Reliability Assessment of a Star Structured Electronic System

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ABSTRACT: *In this work we have studied the reliability assessment of a star structured complex electronic system. The normal methods to reliability indices selection and reliability requirements determination are presented. Some specific reliability indices and basic dependences valid for reliability assessment of a star structured electronic system are presented and described.*

Keywords: Structural Reliability, Reliability Indices, Reliability of Complex Systems

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1. Introduction

A complex electronic system (CES) usually comprises a large number of functional units and blocks (i.e. system elements) interconnected in such a way that the system is able to perform a set of required functions including its basic system function as well as all auxiliary functions. The system elements, together with the links between them form the system structure. Often the CES layout is shaped in line with a network topology. For SCADA (Supervisory Control and Data Acquisition) systems most often this is a centralized star topology [5].

The system and its elements have operational modes including normal operating modes, test modes and contingency modes induced by failures, faults or operator errors.

In reliability perspective each particular CES structure determines the system reliability characteristics.

The system reliability assessment is a very important task standing in front of reliability engineers during the system design process and also during the system lifespan [1]. Such assessment is able to be performed on the base of a set of reliability indices selected in such a way that it becomes possible the system reliability characteristics to be fully revealed.

2. Selection of Reliability Indices for Reliability Assessment of CES

Selection of a proper set of reliability indices for reliability assessment of a complex electronic system (CES) is a task strongly depending on the type and the purpose of the system and also on the common functional requirements to it.

The indices for reliability assessment of CES can be divided on operational and technical ones depending on the degree of system defragmentation [2]. The operational indices characterize the system from customer point of view, as the technical ones carry some more technological sense.

The technical indices are suitable for to be evaluated statistically. These are necessary for reliability assessment of system elements, i.e. subsystems.

Selecting the reliability indices for reliability assessment of a CES, the rules listed below is advisable to be followed:

- The total number of indices have to be the minimal one;

- The applications of complex reliability indices, presenting combinations of criteria have to be avoided;

- The reliability indices chosen have to carry a simple physical sense;

- The reliability indices chosen have to make possible performance of analytical reliability estimation during system design process;

- The reliability indices chosen have to allow a statistical evaluation of it, based on the reliability tests results or system operational data;

- The reliability indices chosen have to make possible quantitative reliability limits to be set-on.

3. Requirements Regarding Reliability of a CES

For to declare reliability requirements technical objects at three different levels have to be distinguished. These are systems, subsystems and components.

3.1. Requirements Regarding System Reliability

Determination of reliability requirements to a CES can be achieved, following three approaches. It might be based on:

- Expert advice, design engineer experience and practice;

- Prototype analysis, or statistical data for a CES similar as purpose, structure and/or component base to the current one;

- Reliability level which is optimal for the current system.

The latter approach is applicable only in case when:

- The system function is measurable by usage of the same units of measure as the expenditure of its production;

- Reliable data for components reliability are at the disposal;

- The system structure, system functioning and maintenance process are fully determined

In this case the system function can be maximized, as follows:

$$F_k(R) = E_k(R) = C_k - (R) \qquad (1)$$

where R is a system reliability index depending on the k-th variant of the system structure S_k chosen and also on the reliability of elements of the i-th kind - r_i, or:

$$R = R(S_k, r_i, k = 1,...,m, i = 1,...,n) \quad (2)$$

where m is the number of system structure variants; n is the number of system components; $E_k(R)$ is the system function of k-th system variant as a number – value, valid for reliability level R; $C_k(R)$ are expenses for reliability level equal to R of k- th system variant to be ensured [3].

For each fixed k a solution can be found, based on the condition below:

$$\frac{\partial E_k(R)}{\partial R} = \frac{\partial C_k(R)}{\partial R} \quad (3)$$

After that the variant of the highest absolute value have to be chosen from among of the optimal solutions E_k (R).

In cases when the complex effect of the system function is incommensurable with the expenditures, only the former two approaches for system reliability requirements determination are applicable.

3.2. Requirements Regarding Subsystems Reliability

Normally the requirements regarding subsystem reliability are set-up when the system reliability requirements are already at the disposal.

3.2.1. Uniformly Division Approach

If the system consists of a number of N elements, which are similar or identical in complexity and structure it is possible a reliability index given (R) to be equally divided in accordance with the rule below:

$$E_k = \sqrt[N]{R}, \qquad i = 1,...,N. \quad (4)$$

In this case MTTF for the i-th subsystem is approximately equal to:

$$T_i = NT, \qquad i = 1,...,N, \quad (5)$$

where T is the average system MTTF given.

3.2.2 Proportional Division Approach

If n_i is the elements number of i-th subsystem, then:

$$T_i = \sqrt[a_i]{R}, \qquad i = 1,...,N, \qquad a_i = n_i \left(\sum_{1 \le i \le N} n_i \right)^{-1} \quad (6)$$

If the failure rates of the elements (or prototypes) of the j – th type λ_j are known, then this approach is able to be modified by substitution in Equation (6) of a_i by:

3.2.3. Optimal Division Approach

In case when at the time of general system reliability requirements (R) set-up the system structure (S) is known as well as the techniques for subsystems reliability improvement, i.e. functions $R_i(C_i)$, where C_i are the resources spent for provision respective subsystem of desired reliability, it becomes possible an optimal reliability requirements division to be found in two cases as follows:

1. At the maximum of the system reliability index, when the total resource C^0 is limited:

$$\max_C \left\{ R(S, R_i(C_i)) \middle| \sum_{1 \le i \le N} C_i \le C^0 \right\},\ C = (C_1, C_2, ..., C_N) \tag{7}$$

2. At the minimum of the system maintenance spends, when the reliability index value given R^0 is achieved:

$$\min \left\{ C(S, R_i(C_i)) \middle| R^0 \right\} \tag{8}$$

3. Requirements Regarding Components Reliability The reliability requirements for electronic components usually are set-up by experts, or these are based on reliability data obtained during prototypes testing.

4. Reliability Assessment of a Star Structured CES

4.1. The Star Structure as a Monotonous Structure

Some systems, including the star structured systems, have specific characteristics in regard to system reliability. Their reliability characteristics monotonous worsen when the reliability characteristics of system elements are getting worse [7]. The structure of such systems is called monotonous structure.

A simple logical system analysis is necessary to be performed in order to be identified a monotonous structure as such one.

Let introduce a logical random variable, which can take two different values, as follows:

$$x_i = \begin{cases} 1, \text{ if the } i-th \text{ element is functioning}, \\ 0, \text{ if the } i-th \text{ element is failed}. \end{cases} \tag{9}$$

The probability that the *i*-th system element is in a workable condition is determined as a math expectation:

$$p_i = Mx_i \tag{10}$$

An *n*-component vector, denoted as

$$X = (x_1, ..., x_n), \tag{11}$$

would characterize the system condition. When the system structure is fixed, the system condition depends on the condition of its elements (*n* in number). In case when a failed element is occurred, the vector takes the form:

$$X_i = (x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n) \tag{12}$$

or it becomes an (*n*-1)-component vector. The *i*-th component is missing. Generally for all missing components is valid $i \in \alpha$, and the number of vector components is equal to $n - |\alpha|$.

The system condition can also be described by a logical random function. In this case this will be a structural system function. This takes the value as follows:

$$\varphi(X) = \begin{cases} 1, \text{ if the condition } X \text{ is equal to workable system condition}; \\ 0, \text{ if the condition } X \text{ is equal to non-workable system condition}. \end{cases}$$

The probability about the system to be in a workable condition is determined as a math expectation of the structural system function:

$$h = M\varphi(X) \tag{13}$$

This index can be expressed by reliability indices of system elements, as follows:

$$h(p) = h(p_1, ..., p_n) \tag{14}$$

(13)

The system structure is assessed as a monotonous one in cases when the equations listed below are valid for the current system. These are:

$$\varphi(1) = 1,\ where\ 1 = (1,1,...,1); \tag{15}$$

$$\varphi(0) = 0,\ where\ 0 = (0,0,...,0); \tag{16}$$

$$\varphi(X) \geq \varphi(Y),\ if\ \ X > Y \tag{17}$$

The latter equation aggregates a number of *n* conditions of the type

$$x_i \geq y_i,\ \ for\ \ \ i = 1,...,n,$$

and at least one of it have to be strictly fulfilled.

Taking into consideration the arguments expressed so far it becomes clear that a star structured systems have to be assessed as a monotonous one. This conclusion gives the opportunity a logical structural reliability analysis to be performed in regard to a star structured CES.

The reliability function of such system can be presented as:

$$h(p) = p_i M\varphi(X_i,\ \ \ x_i = 1) + q_i M\varphi(X_i,\ \ \ x_i = 0) \tag{18}$$

where $M\varphi(X_i,\ \ x_i = 1)$ is the probability the system to work proper under condition that the *i*-th element is absolutely reliable.

$M\varphi(X_i,\ \ x_i = 0)$ is the probability of the same, but under condition that the *i*-th element is definitely failed. The reliability system function can also be similarly expanded in regard to two system elements *i*-th and *j*-th ones [6]. In this case the system reliability function takes the form as follows:

$$\begin{aligned} h(p) = {} & p_i p_j M\varphi(X_{ij},\ x_i = 1,\ x_j = 1) + \\ & + p_i q_j M\varphi(X_{ij},\ x_i = 1,\ x_j = 0) + \\ & + q_i p_j M\varphi(X_{ij},\ x_i = 0,\ x_j = 1) + \\ & + q_i q_j M\varphi(X_{ij},\ x_i = 0,\ x_j = 0) \end{aligned} \tag{19}$$

The latter equation gives the opportunity to assess the structural reliability of a star structured CES by taking into consideration the peculiarity of the star topology, i.e. nonequality of elements positioned at different hierarchic levels of this structure. It becomes obligatory the system reliability function to be expanded in regard to these system elements, which stand at the structural hierarchic levels higher that the lowest one, or by the other words which are not peripheral system elements. Their influence and impact on the system reliability can be fatal, because a failure of each of it can cause a failure of a system branch or a total system failure (when the system element at the highest level failed).

4.2. The Star Structured CES as a System with an Additive Factor of Effectiveness

The structure of some specific electronic systems (star structured systems are also included) consists of functional redundancy [4]. This makes the system able to functioning even in case when one or some partial failures are been occurred. Then the system continues to work with decreased quality and efficiency of functioning, but it is not totally failed.

For qualitative assessment of functioning of such systems it is advisable a quantitative index to be introduced, i.e. quality of system functioning. This takes into consideration the influence and also the impact of partial failures on system functioning. The effectiveness of system functioning is a quantitative characteristic of quality and quantity of work performed by the system.

Star structured CES are systems which are characterized by a relatively simple effectiveness factor. Each peripheral element of such system brings its separate and independent contribution to the effectiveness of the entire system. These systems are known also as systems with an additive effectiveness factor. Such behavior is typical for most of the SCADA systems.

If the contribution of the *i*-th element to the system effectiveness is φ_i, then the system effectiveness of a system intended for short term of operation at a time *t*, would be:

$$E = \sum_{1 \le i \le n} \varphi_i r_i(t) \tag{20}$$

where $r_i(t)$ is the probability the *i*-th element to be in a workable condition at the moment of *t*.

Consider a CES which is built up in line with a centralized star topology [8]. The system structure also can be described as a star within a star. This is shown on Figure 1. The system is spread over two sites of service. The system structure consists of a number of $n+1$ elements and three hierarchic levels. One of the system elements is positioned at the -st (the highest) hierarchic level, two elements are at the -nd level and a number of $n-2$ peripheral (end) elements are positioned at the III-th (the lowest) level. A number of $m-2$ of it are installed at the first service site and the rest of it (a number of $n-m$), are respectively installed at the second site of service.

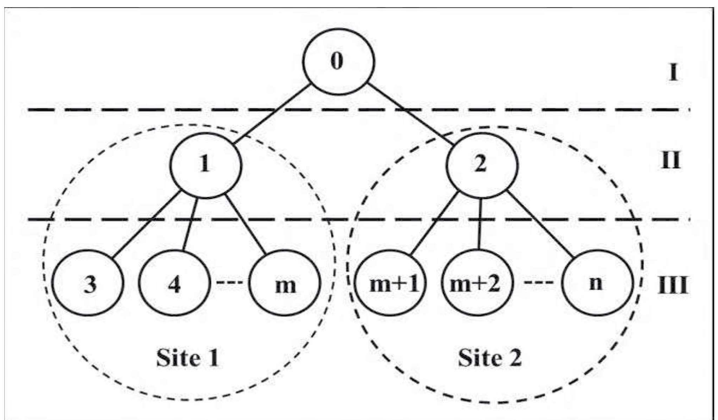

Figure 1. Star structured CES topology

Obviously the peripheral elements would be able to function effective only in case when the elements positioned at the I-st and II-nd level are in a workable condition. If K_i is availability of the *i*-th element, then for the system effectiveness is valid:

$$E = K_0\left(K_1\sum_{i=3}^{m} K_i\varphi_i + K_2\sum_{i=m+1}^{n} K_i\varphi_i\right) \tag{21}$$

In case when all peripheral elements at a site of service are similar, or identical and bring an equal contribution to the system effectiveness (which is typical for most of the SCADA systems), the system effectiveness can be presented as:

$$E = K_0\left[K_1(m-2)\varphi_1\sum_{i=3}^{m} K_i + K_2(n-m)\varphi_2\sum_{i=m+1}^{n} K_i\right] \tag{22}$$

Consider the same system but intended for a long-term operation. Let the contribution of the *i*-th element to the system effectiveness is $\varphi(t)$. In case of a failure at the moment $t \le t_i \le t_0$, for the system effectiveness is valid:

$$E(t, t+t_0) = \sum_{i=1}^{n}\left[r_i(t, t+t_0)\varphi_{0i} + \int_{t}^{t+t_0} f_i(x_i)\varphi_i(x_i)dx_i\right], \tag{23}$$

where φ_{0i} is the contribution of *i*-th element to the entire system effectiveness if it was in a workable condition during the time interval $(t, t+t_0)$.

Now it is possible to determine the effectiveness of the SCADA system already described. Let it was intended for a long term operation. The failure rate of the *i*-th system element is denoted as λ_i. The reliability function of each element allows an exponential distribution. In this case the average system effectiveness at a random moment of time *t,* can be determined as:

$$E(t) = e^{-\lambda_0 t}\left[e^{-\lambda_1 t}\sum_{i=3}^{m}\varphi_i e^{-\lambda_i t} + e^{-\lambda_2 t}\sum_{i=m+1}^{n}\varphi_i e^{-\lambda_i t}\right] \qquad (24)$$

The equation above gives the opportunity to determine the system effectiveness by the specific failure rate and also by the individual contribution factor of each system element. The latter depends not only by the element function but also by the characteristics of the object served.

5. Conclusion

The main problem in reliability assessment of a star structured CES appears to be the evaluation of the contribution of each peripheral element to the entire system effectiveness, even in cases when this contribution is similar or identical for all peripheral elements or only for these installed at the same site of service. These might be estimated upon an expert advice for each specific CES application. The rest reliability indices like the elements availability and the elements failure rate can be evaluated using data obtained by testing of prototypes, or might be estimated upon data for similar or identical elements at disposal. It is also possible for this purpose to be used data obtained during operation of the same or similar elements for long enough time. Based on this it becomes possible to estimate system effectiveness for a star structured CES for each specific application.

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