The Outage Capacity Evaluation of Wireless Communication Channel

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ABSTRACT: *We have presented the outage capacity evaluation of wireless communication channel, in the fading and shadowing phenomena. The capacity analysis over gamma shadowed Weibull fading channel is presented. In addition to ergodic capacity examination, the outage capacity appropriate as a metric of channel with slow signal intensity fluctuations is observed. The case of possible random blockage on the transmission path, taking into consideration both capacity metrics, is also analysed. We did the experimentation and presented the derived analytical expressions, which shown and influence of various performance parameters on outage capacity.*

Keywords: Ergodic Capacity, Generalized Fading Channel, Outage Capacity, Random Blockage

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1. Introduction

The properties of propagation channels are usually such that the disturbances as fading and shadowing phenomena should be taken in consideration when estimation of performance metric of wireless communication is required [1]. Very often these two nuisances affect the propagation channel simultaneously. Given this fact, there are various fading/shadowing models proposed in literature. As the most suitable for describing shadowing effects is lognormal distribution experimentally gained distribution [1], [2]. Unfortunately, it showed to be inadequate for analytical analysis, so upon short notice it had been replaced by gamma distribution. According to this, the Rayleigh-gamma [2], Nakagami-gamma [3], Weibull-gamma [4] and extended generalized K (EGK) [5] models were proposed, as general composite fading models suitable to determine fading/shadowing deleterious effects.

When the fluctuations of signal intensity, during transmission, are fast i.e. when signal intensity samples are independent at two successive symbols, the ergodic (average) channel capacity is the most appropriate as performance metric. In that case, ergodic capacity determines the maximum data transmission rate over a channel under condition that error probability is arbitrary small. The ergodic capacity of Nakagami-gamma fading channels was analysed in [3], applying different adaptation policies. Analytical derivation of bit error rate (BER) for some binary modulations, the ergodic capacity and the second order statistics analysis over EGK fading environment was presented in [5]. Authors in [6], in addition to the expression for evaluating ergodic capacity of correlated Rician fading channels influenced by additive white Gaussian noise (AWGN), have determined outage capacity. In the case of quasi-static channels (when the signal fluctuations are slow), the outage capacity is a more suitable metric. Outage capacity is a maximum data transmission rate under a specified outage probability.

An extensive analysis of Weibull-gamma radio-wave propagation was proposed in [7] with brief average capacity examination under various types of transmission policies. Closed-form expressions for the capacity of multiple-inputmultiple- output systems at low signal-to-noise ratio (SNR) regime, again over Weibull-gamma fading channels, are offered in [8].

Regarding detailed analysis of signal transmission over radio channel, it is important to observe the consequences of possible obstacles that cross the propagation path. Objects, either stationary or moving, can cause permanently/temporary communication interruption between the transmitter and receiver. The blockage caused by densely located buildings in urban areas was analysed in [9]. In [10], both the average and outage capacity when the transmission path is blocked over generalized EGK fading channels are evaluated.

In this paper, we present the mathematical formulation of ergodic and outage capacity with link blockage when multipath Weibull fading is superimposed on gamma distributed shadowing. According to proposed analysis with linearly described link blockage, numerical results are given. On the basis of analytical as well as numerical results, it is possible to estimate the maximum data error free transmission rate for fixed value of outage probability in the case of wide range of multipath fading and shadowing parameters.

2. Analysis Background

We assume that signal experience Weibull-gamma fading. When the signal envelope, R, is Weibull distributed, the corresponding probability density function (pdf) is given by [4]

$$p_{R/y}(R/y)=\beta\left(\frac{\Gamma(1+2/\beta)}{y}\right)^{\beta/2} R^{\beta-1}\exp\left(-\left(R^2\Gamma(1+2/\beta)/y\right)^{\beta/2}\right),\quad R\geq 0 \tag{1}$$

where β is the multipath fading severity parameter, $\Gamma(.)$ denotes the Gamma function [11, eq. (8.310[7]/1))], and $y = E(R^2)$ is the average fading power ($E(.)$ is the expectation ratio). When shadowing effects are present in the propagation channel, y is also random process. For the proposed scenario y is described by gamma pdf as $p_y(y) = y^{\alpha-1}exp(-y/\Omega)/(\Gamma(a)\Omega^{\alpha})$ with α being the shadowing severity parameter and $\Omega = E(y^2)$. In the propagation channel where fading and shadowing occur simultaneously, the average pdf of the Weibull-gamma random variable can be determined as $p_R(R)=\int_0^{\infty} p_{R/y}(R/y)\,p_y(y)\,dy$. According to this, and representing the exponential functions in terms of the Meijer's G functions [12, eqs. (8.4.3.1), (8.4.3.2)], utilizing [13, eq. (2.8.12)] we get [14, eq. (4)]

$$p_R(R)=\frac{R^{\beta-1}}{\Gamma(\alpha)\Omega^{\beta/2}}\left(\Gamma\left(1+\frac{2}{\beta}\right)\right)^{\beta/2} \times H_{0,2}^{2,0}\left(\left(\frac{R^2\Gamma(1+2/\beta)}{\Omega}\right)^{1/2}\middle|\begin{matrix}-\\(0,1/\beta),(\alpha-\beta/2,1/2)\end{matrix}\right) \tag{2}$$

Since the instantaneous SNR per bit is $\gamma=R^2E_b/N_0$, with E_b/N_0 being the energy-per-bit to noise spectral density ratio, the corresponding average SNR will be $\overline{\gamma}=\Omega E_b/\alpha N_0$. So, after some simple algebra, the pdf of the instantaneous SNR becomes

$$p_\gamma(\gamma)=\frac{\gamma^{\beta/2-1}\Xi^{\beta/2}}{2\Gamma(\alpha)}H_{0,2}^{2,0}\left((\Xi\gamma)^{1/2}\middle|\begin{matrix}-\\(0,1/\beta),(\overline{\alpha}-\beta/2,1/2)\end{matrix}\right), \tag{3}$$

with $\Xi=\alpha\Gamma(1+2/\beta)/\overline{\gamma}$ and $H_{p,q}^{m,n}\left(z\middle|\begin{matrix}(a_i,A_i)_{1,p}\\(b_j,B_j)_{1,q}\end{matrix}\right)$ denoting the Fox's H function, where p, q, m, n are integers such that $0\leq m\leq q$,

$0 \le n \le p$; a_i, $b_j \in C$; C is the set of complex numbers, and A_i, $B_j \in R^+ = (0, \infty)$, ($i = 1, ..., p; j = 1,..., q$)[13].

3. Ergodic and Outage Capacity Evaluation in the Presence of Random Blockage

In this Section, we present analytical expressions of ergodic and outage capacity of Weibull-gamma fading channel and extend the analysis to the scenario with blocked transmission path.

On the basis of definition form of normalized ergodic capacity as $C_{erg} / B = \int_0^\infty \log_2(1+\gamma) p_\gamma(\gamma) d\gamma$, relaying on the transformation of log function [12, eq. (8.4.6.5)] into Fox's H function and applying [13, eq. (2.8.12)], after some algebra we derived expression for ergodic capacity of Weibull-gamma fading channel in the following way

$$\frac{C_{erg}}{B} = \frac{\Xi^{\beta/2}}{2\ln(2)\Gamma(\alpha)} \times H_{2,4}^{4,1}\left(\Xi^{1/2} \left| \begin{matrix} (-\beta/2,1/2),(1-\beta/2,1/2) \\ (0,1/\beta),(\alpha-\beta/2,1/2),(-\beta/2,1/2),(-\beta/2,1/2) \end{matrix} \right. \right) \quad (4)$$

with B being the transmission bandwidth. The magnitude $Cerg$ is also called average capacity based on fact that it is obtained by averaging the instantaneous capacity, $C = B \log_2 (1+\gamma)$, of an AWGN channel over the pdf of SNR (3).

In case of the propagation path is somehow blocked by obstacle, the evaluation of ergodic capacity under the same statistics assumptions can be performed following

$$\frac{C_{ergblocked}}{B} = \int_0^\infty \log_2(1+\gamma) p_E(\gamma) d\gamma, \quad (5)$$

with $p_E(\gamma)$ denoting the pdf of instantaneous SNR when the path is blocked, formulated as [10]

$$p_E(\gamma) = p_s \delta(\gamma) + (1-p_s) p_\gamma(\gamma). \quad (6)$$

Probability p_s in (6) is the probability of blockage and δ (.) is Dirac delta function [15, eq. (14.03.02.0001.01)]. Substituting (6) in (5) and involving (3), integral of the form $I = Bp_s \int_0^\infty \log_2(1+\gamma)\delta(\gamma) d\gamma$ equals zero utilizing the relation [15, eq. (14.03.21.0003.01)]. The remaining part of this integral is solved based on the same procedure as in solving (4). So, the expression for evaluating the average capacity of Weibull-gamma fading channel with possible blockage becomes

$$\frac{C_{ergblocked}}{B} = (1-p_s) \frac{C_{erg}}{B}. \quad (7)$$

It is obvious that blockade at transmission path degrades the ergodic channel capacity.

The ergodic capacity is a useful metric when duration of a symbol is much larger than the time when the channel samples are significantly correlated. In the case of quasi-static fading channel, i.e. the slow-varying fading channel, where the instantaneous SNR remains constant over duration of large number of symbols, the more useful metric is outage capacity. The outage capacity, C_{out}, is defined as the capacity guaranteed for a probability rate (1-r) where

$$r = \Pr[C < C_{out}]. \quad (8)$$

To derive the expression for outage capacity, we relay on the pdf of the intstantaneous SNR. The instantaneous SNR is modeled as a random variable, so as a consequence the instantaneous channel capacity C is a random variable as well. Thus, relaying on (3) and the transformation of variables as $p_C(C) = \left. \frac{p_\gamma(\gamma)}{\frac{dC}{d\gamma}} \right|_{\gamma = 2^{C/B} - 1}$, we get the target pdf in the following form

$$p_C(C) = \frac{\ln(2)2^{C/B-1}}{B\left(2^{C/B}-1\right)\Gamma(\alpha)}\Xi^{\beta/2}$$
$$\times H_{0,2}^{2,0}\left(\left(\Xi\left(2^{C/B}-1\right)\right)^{1/2}\middle|\begin{matrix} - \\ (0,1/\beta),(\alpha-\beta/2,1/2)\end{matrix}\right). \quad (9)$$

Furthermore, the probability of outage can be estimated as

$$r_{blocked} = \int_0^{C_{out}} p_{Cblocked}(C)\,dC\,. \quad (10)$$

Substituting (9) in (10), making a change of variables as $2^{C/B} - 1 = t$ and utilizing [13, eq. (2.8.17), (2.1.9)] we get the following closed-form

$$r = \frac{\Xi^{\beta/2}}{2\Gamma(\alpha)}$$
$$\times H_{1,3}^{2,1}\left(\left(\Xi\left(2^{C_{out}/B}-1\right)\right)^{1/2}\middle|\begin{matrix}(1-1/\beta,1/2) \\ (0,1/\beta),(\alpha-\beta/2,1/2),(-\beta/2,1/2)\end{matrix}\right). \quad (11)$$

For the case of possible blockage, the probability of outage can be evaluated as

$$r_{blocked} = \int_0^{C_{out}} p_{Cblocked}(C)\,dC\,. \quad (12)$$

To obtain $p_{Cblocked}$ (C), one should used transformation $p_{Cblocked}(C) = \left.\frac{p_E(\gamma)}{\frac{dC}{d\gamma}}\right|_{\gamma = 2^{C/B}-1}$. Considering (6) and (3), with proposed transformation, (12) becomes

$$r_{blocked} = \frac{\ln(2)}{B}\int_0^{C_{out}} 2^{C/B}\left(p_s\delta\left(2^{C/B}-1\right)+\left(1-p_s\right)p_\gamma\left(2^{C/B}-1\right)\right)dC. \quad (13)$$

Relaying on [15, eq. (14.03.21.0001.02)] and regarding primary definition of Heaviside step function [15, eq. (14.03.21.0001.02)], with appropriate change of variables, the first integral in (13) equals p_s. To solved the other part of integration in (13) we used the similar procedure as in deriving (7), so the $r_{blocked}$ can be rewritten and evaluated as

$$r_{blocked} = p_s + \left(1-p_s\right)r\,. \quad (14)$$

Evaluation of channel capacity with the help of analytical results (4), (7), (11), (14), derived in a form of Fox's *H* functions, can be performed in *Mathematica* software package utilizing program given in [10, Appendix].

4. Numerical Results

Figure 1 shows ergodic and outage capacity dependence on the average SNR for fixed value of probability of outage (r=0.01) and in environments with different fading severity parameter. This figure confirms that the ergodic capacity is the highest attainable channel capacity. In addition, results show that both, ergodic and outage capacity improve with fading parameter increasing (i.e. when better channel conditions occur). Still, the effect of fading severity parameter is more significant for outage than ergodic capacity. For instance, in order to achieve ergodic capacity of 2b/s/Hz, the penalty in average SNR of only 2.3dB can be noticed when fading severity parameter decrease from $\beta = 4.5$ to $\beta = 1.5$, while the penalty in average SNR of even 16dB should be paid to achieve the same value of outage capacity under the similar shifting of fading severity parameterβ.

The ergodic capacity of Weibull-gamma fading channel for different values of probability of blockage is presented in Figure 2. As expected, the best performance i.e. the highest value of ergodic capacity is obtained when there is no blockade between the transmitter and the receiver (p_s = 0.01 in Figure). When the probability of blockage increases, the system performance degrades. The system failure occurs in the case of complete path blockade (p_s = 0.9 in Figure). The effect of possible random blockage on capacity values is the most obvious in the range from medium to large SNR values

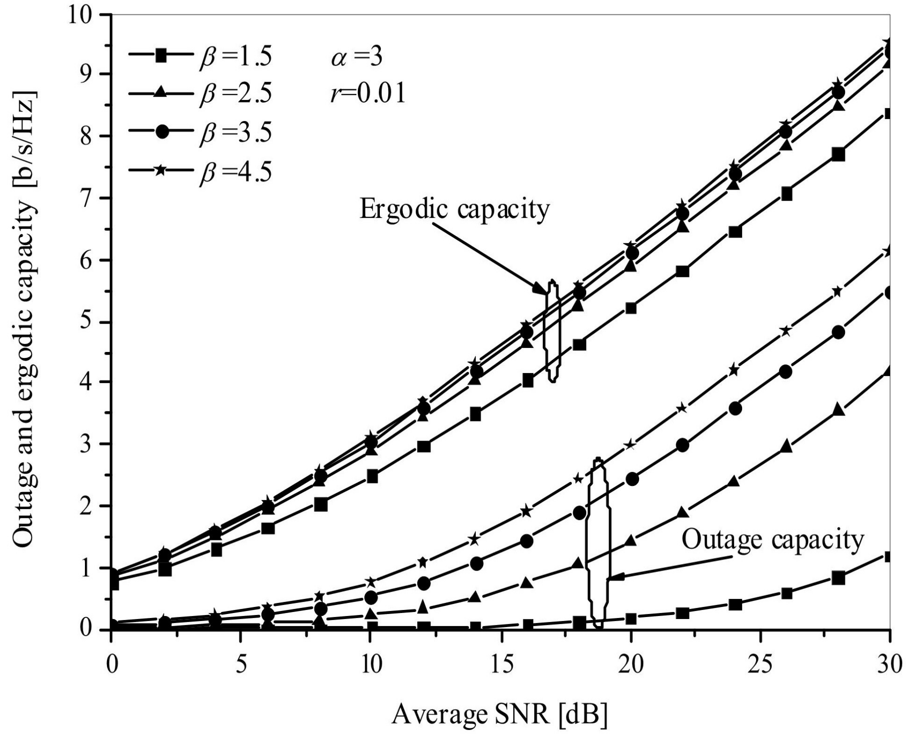

Figure 1. Ergodic and outage capacity versus the average SNR for different values of fading severity parameter

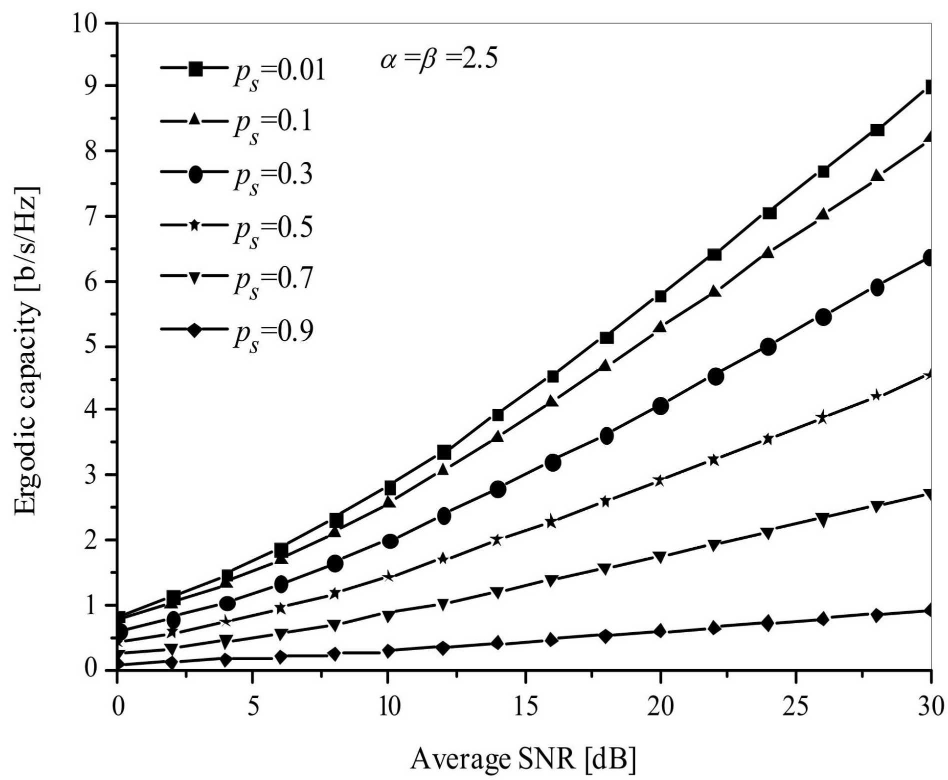

Figure 2. Ergodic capacity versus the average SNR for different values of probability of blockage

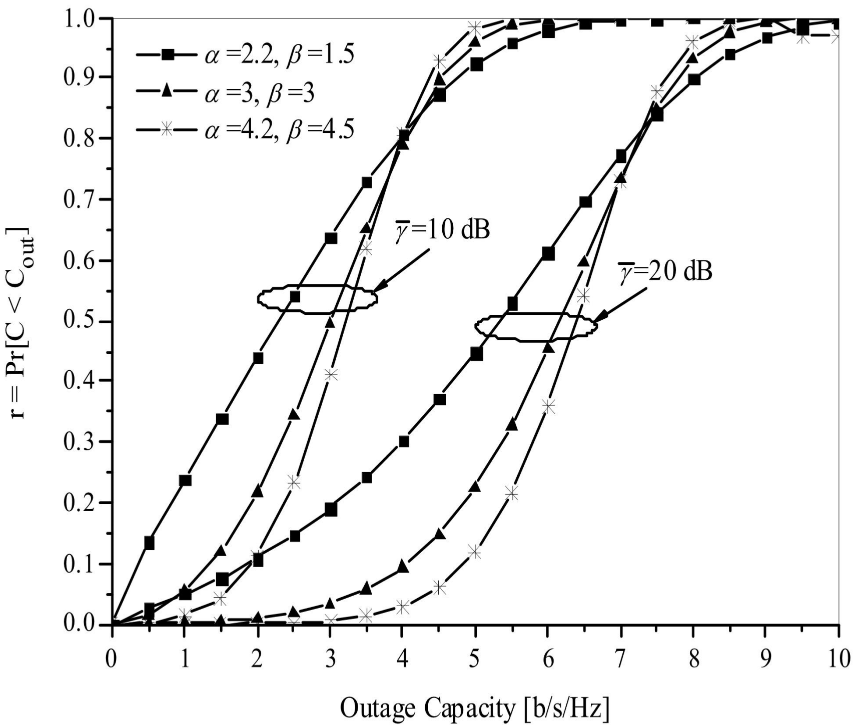

Figure 3. Probability of channel capacity being less than the outage capacity for different fading/shadowing conditions

Figure 3 presents the probability of the instantaneous channel capacity being below the outage capacity, r, for different values of fading/shadowing parameters. We can notice that the probability $r = \Pr[C < C_{out}]$ increases as the channel fading/shadowing conditions get worse (i.e. as α and β decreases), up to 0.8. Beyond this point, r grows faster for lighter fading/shadowing conditions.

5. Conclusion

In this paper an analytical framework for evaluating ergodic and outage capacity of randomly blocked Weibull-gamma fading channels have been proposed. Linking outage capacity with probability of outage a new expression has been derived for the case of possible link blockage. The obtained analytical results are applicable in analysing of variety fading/shadowing propagation conditions. The results have illustrated that the effect of random blockage on capacity is more pronounced in the range from medium to large SNR values. In addition, the effect of multipath fading severity showed to be more evident for outage than for ergodic capacity gain. At last, we confirmed that the ergodic capacity is the highest attainable channel capacity.

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