and industrial).

A number of methods for the inversion of single scattering elastic lidar equation have been developed and improved. Each one of these approaches require the use of prior information or adoption of physically justified assumptions. Thus reaching the lidar inversion.

One of the first proposed methods for solving lidar equation, used today, is slope method. For its use it is assumed homogeneous atmosphere, i.e. volume extinction and backscatter coefficients are accepted constant along the entire sounding path. This method is very convenient to calculate the average value of the extinction coefficient.

In relatively clean atmosphere can be applied close boundary solution. This method works in the forward direction, suggesting independent measurement or prior knowledge of the extinction coefficient at the start of the measurement range.

Another approach to inversion is the optical depth solution. In this method, it is necessary independent non-lidar measurement of total optical depth of lidar measurement range. By determination of the transmission term in the lidar equation it is possible calibration of the lidar system.

The most widely used method for inverting elastic lidar returns today is the backward inversion method. In this method the extinction coefficient at the far boundary is assumed to be known. The signal is inverted backward, toward the instrument. This method is part of our overall recovery algorithm of the extinction coefficient profile. Below will present its improvement.

The basic form of the single scattering elastic lidar equation, describing a monostatic monochromatic lidar is [4], [9]:

$$S(z) = A\beta(z)\exp\left[-2\int_0^z \alpha(z')dz'\right] \qquad (1)$$

where $S(z)$ is the range-normalized signal, A is the instrumentation constant, α and β are the volume extinction and backscatter coefficients.

Finding the solution of the lidar inverse problem is an inherently incorrect problem in the sense that the solution is not unique and is unstable. It is a common practice to assume a power law relationship between β and $\alpha^{(\beta = C\alpha^K)}$. Then the solution of (1) with respect to α ($K = 1$) can be written in the form [8, 9]:

Where z_m is a specific distance at the far end of the sounding trace and δz is the data sampling interval. The lidar inversion solution is stabilized if one chooses the backward procedure of Klett [9, 10]. This requires the determination of the boundary

$$\alpha(z_i) = \frac{S(z_i)}{\frac{S(z_m)}{\alpha(z_m)} \pm 2\sum_{j=m}^{i} S(z_j)\delta z}, \qquad i = 0, 1, 2, ..., n. \tag{2}$$

value of $\alpha(z_m)$ which cannot be obtained directly from the experimental data. Knowing the correct $\alpha(z_m)$ is important since in practice it is the most significant source of errors. Ferguson, further, employed an iteration scheme to determine $\alpha(z_m)$ whose initial value is chosen from visibility data and Mulders showed a procedure requiring much less computing time. Evans utilized an appropriate calibration of the lidar system in conjunction with a simple modification of Klett's method. Yee developed a technique for inversion of the lidar equation that permits objective incorporation of prior information (made available by an alternative means) for the extinction function and of additional information encoded in the lidar data.

We will now derive the formulas for the point z_m where the S-function has a minimum (Figure 1a).

First, we approximate $\alpha(z)$ (Figure 1b) in the vicinity of point z_m by a second order polynomial:

$$\alpha(\xi) = a_0 + a_1\xi + a_2\xi^2, \quad \xi = z - z_m, \quad \xi \in [-\Delta z_1, \Delta z_2] \tag{3}$$

For the coefficients a_0 and a_1 we can write

$$a_0 = \alpha(0) = \alpha(z_m), \quad a_1 = \alpha'(0) \tag{4}$$

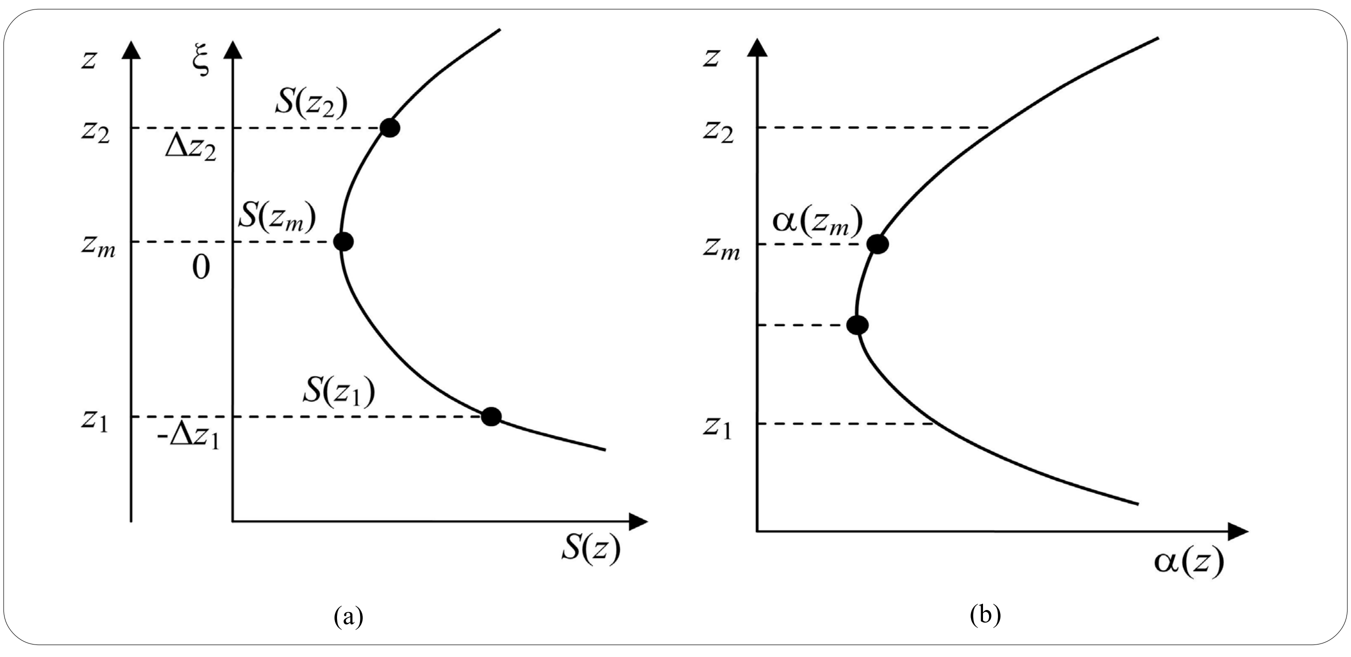

Figure 1. General view of the portion of the S-function used (a) and the corresponding α(z) portion (b)

Using the above assumptions and substitutions, we can reduce (1) to:

Differentiating the below expression, and taking into account that $S'(0) = S'(z_m) = 0$ and $\alpha'(0) = 2a^2(0)$ we arrive at:

We then substitute (3) in (5) and make use of (4) and (6) to obtain an analytical approximation of the S-function in the interval $\xi \in [-\Delta z_1, \Delta z_2]$ ($\Delta z_1 = z_m - z_1$ and $\Delta z_2 = z_2 - z_m$):

$$S(\xi) = A_0 \alpha(\xi) \exp\left[-2 \int_0^{\xi} \alpha(\xi') d\xi' \right],$$
$$A_0 = AC \exp\left[-2 \int_{-z_m}^{0} \alpha(\xi') d\xi' \right]. \tag{5}$$

$$a_1 = 2\alpha^2(z_m) \tag{6}$$

$$S(\xi) = A_0 \left[\alpha(z_m) + 2\alpha^2(z_m)\xi + a_2 \xi^2 \right] \times$$
$$\times \exp\left\{ -2 \left[\alpha(z_m)\xi + \alpha^2(z_m)\xi^2 + \frac{1}{3} a_2 \xi^3 \right] \right\}. \tag{7}$$

In order to determine $\alpha(z_m)$ based on lidar data for $S(z)$, we substitute ξ in (7) by 0, $-\Delta z_1$ and Δz_2, successively, and form the ratios:

$$\sigma_1 = \frac{S(-\Delta z_1)}{S(0)} = \left(1 - 2x + \frac{y}{x} \right) x$$
$$x \exp\left(2x - 2x^2 + \frac{2}{3} y \right) \tag{8}$$

$$\sigma_2 = \frac{S(\Delta z_2)}{S(0)} = \left(1 + 2kx + k^2 \frac{y}{x} \right) \times$$
$$\times \exp\left(-2kx - 2k^2 x^2 - \frac{2}{3} k^3 y \right) \tag{9}$$

where $k = \Delta z_2 / \Delta z_1$, $x = \alpha(z_m)\Delta z_1$, $y = a_2 \Delta z_1^3$.

Since z_m, z_1, z_2, $S(z_1)$, $S(z_2)$ and therefore, σ_1 and σ_2, can be determined using the lidar data $S(z_i)$ (after an appropriate smoothing of the latter), the set of equations (8) and (9) gives us the possibility to calculate $\alpha(z_m)$ and a_2. The way of defining σ_1 and σ_2, makes it obvious that is only sufficient to have the values of the S-function in relative units, and that the value of A_0 (respectively C) is of no significance.

The choice $\Delta z_1 = \Delta z_2 = \Delta z$ allows us to reduce set (8), (9) to a single transcendental equation. Based on the lidar-registered S-function, it is also possible to determine $\alpha(z_m)$ by choosing the points z_1 and z_2 (Figure 1) unilaterally with respect to z_m. A further possibility is to assume $\sigma_1 = \sigma_2 = \sigma$, calculate $k = \Delta z_2 / \Delta z_1$, and then use (8) and (9) to find x, respectively $\alpha(z_m)$.

3. Numerical Experiment and Results

We will now apply the technique developed to recover the profile $\alpha(z)$ in the case of a multilayer distribution of the atmospheric aerosol. We will use a model profile with optical thickness $\tau = 0{,}909$ and define the values of the profile $\alpha_{mod}(z)$ at 51 points z_i ($i = 0, 1, \ldots, 50$) with $\delta z = 0{,}01$ km. We then calculate the respective S-function in relative units:

$$S(z_i) = \alpha_{\text{mod}}(z_i)\exp\left\{-\delta z\sum_{j=1}^{i}\left[\alpha_{\text{mod}}(z_{j-1}) + \alpha_{\text{mod}}(z_j)\right]\right\},$$
$$S(z_0) = \alpha_{\text{mod}}(z_0). \quad (10)$$

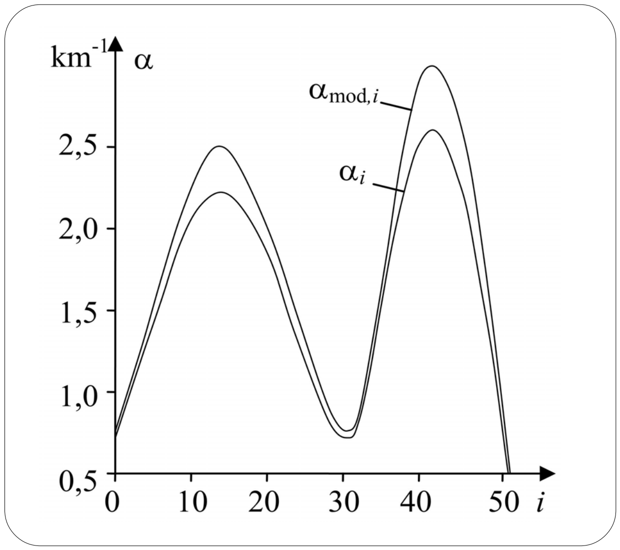

Figure 2. Comparison between the inversion profile $\alpha(z)$ and the model profile $\alpha_{mod}(z)$

We then proceed to find z_m, respectively $S(z_m)$, using the abscissa values of the $S(z_i)$ minimum. If $z_m/\delta z$ is not an integer, we discretize the S-function again, but keep the δz value such that z_m coincides with the abscissa of one of the $S(z)$ samples. Further, we determine the respective pairs $S(z_m \pm q.\delta z)$, q = 3, 4, 5,... Having calculated $\sigma_{1,q}$, $\sigma_{2,q}$, we determine x_q and, respectively, $\alpha_q(z_m) = x_q / q.\delta z$). The next step is to find the averaged with respect to q value of $\alpha(z_m)$. The latter is then substituted in algorithm (2). Finally, using the entire arrays of "lidar-registered"data $S(z_i)$, we recover the profile $\alpha(z_i)$ within the ranges $[z_0, z_n]$.

Figure 2 presents a comparison of the recovered profile $\alpha(z_i)$ with the model profile $\alpha_{mod}(z_i)$. The curves illustrate the satisfactory accuracy and stability of the overall recovery of the $\alpha(z_i)$ profile.

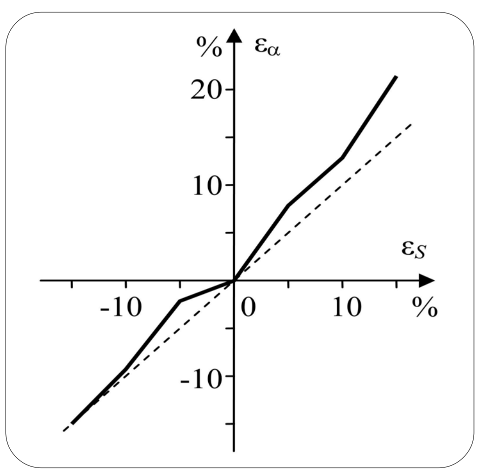

Figure 3. Sensitivity of $\alpha(z_m)$ to noise fluctuations in $S(z_m)$

We carried out a numerical experiment. It demonstrated that the method proposed does not amplify the noise variations $\varepsilon_S = \Delta S(z_m) / S(z_m)$ of the input data, i.e, $\varepsilon_\alpha = \Delta\alpha(z_m) / \alpha(z_m) \approx \varepsilon_S$ a (Figure 3).

4. Conclusion

We describe a procedure for approximate solution of the lidar equation. It is based on using lidar data to determine the extinction coefficient at a point where the *S*-function has a minimum. The solution is not constrained by the type of aerosol stratification investigated. The technique is tested by means of a numerical experiment on model profiles $\alpha_{mod}(z)$. The solutions thus obtained satisfy the requirements of atmospheric remote sensing investigations.

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