

# Physical Models and Programming of Objects for the Study of Oscillation in The Nonlinear Control Systems



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**ABSTRACT:** *With the help of this research, we tried to understand empirically how the oscillation in the nonlinear control systems work. To systematically do the performance testing we studied the models for determining the oscillation. We intend to apply it to the learning process in the course as it is used to work the effect of the activities and processes for students learning. We have employed the programming and physical model of objects.*

**Keywords:** Nonlinear Control, Harmonic Linearization, Real Time Control, Matlab Instrumentation Control

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## 1. Introduction

**Objectives of Development:** Designing and realizing laboratory stand for demonstration and training in automation that integrates two basic methods for synthesis and analysis of control systems - simulation modeling and physical modeling.

**Application of Laboratory Stand:** Computer-based learning of basic principles of control, setting regulators, synthesis of

control algorithms, studying effects such as non-linearity, saturation, “wind-up”, noise, factors with adverse influence, etc., inherent to real objects for automation. Examines is the method of harmonic linearization (*describing function method*).

**Main Advantages:**

- Create a flexible environment, combining the advantages of simulation and physical modeling.
- Reducing the level of abstraction typical of simulation modeling.
- Opportunity for graphical visualization and data recording.
- Intensification of training related to the possibility of realization of a large number of experimental settings.

**2. Materials and Methods**

In the literature there are a significant number of examples of the use of real-time control system in the automation training [1-3,6-8].

Oscillating behavior are typical of nonlinear systems. In these systems usually contain positional elements, such as controlling devices, secondary measuring means or actuators, which are characterized by simple realization, but reliability and low grade value. In most cases, non-linearity may be presented with straightforward types non-linearly characteristic - “dead zone”, “saturation”, positional elements and others. The auto-oscillations sufficiently objective is characterized by an amplitude ( $A$ ) and frequency of oscillations ( $\omega_a$ ). The value of  $A$  determines the accuracy of the regulation process and the frequency characterizes the intensity of the switching actuator. By considerations of reliability of the actuator in the synthesis of the system  $\omega_a$  is minimized. In many cases also, the parameters of oscillation allow to define the conditions for the stability of the system. The most common method for the study of oscillations of non-linear systems is the method of harmonic linearization. The essence of the method is presented in the following steps.

**2.1. Decoupled Non-linear Feedback System**

It is necessary the transformation of the structural scheme of the system, so that the one summarized non-linear element and the remainder a linear unit (Figure 1).

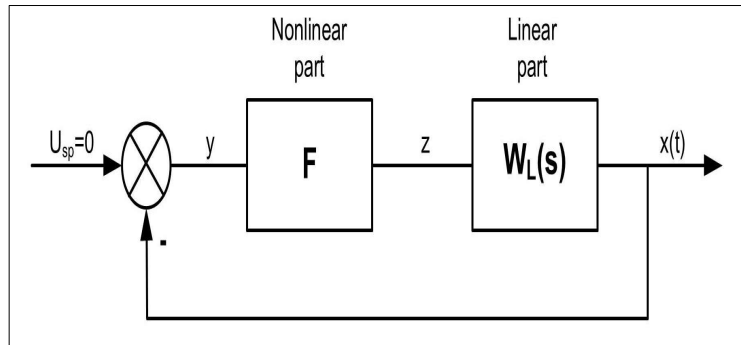


Figure 1. An equivalent circuit

In the transformation is not allowed:

- Exchange of linear element with non-linear and conversely (without time delay). This is necessitated by the invalidity of the principle of commutative property.
- Moving the non-linear unit through summing node. It is necessary failure to of the principle of superposition.

**2.2. Verification of the Hypothesis of Low-pass Filter of the Linear Part**

Perform a amplitude-frequency response of the linear part. Let there be the kind shown in Figure 2.

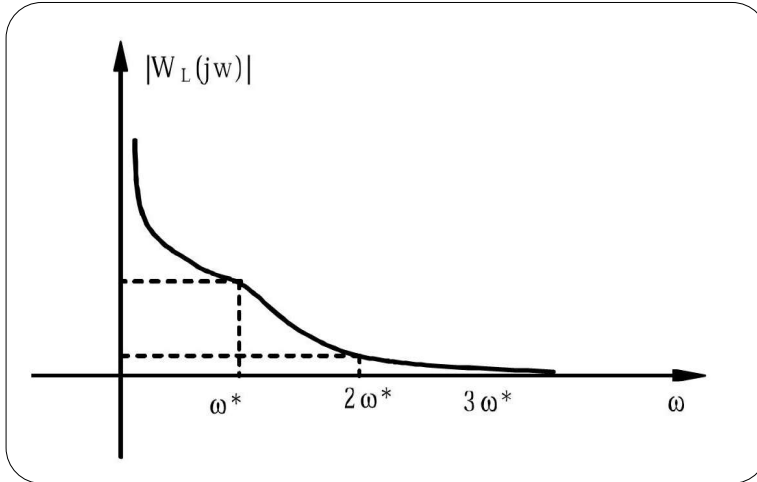


Figure 2. Magnitude of frequency response

If  $\omega^*$  is the frequency of the first harmonic (fundamental frequency) of oscillation, higher harmonic multiples of it with  $2\omega^*$ ,  $3\omega^*$ , ... hypothesis for low-pass filter is confirmed in the case of satisfying the condition  $|W_L(j\omega^*)| \gg |W_L(jn\omega^*)|$ ,  $n = 2, 3, \dots$

### 3. Obtaining Coefficient on Harmonic Linearization

This is for structural scheme of the type at Figure 1.

The linear part has the form (in autonomous system  $U_{sp} = 0$ ):

$$y = K_L(s)z,$$

When an oscillator process (if  $y$  is harmonic oscillation), passing through the non-linear element signal remains periodic not strictly harmonious, but with the same period and may be decomposed in order for Fourier harmonics with frequency components at integer multiples  $n\omega^*$ . Moreover, suppose that the linear dynamic system is a filter of low frequencies and missed only the first (hypothesis filter). If this assumption is not performing, errors of the method of harmonic linearization may be significant.

$$\text{Let } z = F(y); y = A \sin \omega t.$$

We present  $z$  in the order of Fourier:

$$z = C_0 + D_1 \sin \omega t + C_1 \cos \omega t + D_2 \sin 2\omega t + C_2 \cos 2\omega t + \dots \quad (1)$$

In symmetric non-linearity  $F(-y) = -F(y)$  is applied:

$$C_0 = \frac{1}{2\pi} \int_0^{2\pi} F(A \sin \omega t) d(\omega t) = 0.$$

When filtering harmonics of high order is necessary to determine only the coefficients  $D_1$  and  $C_1$  in equation (1).

After a substitution:

$$z = F(y) = F(A \sin \omega t) = D_1 \sin \omega t + C_1 \cos \omega t = D_1 \sin \omega t + C_1 \frac{d \sin \omega t}{dt} \frac{1}{\omega} = \left( D_1 \sin \omega t + \frac{C_1}{\omega} s \sin \omega t \right) \frac{A}{A} = \left( \frac{D_1}{A} + \frac{C_1}{A\omega} s \right) A \sin \omega t$$

As a result, we obtain the equivalent transfer function of the non-linear element:

$$W_{NE} = \frac{F(A \sin \omega t)}{A \sin \omega t} = \frac{D_1}{A} + \frac{C_1}{A \omega} s = q_0 + q_1 \frac{s}{A \omega} \quad (2)$$

or  $(z = q_0 y + q_1 \frac{sy}{A \omega})$

where:

$$q_0 = \frac{D_1}{A} = \frac{1}{\pi A} \int_0^{2\pi} F(A \sin \varphi) \sin \varphi \cdot d\varphi,$$

$$\varphi = \omega t;$$

$$q_1 = \frac{C_1}{A} = \frac{1}{\pi A} \int_0^{2\pi} F(A \sin \varphi) \cos \varphi \cdot d\varphi. \quad (3)$$

Thus the non-linear equation for  $z$  is replaced by the approximate equation (2) for the first harmonic. The coefficients  $q_0$  and  $q_1$  are called coefficient of harmonic linearization. They may depend on the amplitude and the frequency.

Displayed linearization coefficients for typical nonlinearities can be found in reference [4]. For the nontypical should be derived from (3).

#### 4. Analysis

The most commonly used methods for analyzing the method of harmonic linearization are [4]:

- Determining the parameters of oscillation using the criterion of Routh–Hurwitz.
- Determining the parameters of oscillation and stability using the criterion of Nyquist and derivatives - method of Goldfarb and Kochenburger.
- Determining the parameters of oscillation and stability using the criterion of Mihaylov.

#### 5. Laboratory Experiments

The principal scheme of laboratory stand which is realized is presented in Figure 3.

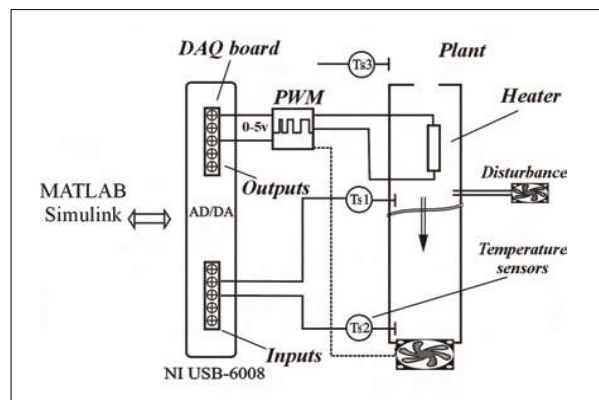


Figure 3

Figure 4 shows the existing model.



Figure 4. Physical model and communication module

The physical model communicates with the computer system and software, by means of a device for data collection (Data Acquisition, DAQ) - NI USB-6008 devices mainly include analog-to-digital (A/D) and digitalto- analog (D/A) converters, which are generally known as data converters.

Figure 5 shows scheme realizing the function of the a ontrol device, in this case PID - regulator, which performed the necessary simulations and analyzes in the middle of MATLAB / SIMULINK. In particular - setting the parameters of the regulator.

Training example: a system of third order with twoposition relay.

1. Object

$$G = tf(21.6, [187\ 108\ 147\ 1], 'OutputDelay', 4)$$

Transfer function:

$$\exp(-4*s) * \frac{21.6}{187 s^3 + 108 s^2 + 147 s + 1}$$

2. Non-linear element - two-position polarized relay with hysteresis:

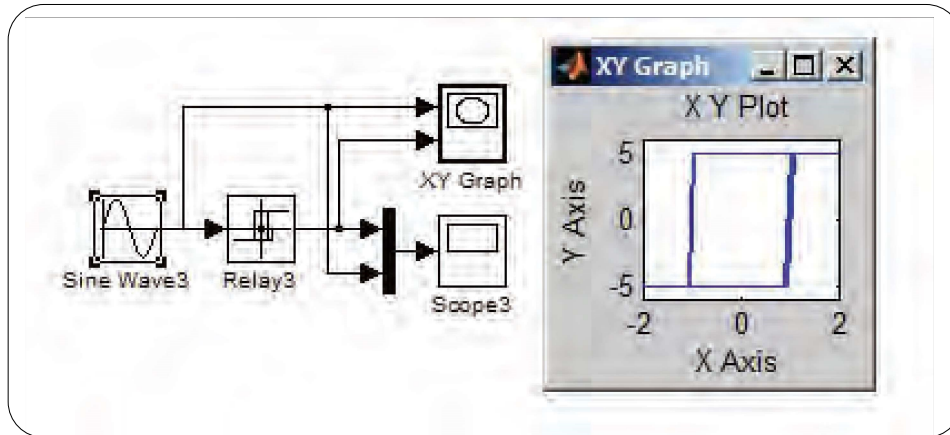


Figure 5

3. Receive coefficients of harmonic linearization

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A = 1:0.0001:20;
Wnon = ((4*5)./(pi.*A)).*sqrt(1-((1.^2)./(A.^2)))...
+j*(-4*5)./(pi.*(A.^2));
Z = -1./(Wnon);
Re = real(Z);
Im = imag(Z);
plot (Re, Im, b*)
hold on

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4. Plotting Nyquist on the linear part and Goldfarb  $(-1/W_{NE}(A))$ .

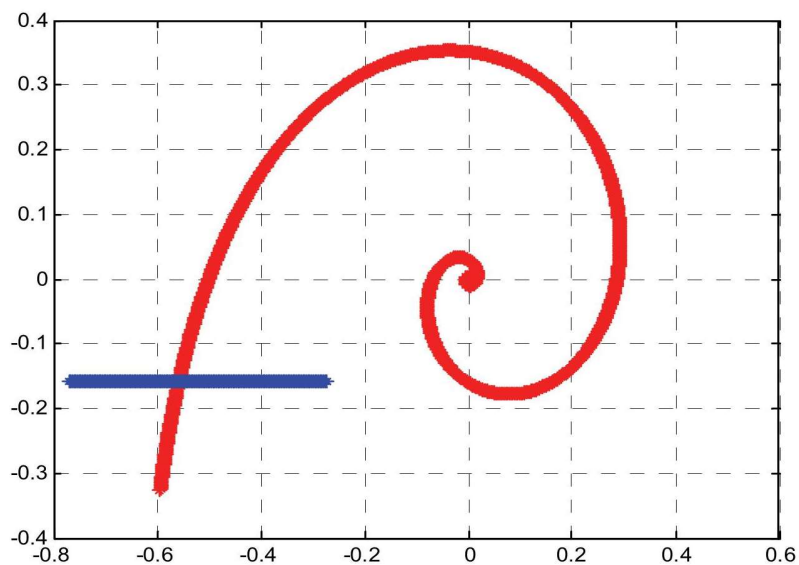


Figure 6

5. Defining the parameters of oscillation in the intersection point on hodographs.
6. Plotting model in Simulink, confirm the results of the presence of oscillator regime and evaluation of error for linearization.

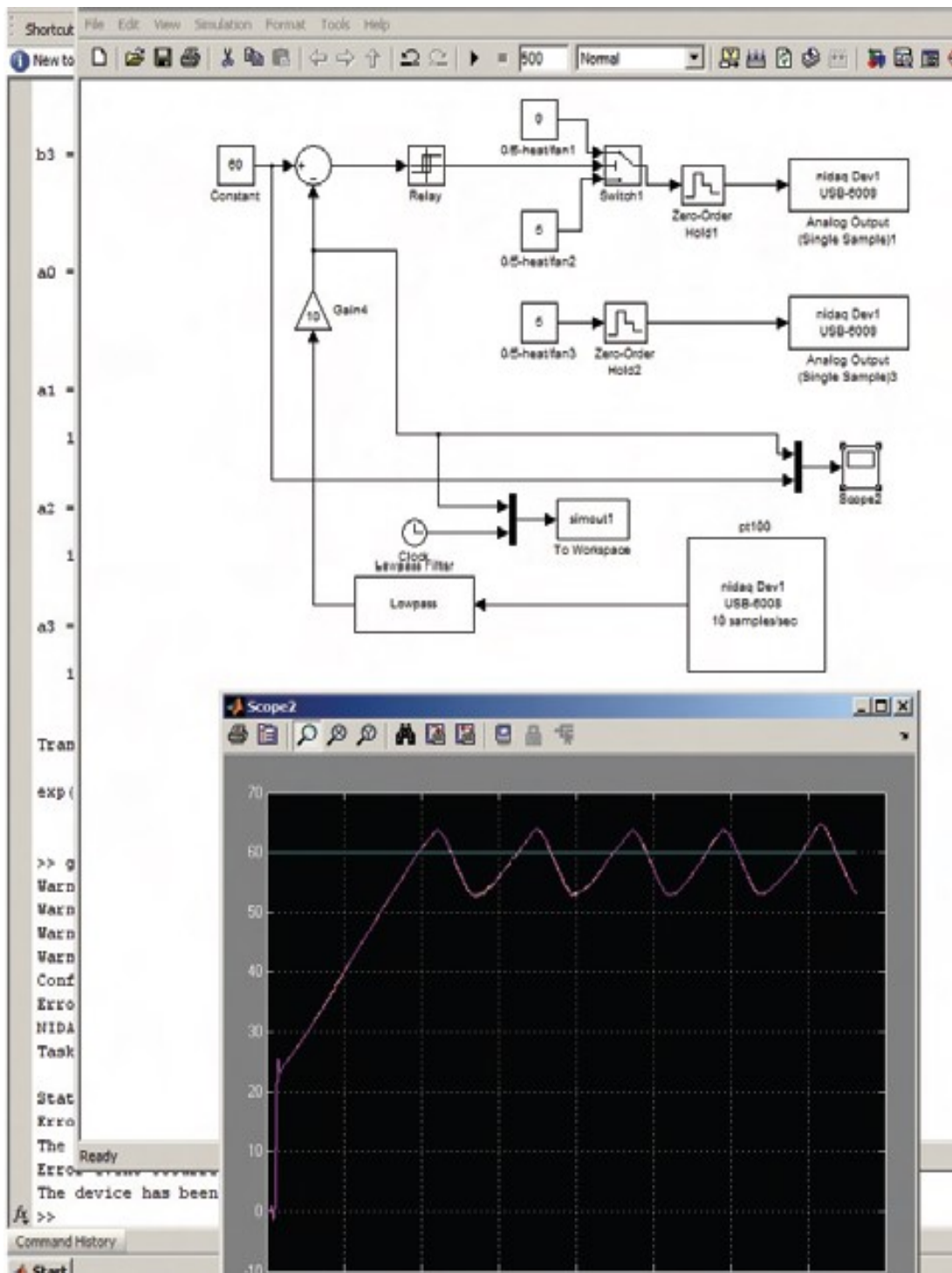


Figure 7. Real-time diagram with physical plant and SIMULINK controller

Other schemes (tasks or “scenarios”) that are made with laboratory stand are:

- Removing the transfer function on input and disturbance;
- Determination of the inertness on sensor;
- Definition of quality parameters of step response;
- Setting the controller on defined criteria of quality of step response;
- Control with two-position controller;
- Control with three-position controller;
- Synthesis of logical algorithms such as “if-then” eliminate the effects wind-up;
- Design of advanced control algorithms - neural, fuzzy, adaptive robust systems;
- Filtering, normalization and calibration signals from sensors, etc.

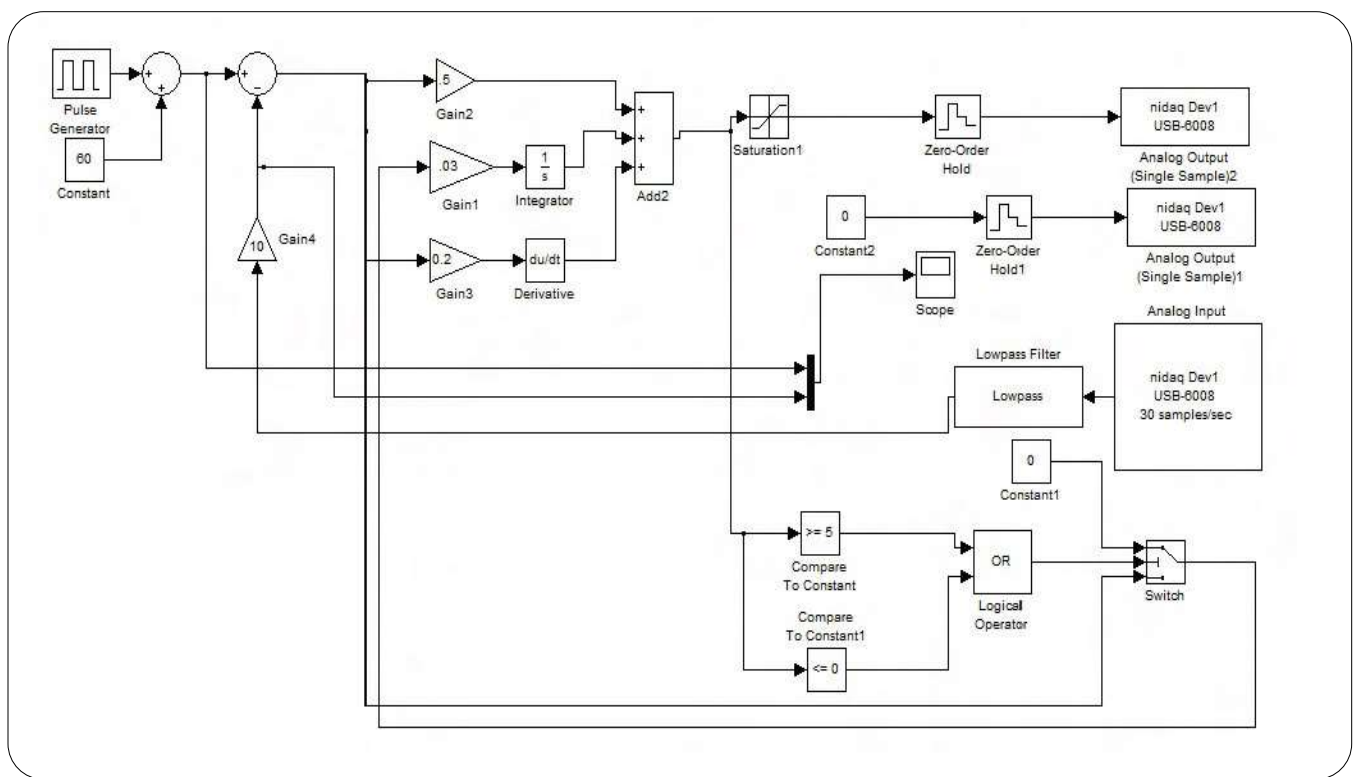


Figure 8. Example simulation scheme for control of the object with PID and actuator saturation

## 6. Conclusion

The idea presented development is led from the need to implement innovative training methods - practical training in close to industrial environments, training base project development, problem-based learning and more. In this case this was achieved using a real physical object in combination with virtual tools for its control.

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