Measuring Free Space of the Atmospheric Turbulences of the Optic Communication Systems

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ABSTRACT: *The criteria used for the free space optics communication system during atmospheric turbulences are measured. These are derived from the closed form of expressions for standard performance measures. We observed the channel capacity used for carrying out the performance analysis over various FSO transmission scenarios. The M model represents the general turbulence it will reduce the other turbulence models received from the bit error rate. While capturing the expressions, the values given in the modulation formats and ergodic cc are also assessed like the number of large-scale cells of the scattering process. The mean power of the scatter components of the fading coupled with the line off-sight component is also studied.*

Keywords: Free Space Optics, M-distribution Model, Bit Error Rate, Ergodic Channel Capacity

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1. Introduction

Free space optics (FSO) transmission has emerged recently as efficient solution for obtaining secure, high data rate, wide bandwidth communication, due to its lack of licensing requirements, non-susceptibility to interferences and cost-effectivness [1]. However, mentioned advantages of FSO transmission are significantly reduced by the occurrence of scintillation in the optical channel. Variations of the refractive index, atmopsheric turbulence, lead to irradiance fluctuations of the received optical signal-scintillation. Channel modelling is very important in order to carry out accurate estimation of system performances in the FSO link designing process. In that purpose, extensive research has been conducted, and numerous mathematical models for the probability density function (PDF) of the irradiance are proposed for various fluctuation scenarios [2-8]. First, models which describe the weak irradiance fluctuation conditions such are *K*-distribution, Nakagami-*m*, the normalized *K*-distribution, as well as a lognormal distribution [3] are proposed. Mathematical model which describes wide range of turbulence conditions is Gamma-Gamma model [4]. In order to describe the fluctuations of widest possible level Gama-Gamma model has long been the most convenient and most described in the literature. As a result of the quest for obtainig a general and mathematically simpler model to describe the widest possible range of fluctuations, in the paper [9] is presented a generalized ú model. This model is applicable to a large number of FSO fluctuation conditions. Because of its generality, the generalized ú model can be reduced to large number of other known models, such are Rice-Nakagami [2], Gamma [4], HK, Gamma-Gamma, Gama-Rician, lognormal [9]. In this paper, performance analysis of FSO transmission over atmospheric channel modeled with generalized ú model will be carried out. Closed form expressions will be derived for bit-error rate (BER) over Binary Differentially Phase Shift Keying (BDPSK) modulation schemes. Further, closed form expressions for ergodic channel capacity (CC) will also be derived. Based on these expressions, average BER and ergodic CC values will be evaluated, graphically presented and discussed in the function of FSO link parameters.

The remainder of this paper is structured as follows: section 2 describes system model; section 3 provides performance analysis; numerical results are presented in section 4; conclusion remarks are given in section 5.

2. System Model

FSO channel modeled by generalized ú distribution, describes the FSO transmission carried out through three signal components, line-of-sight (LOS) component U_L and two scattered components, denoted as U_S^{cop} and U_S^{ind}. The first scattered component U_S^{cop}, is assumed to be coupled to the LOS component, representing quasi-forward optical signal. Component denoted as U_S^{ind} is statistically independent of first two components and denotes optical field scattering, occurring from the energy which is scattered by the off-axis eddies. Composed optical field can now be represented as [9]:

$$U = (U_L + U_S^{cop} + U_S^{ind}) \exp(\chi + jS) \tag{1}$$

with χ and S being real random variables representing the log-amplitude and phase fluctuations of the optical field. LOS components is defined as $U_L = \sqrt{G}\sqrt{\Omega}\exp(j\phi_L)$, while scattered components are defined as $U_S^{cop} = \sqrt{\rho}\sqrt{G}\sqrt{2b_0}\exp(j\phi_C)$ *and* $U_S^{ind} = \sqrt{(1-\rho)}U_S^{'}$ with parameter $\Omega = E\left[|U_L|^2\right]$, total scattered component denoted as $2b_0 = E\left[\left|U_S^{cop}\right|^2 + \left|U_S^{ind}\right|^2\right]$. Parameter ρ, $0 \le \rho \le 1$, denotes the factor expressing the amount of scattering power coupled to the LOS component and depends on the propagation path length, while U'_S denoting circular Gaussian complex random variable, and G denoting Gamma random process with unit mean value. Constants ϕ_L and ϕ_C denote deterministic phases of the LOS and coupled-to-LOS scatter components.

Received irradiance can be expressed as:

$$I = \left|U_L + U_S^{cop} + U_S^{ind}\right| \exp(2\chi) = YX \tag{2}$$

where $X = exp(2\chi)$ denotes large-scale fluctuations and $Y = \left|U_L + U_S^{cop} + U_S^{ind}\right|$ denotes small-scale fluctuations, distributed according to (20) and (21) from [9]. It is further shown that received irradiance is distributed according to Generalized *M* as:

$$f(I) = A\sum_{k=1}^{\beta} a_k I^{\frac{\alpha+k}{2}-1} K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta I}{\gamma\beta+\Omega^{'}}}\right) \tag{3}$$

where,

$$\begin{cases} A \triangleq \dfrac{2\alpha^{\frac{\alpha}{2}}}{\gamma^{1+\frac{\alpha}{2}}\Gamma(\alpha)}\left(\dfrac{\gamma\beta}{\gamma\beta+\Omega'}\right)^{\beta+\frac{\alpha}{2}} \\ a_k \triangleq \dbinom{\beta-1}{k-1}\dfrac{(\gamma\beta+\Omega')^{1-\frac{k}{2}}}{\Gamma(k)}\left(\dfrac{\Omega'}{\gamma}\right)^{k-1}\left(\dfrac{\alpha}{\beta}\right)^{\frac{k}{2}} \end{cases} \tag{4}$$

where $K_\nu(\cdot)$ [10, (8.432.2)] denotes the modified Bessel function of the second kind where, Γ(.) denotes the Gamma function [10, (8.310.1)] and $\binom{n}{k}$ represent the binomial coefficient.

Parameter α represents the effective number of large-scale cells of the scattering process, while parameter β represents the effective number of small-scale effects, in the same form as was explained. Also stands ‘ $\Omega' = \Omega + \rho 2b_0 + 2\sqrt{2b_0\Omega\rho}\cos(\phi_L - \phi_C)$, $\gamma = 2b_0(1-\rho)$.

After setting corresponding values, to parameters ρ, γ, Ω', α, β , *X, G* and U_L this general ú distribution model can be reduced to other well-known irradiance models, i.e.: a) by setting $\rho = 0$, and $Var[|U_L|] = 0$ M model reduces to Rice- Nakagami model; b) by setting $\rho = 0$, and γ = 0 it reduces to Gamma model; c) by setting Var[G] = 0, $\rho = 0$, and $X = \gamma$ it reduces to HK distribution model; d) by setting $\rho = 1$, γ =0 and Ω’=1 it reduces to Gamma-Gamma distribution; e) by setting $Var[|X|] = 0$, it reduces to Shadowed-Rician distribution; f) by setting $\rho = 0$, Var[|UL|] = 0, and $\gamma = 0$ it reduces to Log normal model; g) by setting Ω = 0 and $\rho = 0$ or β = 1 it reduces to K distribution; h) by setting Ω = 0, $\rho = 0$, and α = ∞ it reduces to Exponential distribution; and i) by setting β = ∞ it reduces to Gamma-Rician distribution.

3. Performance Analysis

The average BER can be evaluated directly by averaging the conditional BER, $\bar{P}_e$, depending on the type of modulation, i.e.:

$$P_e = \int_0^{+\infty} f_I(I)\, \bar{P}_e\, dI \tag{5}$$

and after substituting $\bar{P}_e$ expression for BDPSK modulation scheme, it yields:

$$P_e = \int_0^{+\infty} \frac{1}{2} erfc\left(\frac{P_T}{\sigma_N} I\right) f_I(I)\, dI \tag{6}$$

In order to evaluate BER performance values of the BDPSK FSO system in *M* turbulence induced fading channel, after expressing integrands from (6) in terms of the Meijer’s G-function as: $e^{-x} = \frac{1}{\sqrt{\pi}} G_{0,1}^{1,0}\left[x \,|\, {}^{-}_{0}\right]$ [11, (8.4.3/1)], $K_\nu(x) = \frac{1}{2} G_{0,2}^{2,0}\left[\frac{x^2}{4} \,|\, {}^{-}_{\frac{\nu}{2},-\frac{\nu}{2}}\right]$, and we obtain:

$$P_e = \frac{1}{8} A \sum_{k=1}^{\beta} a_k \int_0^{+\infty} I^{\frac{\alpha+k-1}{2}} G_{0,2}^{2,0}\left[\frac{\alpha\beta I}{\gamma\beta+\Omega'} \,\middle|\, {}^{-}_{\frac{\alpha-k}{2},-\frac{\alpha-k}{2}}\right] G_{0,1}^{1,0}\left[\frac{P_T}{\sigma_N} I \,\middle|\, {}^{-}_{0}\right] dI \tag{7}$$

Using [11, (2.24.1/1)] along with [11, (8.2.2/14)], the closed form of the average BER for the BDPSK FSO system over *M* turbulence induced fading can be expressed as:

$$P_e = \frac{1}{4\pi} A \sum_{k=1}^{\beta} a_k \left(\frac{\alpha\beta}{\gamma\beta + \Omega'} \right)^{-\frac{\alpha+\beta+1}{2}}$$

$$G_{2,1}^{1,2} \left[\frac{P_T}{\sigma_N} \frac{\gamma\beta + \Omega'}{\alpha\beta} \middle| \begin{matrix} \frac{1-2\alpha}{2}, \frac{2-2k}{2} \\ 0 \end{matrix} \right] \tag{8}$$

where $G_{p,q}^{m,n}[\cdot]$ denotes the Meijer's G-function [10, (9.301)].

Assuming that the FSO channel is memoryless, stationary and ergodic with perfect channel state information (CSI) available at both the transmitting lasers and the aperture receivers, the normalized ergodic CC can be defined as:

$$\frac{C}{B_{FSO}} = \int_0^{+\infty} log_2(1+I)\, f_I(I)\, dI \qquad \text{(bit/s/Hz)} \tag{9}$$

where B_{FSO} is the channels bandwidth.

In order to derive ergodic CC of the FSO system over ú turbulence induced fading, integrands from (10) should be expressed in terms of the Meijer's G-function $(1+x) = G_{2,2}^{1,2}\left[x \middle| \begin{smallmatrix} 1,1 \\ 1,0 \end{smallmatrix} \right]$ [11, (8.4.6/5)], $K_\nu(x) = \frac{1}{2} G_{0,2}^{2,0}\left[\frac{x^2}{4} \middle| \begin{smallmatrix} - \\ \frac{\nu}{2}, -\frac{\nu}{2} \end{smallmatrix} \right]$ [11, (8.4.23/1)], and we obtain:

$$\frac{C}{B_{FSO}} = \frac{1}{8\pi \log 2} A \sum_{k=1}^{\beta} a_k \int_{k=1}^{+\infty} I^{\frac{\alpha+k-1}{2}}$$

$$G_{0,2}^{2,0}\left[\frac{\alpha\beta I}{\gamma\beta + \Omega'} \middle| \begin{matrix} - \\ \frac{\alpha-k}{2}, -\frac{\alpha-k}{2} \end{matrix} \right] G_{2,2}^{1,2}\left[\frac{P_T}{\sigma_N} I^2 \middle| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right] \tag{10}$$

Using [11, (2.24.1/1)] along with [11, (8.2.2/14)], the closed form of the FSO CC in M turbulence induced fading can be expressed as:

$$\frac{C}{B_{FSO}} = \frac{1}{8\pi \log 2} A \sum_{k=1}^{\beta} a_k 2^{\alpha+k} \left(\frac{\alpha\beta}{\gamma\beta + \Omega'} \right)^{-\frac{\alpha+\beta+1}{2}} G_{6,2}^{1,6}\Bigg[16$$

$$\frac{P_T}{\sigma_N} \left(\frac{\gamma\beta + \Omega'}{\alpha\beta} \right)^2 \middle| \begin{matrix} 1, 1, \frac{1-2\alpha}{4}, \frac{2-2\alpha}{4}, \frac{1-2k}{4}, \frac{2-2k}{4} \\ 1, 0 \end{matrix} \Bigg] \tag{11}$$

4. Numerical Results

In this section, numerical results for BER of BDPSK FSO communication systems over M turbulence induced fading channels and for CC are presented.

Figures. 1-3 illustrate obtained BER values for the BDPSK FSO system in M turbulence induced fading. At Figure 1. is shown the influence of α and β parameters on obtained BER. Obtained BER values are lower when parameter β is increasing and/or when parameter α is increasing. At Figure 2. is the influence of Ω' and γ parameters on obtained BER. The performances are better, obtained BER values are lower when parameter Ω' is taking higher values and/or when parameter γ takes lower values. At Figure 3. is shown the dependence between ρ and γ parameter values and obtained BER.

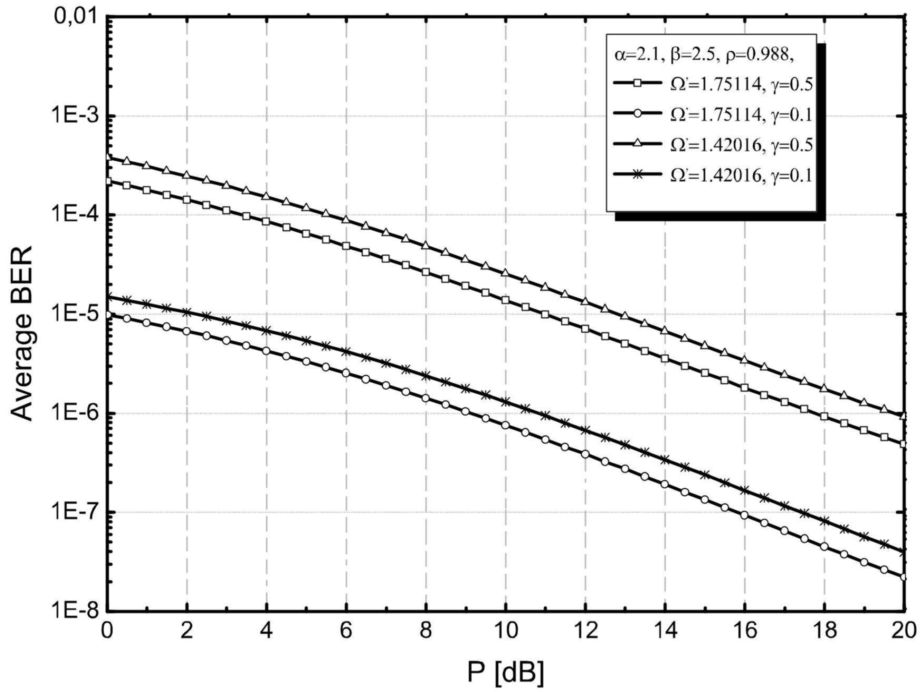

Figure 1. BER of the BDPSK FSO system over M turbulence induced fading when ρ, Ω' and γ are constant

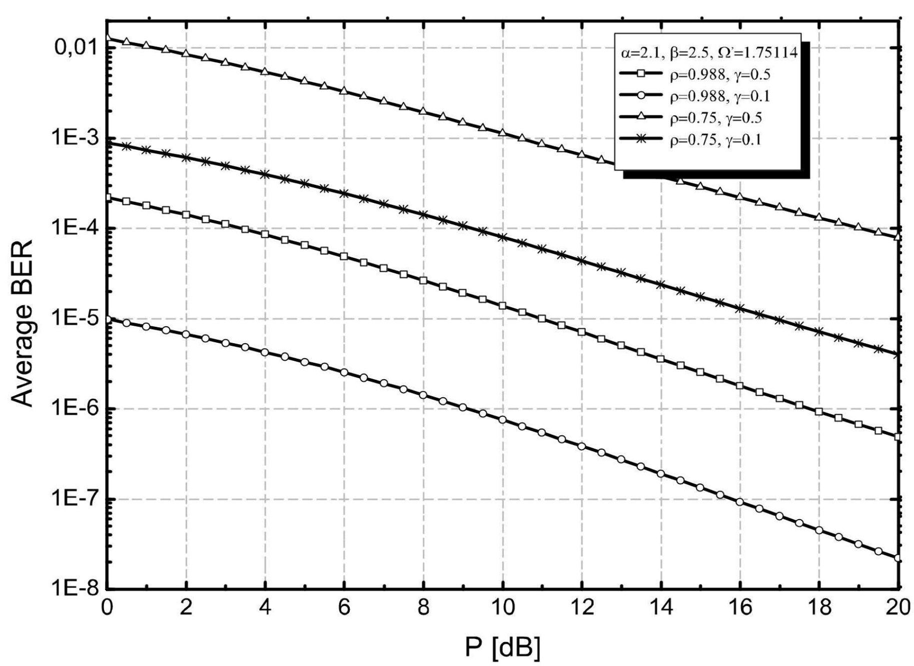

Figure 2. BER of the BDPSK FSO system over M turbulence induced fading when α, β and ρ are constant

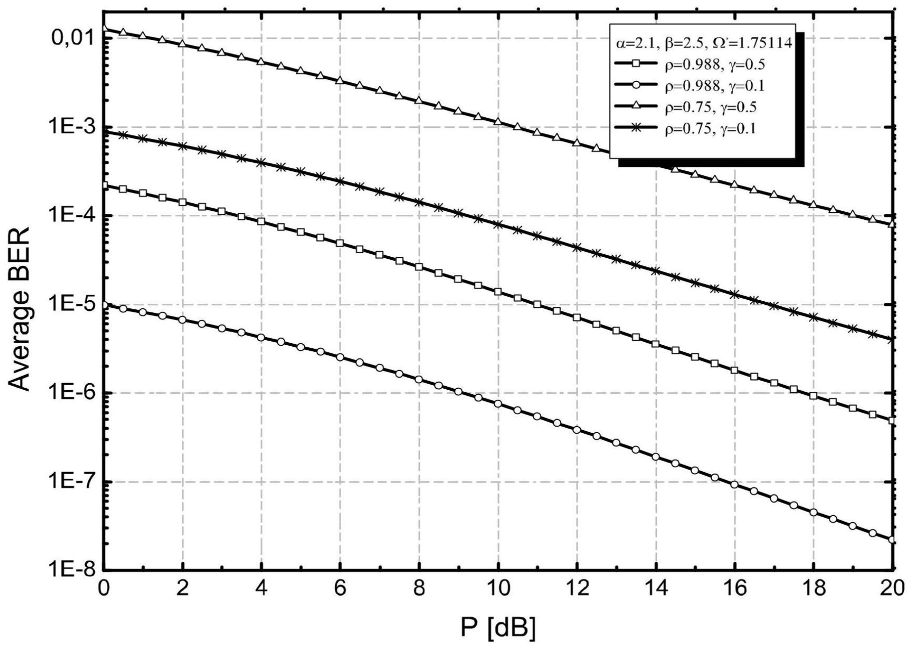

Figure 3. BER of the BDPSK FSO system over M turbulence induced fading when α, β and Ω' are constant

Figure 4. illustrate the normalized CC of the FSO system over M turbulence induced fading. At Figure 4. is shown dependence between values of α and β parameters and obtained normalized CC. The normalized CC is higher when parameter ± increase. Also is visible that when parameter ² increases normalized CC increase.

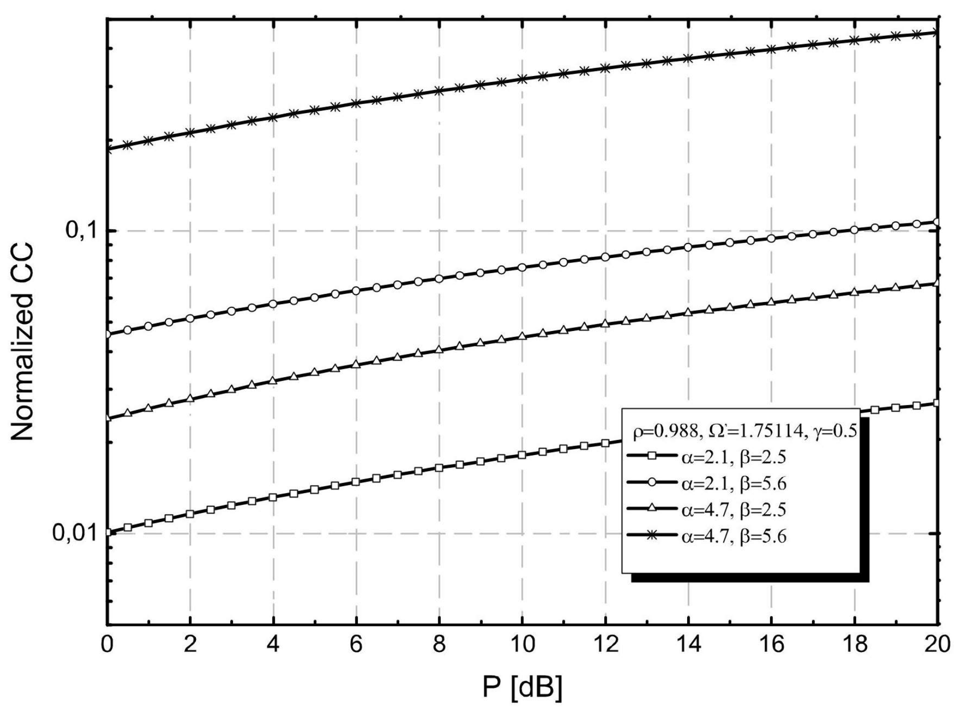

Figure 4. Normalized CC of the FSO system over M turbulence induced fading when Á, ©' and ³ are constant

5. Conclusion

In this paper FSO transmission over ú distributed atmospheric channel was analyzed through standard performance criterions such are BER and ergodic CC. Capitalizing on closed form expressions for BDPSK BER and ergodic CC derived in this paper, observed FSO link performances were analyzed in the function of FSO transmission parameters, such are number of large-scale cells of the scattering process, amount of irradiance, amount of scattering power coupled to LOS component and average power of the total scatter components. Major contribution of this work is its generality, because results from this paper could be applied to various other FSO transmission scenarios.

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