Atmospheric Pollution Measurements using the Outage Probability FSO System

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ABSTRACT: *With the help of atmospheric pressure we have measured the turbulence for which we used the outage probability of FSO system. The optimal system parameters are also adopted. We derived the outage probability in the presence of atmospheric turbulence and used many numeric parameters for identifying the turbulent channels. We are able to measure the outage probability of the FSO system with the help of optimal system parameters and we have presented the various simulations.*

Keywords: FSO, Gamma-gamma, Exponentiated Weibull, Outage Probability, Log-normal Distribution

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1. Introduction

FSO (Free Space Optics) systems are high speed wireless optical communication systems that can be used as a backup of the traditional wireless technologies or as a last-mile solution in the high-speed local area networks (LAN). Their main disadvantage is the unpredictable reliability, as laser beam propagation through the atmospheric channel is highly influenced by various random atmospheric factors [1, 2].

One of the main factors that causes degradation in FSO availability is atmospheric turbulence [4 – 12]. Large turbulent eddies cause fluctuations in the initial beam direction, and small turbulent eddies cause scintillation. In this paper we evaluate the FSO channels by investigating their outage probability caused by atmospheric turbulence. We will study mainly the influence of turbulence induced fading (caused by scintillation) on the FSO system performance.

Over the years different statistical models have been proposed to describe the optical channel characteristics with respect to the atmospheric turbulence. The most commonly used are the log-normal and gamma-gamma distributions [13]. They showed to be suitable for modelling optical channels for weak-to-moderate and moderate-to-strong atmospheric turbulence. The gamma-gamma model shows better fit when used for modelling moderate-to-strong atmospheric turbulence channels. In the last few years the exponentiated Weibull is preferred for modelling moderate-to-strong turbulence channels, when aperture averaging is used [4, 6].

In this work we derive closed form expressions for calculating the outage probability of FSO links over atmospheric turbulence-induced fading channels modelled by the aforementioned distributions.

The rest of the paper is organized as follows: in Section 2 we present a short theoretical explanation of the atmospheric turbulence, introduce the channel models and the derivation of some system parameters needed for calculating the outage probability. In Section 3 the simulation results and some analysis of the results are presented.

2. Theory

2.1. Atmospheric Turbulence

In general atmospheric turbulence is caused by inhomogeneties in both temperature and pressure in the atmosphere, causing air cells or air pockets that are differently heated. This results in changes in the index of refraction, which in turn changes the path that the optical beam takes while it propagates through the atmospheric channel. Because the air pockets are not stable in time or space, the change of the index of refraction is random. A good measure of atmospheric turbulence is the refractive index structure coefficient C_n^2.

There are three main effects that optical beams experience when propagating through turbulent atmosphere. First, the laser beam direction can deviate randomly through the changing refractive index (beam wander). Second, the phase front of the optical beam can vary, producing intensity fluctuations or in other words scintillation. Third, the optical beam can spread more than diffraction theory can predict [13 - 14].

Beam Wander

This occurs in the presence of large turbulent eddies, or cells of turbulence that can be equal to or larger than the beam diameter. The radial variance σ_r can be presented as a function of wavelength λ and distance, z, as follows:

$$\sigma_\mathrm{r} = 1.83 C_\mathrm{n}^2 \lambda^{-1/6} z^{17/6} \tag{1}$$

This relationship implies that longer wavelengths are less affected by large turbulence eddies (they will have less beam wander).

Scintillation

FSO system performance is most affected by scintillation. The random interference with the wave front can cause peaks and dips, resulting in receiver saturation or signal loss. This is presented on Figure 1:

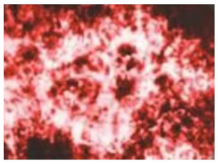

Figure 1. Scintillation effect on the optical intensity

The scintillation causes redistribution of the optical intensity in the receiver's plane making it from normally distributed to log-normal or gamma-gamma distributed. Scintillation effects are characterized by the variance σ_I as shown in Equation 2:

$$\sigma_\mathrm{I}^2 = 1.23 C_\mathrm{n}^2 k^{7/6} z^{11/6}, \tag{2}$$

where $k = 2\pi/\lambda$. Equation 2 shows that larger wavelengths would experience smaller variance, all other factor being equal.

Beam Spreading

This is characterized by the effective radius Áz, eff, the distance from the center of the beam ($z = 0$) to where the relative mean intensity has decreased by 1/e. The effective radius is given by the expression:

$$\rho_{\mathrm{z,eff}} = 2.01\left(\lambda^{-1/5} C_\mathrm{n}^{6/5} z^{8/5}\right) \tag{3}$$

2.2. System Parameters Calculation

The needed system parameters can be calculated as follows. First we need to calculate the desired bit error rate (*BER*), so that the FSO system can function properly [11]:

$$BER = \frac{1}{2} erfc\left(\frac{SNR}{2\sqrt{2}}\right) \tag{4}$$

From Equation (4) we can easily derive the value of the needed signal to noise ratio (*SNR*) required to maintain *BER*. Having the *SNR* defined, the minimal intensity needed at the plane of the receiving aperture can be calculated using the equation:

$$I_{\min} = \frac{\Phi_{\mathrm{PD}|SNR=const}}{\pi \tau_{\mathrm{r}} R_{\mathrm{r}}^2}, \tag{5}$$

where Φ_{PD} is the optical power at the photo detector's aperture, needed to keep the *BER* calculated with Eq. (4). This optical power can be derived by:

$$\Phi_{\mathrm{pd}} = \frac{1}{2}\left[\frac{SNR^2 . C_{\mathrm{I}} . e^-}{R_{\mathrm{I}}} + \left(\left(-\frac{SNR^2 . C_{\mathrm{I}} . e^-}{R_{\mathrm{I}}}\right)^2 + \frac{4SNR^2 . C_{\mathrm{I}}}{R_{\mathrm{I}}}\left(\frac{2k_{\mathrm{B}} . T . A}{R_{\mathrm{I}} . R_{\mathrm{Fb}}} + e^- . \Phi_{\mathrm{B}}\right)\right)^{\frac{1}{2}}\right] \tag{6}$$

where $R_{\mathrm{I}} = 8.06.10^5 \eta(\lambda_0)\lambda_0$ and $\eta(\lambda_0)$ are the integral sensitivity and the quantum efficiency of the photodetector, C_I is the channel capacity. A is a constant of the receiver; R_{FB} is the value of the resistor in the feedback of the preamplifier and $e^- = 1{,}602.10^{-19}C$ is the electric charge of the electron.

The background optical flux Φ_{B} is defined by the brightness of the background radiation $L_{\lambda\mathrm{B}}$, the transmission wavelength of the interference filter before the photodetector $\Delta_{\lambda\mathrm{F}}$, the losses in the transmitting antenna τ_{t}, and the parameters of the receiver: radius of the receiver's aperture R_r and its angular width θ_r:

$$\Phi_{\mathrm{B}} = \pi^2 \tau_{\mathrm{r}} L_{\lambda,\mathrm{B}} R_{\mathrm{r}}^2 \theta_{\mathrm{r}}^2 \Delta\lambda_{\mathrm{F}} \tag{7}$$

Having Eqs. (5) and (6), the optimal system parameters for maintaining I_{min} and respectively *BER* can be calculated. The optimal beam radius $\rho_{z,\,\mathrm{opt}}$ is:

$$\rho_{\mathrm{z,opt}} = \sqrt{\frac{2\tau_{\mathrm{a}}\tau_{\mathrm{t}}\Phi_{\mathrm{L}}}{\pi e I_{\min}}}, e = 2.7182 \tag{8}$$

and the optimal beam divergence angle $\theta_{t,\,opt}$ is

$$\theta_{\mathrm{t,opt}} = \frac{1}{z}\sqrt{\frac{2\tau_{\mathrm{a}}\tau_{\mathrm{t}}\tau_{\mathrm{r}} R_{\mathrm{r}}^2 \Phi_{\mathrm{L}}}{e\Phi_{\mathrm{PD}|SNR=const}}} \tag{9}$$

where τ_a is the atmospheric transparency and τ_r denotes the losses in the receiver's antenna. Φ_L is the transmitted optical power.

These optimal system parameters (Eqs. (8) and (9)) allow for the compensation of the errors in the FSO system caused by random angular vibrations in the transmitting antenna. These angular vibrations correspond to linear misalignments between the optical beam axis and the center of the receiving antenna. The maximal linear misalignments that can be compensated using optimal beam radius and divergence angle are:

$$\rho_{\max} = \frac{1}{\sqrt{2}}\rho_{z,\text{opt}}\sqrt{\frac{2\tau_a\tau_t\Phi_L}{\pi\rho_{z,\text{opt}}^2(1-e^{-2})I_{\min}}} \tag{10}$$

2.3. Channel Models

The FSO channel with atmospheric turbulence is well described by log-normal and gamma-gamma distributions. The optical intensity distribution at the plane of the receiver is presented with the following expression in the case of lognormal distribution [3, 10, 13]:

$$f(I) = \frac{1}{\sqrt{2\pi}I\sigma_I}exp\left[-\frac{(\ln(I)-\sigma_I^2/2)^2}{2\sigma_I^2}\right], \tag{11}$$

where σ_I is the standard deviation of the log-normal distribution and depends on the channel characteristics.

$$\begin{aligned}\sigma_I^2 = exp\Bigg[&\frac{0.49\delta^2}{(1+0.18d^2+0.56\delta^{12/5})^{7/6}} \\ &+ \frac{0.51\delta^2}{(1+0.9d^2+0.62d^2\delta^{12/5})^{5/6}}\Bigg] \\ &-1\end{aligned} \tag{12}$$

where

$$d = \sqrt{\frac{kR_r^2}{4z}}, k = \frac{2\pi}{\lambda}, \delta^2 = 1.23C_n^2k^{7/6}z^{11/6} \tag{13}$$

In the case of gamma-gamma distribution:

$$f(I) = \frac{2(\alpha\beta)^{\alpha+\beta/2}}{\Gamma(\alpha)\Gamma(\beta)\langle I\rangle}\left(\frac{I}{\langle I\rangle}\right)^{\alpha+\beta/2-1} \mathrm{K}_{\alpha-\beta}\left(2\sqrt{\frac{\alpha\beta I}{\langle I\rangle}}\right), \tag{14}$$

where

$$\begin{aligned}\alpha &= \left[exp\left(\frac{0.49\delta^2}{(1+0.18d^2+0.56\delta^{12/5})^{7/6}}\right)-1\right]^{-1} \\ \beta &= \left[exp\left(\frac{0.51\delta^2}{(1+0.9d^2+0.62d^2\delta^{12/5})^{5/6}}\right)-1\right]^{-1}\end{aligned} \tag{15}$$

In the presence of moderate-to-strong atmospheric turbulence and when aperture averaging is used exponentiated Weibull distribution gives the best fit for the optical intensity distribution [4-6]:

$$f(I,\alpha,\beta,\eta) = \frac{\alpha\beta}{\eta}\left(\frac{I}{\eta}\right)^{\beta-1} exp\left[-\left(\frac{I}{\eta}\right)^{\beta}\right]\left\{1-exp\left[-\left(\frac{I}{\eta}\right)^{\beta}\right]\right\}^{\alpha-1} \tag{16}$$

where α and β are the second scale parameter and the scale parameter respectively and η is the shape parameter of the Weibull distribution

$$\beta = 1.012(\alpha\sigma_I^2)^{-13/25} + 0.142$$
$$\alpha = 3.93\left(\frac{R_r}{\delta}\right)^{-0.519}$$

$$\delta = (1.46C_n^2k^2z)^{-3/5}$$
$$\eta = \frac{1}{\alpha\Gamma(1+1/\beta)g_1(\alpha,\beta)} \tag{17}$$

2.4. Outage Probability Calculation

In this work, by outage probability we mean that the system will work with bit error rate greater than the one calculated with Eq. (4). In other words, if we calculate *SNR* and *I*min to maintain $BER = 10^{-8}$, then by system outage we will consider any time the bit error rate gets greater than 10-8 due to atmospheric turbulence.

Having the optical intensity distribution defined in Eq. (11) through Eq. (17), the outage probability P_{out} will be equal to $P(I < I_{min})$ or, in other words, the cumulative density function CDF or $F(I)$ of the corresponding pdf. This corresponds to:

$$P_{\mathrm{out}} = F(I_{\mathrm{min}}) = \frac{1}{2}erfc\left(-\frac{\ln(I)+0.5\sigma_I^2}{\sqrt{2}\sigma_I}\right) \tag{18}$$

When gamma-gamma distribution is used, the outage probability is [15, 16]:

$$P_{out} = \frac{(\alpha\beta)^{\alpha+\beta/2}}{\Gamma(\alpha)\Gamma(\beta)}(I)^{\alpha+\beta/2} \times G_{1,3}^{2,1}\left[\alpha\beta(I)\middle|\begin{matrix}1-\frac{\alpha+\beta}{2}\\ \frac{\alpha-\beta}{2},\frac{\beta-\alpha}{2},-\frac{\alpha+\beta}{2}\end{matrix}\right] \tag{19}$$

And for the case, when exponentiated Weibull distribution is used [4 - 6], the outage probability is:

$$P_{\mathrm{out}} = F(I_{\mathrm{min}}) = \left\{1 - exp\left[-\left(\frac{I}{\eta}\right)^{\beta}\right]\right\}^{\alpha} \tag{20}$$

3. Simulation Results and Discussion

The numerical simulations are performed using the following FSO system parameters: quantum efficiency of the photodetector material $\eta(\lambda_0) = 0{,}7$; central wavelength $\lambda_0 = 1{,}55\ \mu m$; $T = 300$ K; aperture coefficient $A = 5$; value of the resistor in the feedback of the preamplifier, $R_{Fb} = 1$ kΩ; $\tau_r = \tau_t = 0{,}85$; $R_r = 5{,}5$ cm; transmission wavelength of the interference filter before the photodetector $\Delta_{\lambda F} = 10$ nm; background radiation, $L_{\lambda,B} = 10^{-2}$ *W/m*2.sr.Ang (corresponds to bright day); angular width of the receiving antenna $\theta_r = 5$ mrad and $z = 2$km.

From here-on after as outage probability we will consider the probability for $I < I_{min}$. This doesn't mean that the system will stop working it just indicates that the FSO will work with bit error rate larger than the one calculated with Eq. (4). The outage probability is calculated using the log-normal distribution model for the atmospheric turbulence channel (Eq. (11)). It is chosen because of its mathematical simplicity $C_n^2 = 2.3.10^{-13}$.

Figure 2 shows the outage probability of FSO system using optimal system parameters depending on different values of *SNR*. Figure 3 represents the dependence of the outage probability of FSO system using optimal system parameters on the channel capacity (C_I).

It is observed in Figure 2 that the higher the *SNR* the higher the outage probability. This is because as seen in Equation (5) greater *SNR* values (respectively lower *BER*) require greater value of I_{min}, which means, that because of the turbulent channel (and the scintillation), there is a greater chance the intensity at the plane of the receiver drops below the required I_{min}, needed to keep constant *BER* calculated with Equation (4). In Figure 3 higher channel capacity requires higher values of I_{min}, which again means greater probability that the optical intensity at the plane of the receiver could be lower than the required I_{min}, needed to keep *BER* = const as calculated by Equation (4).

Figure 4 depicts the dependence of the outage probability of FSO system using optimal system parameters on the receiver's aperture R_r:

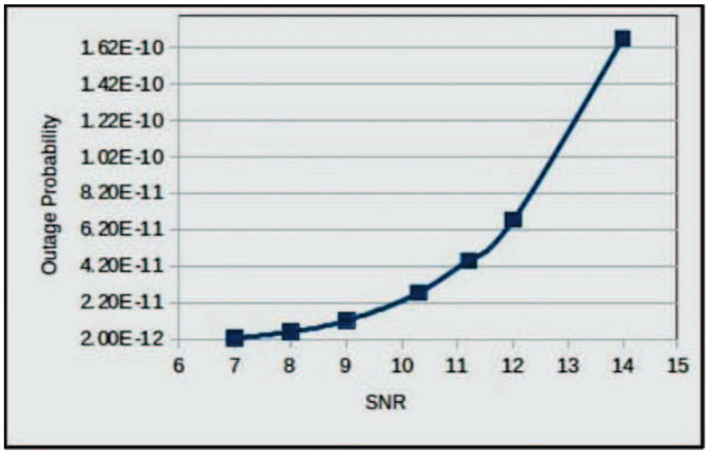

Figure 2. Outage probability of FSO system using optimal system parameters depending on SNR; R_r = 5.5 cm = const; C_I = 1.25 Gbps = const

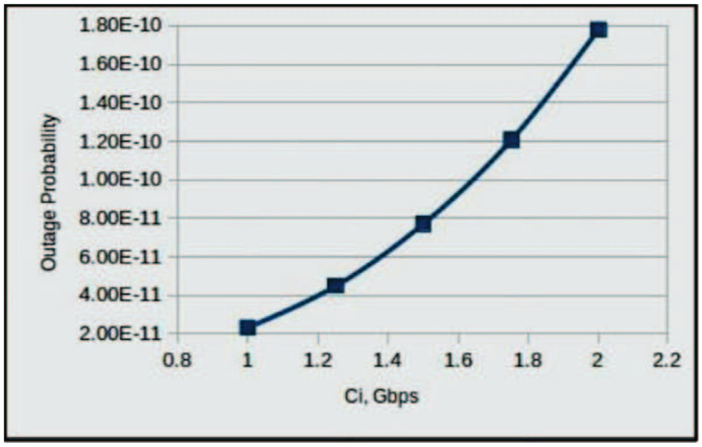

Figure 3. Outage probability of FSO system using optimal system parameters depending on the channel capacity C_I; SNR = 11.2 = const R_r = 5.5 cm = const

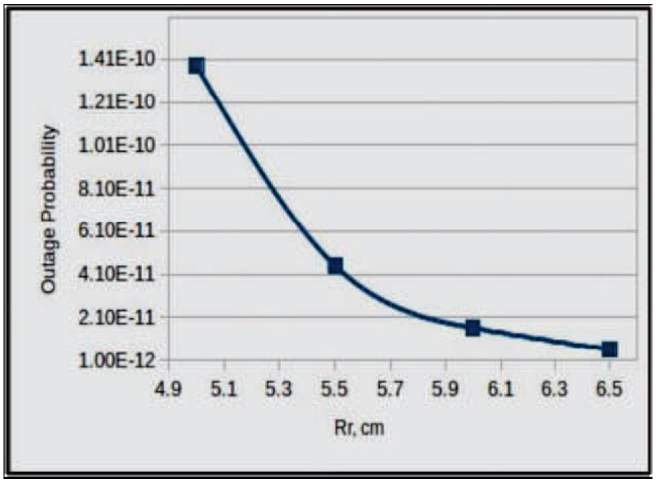

Figure 4. Outage probability of FSO system using optimal system parameters depending on receiver's aperture R_r; SNR = 11.2 = const; C_I = 1.25 GBPS = const

In Figure 4 the bigger the aperture radius (R_r) the more optical power gets in the receiver, so lower values of Imin are required to keep BER = const, which means lower outage probability. It is observed (in Figure 3 and Figure 4), that because the FSO system uses optimal values of the beam radius and the beam divergence angle respectively, there are less fluctuations in the outage probability. It varies in the interval [10^{-11}, 10^{-10}]. This means that using optimal system parameters can guarantee some level of predictability of the FOS system's availabilityin the presence of atmospheric turbulence over the optical channel.

4. Conclusion

In this paper the outage probability of FSO system using optimal system parameters in the presence of atmospheric turbulence is studied. For the purpose various statistical models for turbulent channels were explored. Closed form expressions for the outage probability in the presence of atmospheric turbulence were derived.

The outage probability of the FSO system depending on various parameters was simulated and the results were graphically represented in Figure 2 through Figure 4. The results can be used when designing FSO systems using optimal system parameters.

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