

# New Communication Protocols for Cascaded-integrator-CPMB Finite Impulse Response Filters

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**ABSTRACT:** *We have used new communication protocols for designing the multiplierless cascaded-integrator-cpmb finite impulse response filters. We have used the designs of the cascading simple filters. We found that the responsive behaviour of the resulting filters has improved frequency. We have studied the influence of the cosine prefilter to study the novel CIC FIR filter functions.*

**Keywords:** CIC Filters, FIR Filters, Linear Phase, Multiplierless Structure, Selective Filters

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## 1. Introduction

The term “Cascaded-Integrator-Comb (CIC)” filters was first reported in 1981 by E.B. Hogenauer [1]. He proposed a class of hardware-efficient linear phase finite impulse response (FIR) filters known as CIC or Hogenauer filters.

Because of the disadvantages of a CIC FIR filter such as not flat passband and a high passband droop, it is of a great interest to improve magnitude response characteristic. The idea of modified CIC filters is not new, it is proposed by the authors in [2]-[8]. This paper is focused on the design of novel class of CIC FIR filter functions in the explicit form. The properties of the designed novel class are demonstrated in detailed examples. The ideas of spreading the delays in the comb stages provide excellent results for increasing insertion loss in stopband and small changes in the bandwidth size of the filter function compared to classical CIC filter that has the same number of cascades and the same level of constant group delay. In general, the passband characteristic can be improved by adding a compensator filter in cascade with the original filter [9]-[12]. A cosine prefilter [13]-[14] can be added to improve the stopband characteristic. In this paper, influence of cosine prefilter and compensator blocks to frequency response characteristics of proposed novel CIC FIR filter functions is investigated.

The rest of the paper is organized as follows. Section 2 describes multiplierless CIC filter functions. The cosine prefilter and compensator blocks are elaborated in Section 3 and 4, respectively. In Section 5, proposed novel CIC FIR filter functions are presented and analyzed. Section 6 compares different modified CIC FIR filter functions present in the literature. Influence of added filter blocks (cosine prefilter and compensator) to modified CIC filters is described in Section 7. Concluding remarks of this paper are given in Section 8.

## 2. Multiplierless Linear-phase CIC Filter Functions

The design methodology of conventional CIC FIR filters is well known to the scientific community. The normalized CIC FIR filter function of one section in  $z$ -domain is defined with

$$H(N, z) = \frac{1}{N} \cdot H_c(N, z) \cdot H_i(z). \quad (1)$$

There are two basic building blocks: a comb and an integrator. The filter function of a comb in  $z$ -domain is  $H_c(N, z) = (1 - z^{-N})$ , therefore a difference equation in time domain is  $y_c(n) = x(n) - x(n - N)$ .  $H_i(z) = 1/(1 - z^{-1})$  represents the filter function of an integrator in  $z$ -domain and  $y_i(n) = x(n) + y_i(n-1)$  is its time domain representation.

A poor magnitude characteristic of the CIC filter composed of one section, Equation (1), is improved by cascading several identical CIC filters. The classical CIC FIR filter function of normalized amplitude response characteristic, represented in the  $z$ -domain, is defined as

$$H(N, K, z) = \left( \frac{1 - z^{-N}}{N \cdot (1 - z^{-1})} \right)^K, \quad (2)$$

where  $N$  is the decimation factor, and  $K$  is the number of sections (identical cascaded CIC filters of one section) [1]. They are positive integers.

## 3. Cosine Prefilter

To improve the magnitude characteristic of the CIC filter, the following cosine prefilter introduced in [13]-[14], can be used

$$H_{\cos}(z^N) = 0,125 \cdot (1 + z^{-2N}) \cdot (1 + z^{-N})^2. \quad (3)$$

It is combination of two cosine functions. Because this prefilter only employs unity coefficients, this prefilter is essentially multiplication free.

## 4. Compensator

Passband droop compensation is done by use of a multiplierless  $2N$ -order FIR filter with one free integer parameter  $b$  presented in [12]

$$G(z^N) = B \cdot [1 + A \cdot z^{-N} + z^{-2N}], \quad (4)$$

where  $B = -2^{-(b+2)}$  is a scaling factor ensuring unitary gain at the digital frequency zero, and  $A = -(2^{b+2} + 2)$ . The transfer function of compensator filter is a function of  $z^N$  and has only one coefficient  $A$  which can be realized using additions and shifts.

## 5. Proposed Novel CIC FIR Filter Functions

A modification of classic CIC filters is here proposed. The filter function of normalized amplitude response characteristic of designed novel class of CIC FIR filter functions can be written in non-recursive form as follows

$$H(N, K, L, z) = \left[ \frac{1}{N-3} \cdot \frac{1}{N-2} \cdot \frac{1}{N-1} \cdot \frac{1}{N} \right]^L \cdot \left[ \left( \sum_{r=0}^{N-4} z^{-r} \right) \cdot \left( \sum_{r=0}^{N-3} z^{-r} \right) \cdot \left( \sum_{r=0}^{N-2} z^{-r} \right) \cdot \left( \sum_{r=0}^{N-1} z^{-r} \right) \right]^L.$$

$$\left[ \frac{1}{N+1} \cdot \frac{1}{N+2} \cdot \frac{1}{N+3} \right]^L \cdot \left[ \left( \sum_{r=0}^N z^{-r} \right) \cdot \left( \sum_{r=0}^{N+1} z^{-r} \right) \cdot \left( \sum_{r=0}^{N+2} z^{-r} \right) \right]^L \quad (5)$$

and  $K = 7L$ ,

where  $N$  and  $L$  are free integer parameters. The main idea is instead of using  $K$  times the same CIC core filter with a fixed order  $N$ , to use  $L$  times (where  $L = K/7$ ) a core combined from orders  $N-3, N-2, N-1, N, N-1, N-2$ , and  $N+3$ .

Classical CIC filters have multiple zeros located on the unit circle. With this new idea, zeros are more spread around the stopband region of the unit circle leading to a better stopband attenuation while having the same number of zeros and a very similar circuit complexity. The idea provides an elegant solution to increase the stopband attenuation while having the same complexity.

A recursive form of a novel class of CIC FIR filter functions is

$$H(N, K, L, z) = \left( \frac{1-z^{-(N-3)}}{(N-3) \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N-2)}}{(N-2) \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N-1)}}{(N-1) \cdot (1-z^{-1})} \cdot \frac{1-z^{-N}}{N \cdot (1-z^{-1})} \right)^L \cdot \left( \frac{1-z^{-(N+1)}}{(N+1) \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N+2)}}{(N+2) \cdot (1-z^{-1})} \cdot \frac{1-z^{-(N+3)}}{(N+3) \cdot (1-z^{-1})} \right)^L \quad (6)$$

and  $K = 7L$ .

Frequency response characteristic is obtained by evaluating the filter function in the  $z$ -plane at the sample points defined by setting  $z = e^{j\omega}$ , where  $\omega = 2\pi \cdot f$  [rad/s]. The normalized amplitude response characteristic is defined as the magnitude of the complex filter frequency response  $H(N, K, L, z = e^{j\omega})$ . The magnitude response characteristic is obtained as absolute value of normalized amplitude response characteristic. The linear phase response characteristic is defined as the phase angle of the complex filter frequency response. The constant group delay response characteristic is expressed as

$$\tau(N, K, L, \omega) = (N-1) \cdot 7L/2. \quad (7)$$

### 5.1. Impulse Coefficients

Generally, the FIR filter function of the corresponding nonrecursive implementation has the form passband

$$H(N, K, L, z) = \frac{1}{H_0(N, K, L)} \cdot \sum_{r=0}^{(N-1) \cdot K} h(N, K, L, r) \cdot z^{-r}, \quad (8)$$

where  $H_0(N, K, L) = H(N, K, L, z \rightarrow 1)$  is normalized constant for unit magnitude response at  $f = 0$ . Vector  $\mathbf{h}(N, K, L)$  is the vector of impulse response coefficients  $h(N, K, L, r)$ , where total number of elements in vector is  $M-1$ .  $K+1$  and  $h(N, K, L, r) = (N, K, L, M-r)$ .

The non-recursive implementation of classical CIC filter functions  $H(N, K, z)$ , obtained for even value of integer parameter  $N = 4$  and  $K = 7$ , is observed here. The normalized constant is  $H_{0CIC}(4, 7) = 16384$ . The vector of the impulse response coefficients is

$$\mathbf{h}_{CIC}(4, 7) = \left\{ 1, 7, 28, 84, 203, 413, 728, 1128, 1554, 1918, 2128, 2128, 1918, 1554, 1128, 728, 413, 203, 84, 28, 7, 1 \right\} \quad (9)$$

The non-recursive forms of novel CIC filter functions,  $H(N, K, L, z)$ , obtained for even value of integer parameter  $N$  and  $K = 7$  (obtained  $N$  for  $L = 1$ ), satisfied Eq. (8). The vector  $\mathbf{h}(N, K, L)$  of impulse response coefficients is given in Eq. (10). The constant is  $H_0(4, 7, 1) = 5040$ .

$$h(4, 7, 1) = \left\{ \begin{array}{l} 1, 6, 20, 49, 98, 169, 259, 359, 455, 531, 573 \\ 573, 531, 455, 359, 259, 169, 98, 49, 20, 6, 1 \end{array} \right\} \quad (10)$$

The normalized constant for proposed filter class is smaller than that of classical CIC FIR filters. Also, impulse response coefficients of proposed filter class have smaller values in comparison to those of classical CIC filters.

## 5.2. New Filter Functions and their Properties

In Table 1, passband and stopband cut-off frequencies,  $f_{cp}$  and  $f_{cs}$ , maximum attenuation in the passband, chosen as  $\alpha_{max} = 0.28\text{dB}$ , minimum attenuation in the stopband,  $\alpha_{min}$  [dB], and group delay,  $\tau(f)$  [s], of the classical CIC filter functions  $H(N, K, z)$  from Equation (2), are given. These parameters, as well as stopband cut-off frequency  $\hat{f}_{cs}$  and attenuation in the stopband area  $\hat{\alpha}_{min}$  [dB] of the novel CIC filter functions  $H(N, K, L, z)$  from Equation (5), are listed in Table 2. The novel CIC filter functions have two peaks in the transition area of the classical filter (on frequency between the passband and stopband cut-off frequencies) as can be seen in Figure 2. The attenuation of the lowest peak in the stopband region is assigned as  $\alpha_{min}$  [dB] and given in Table 2. Note that the attenuations of the novel functions in the stopband area  $\hat{\alpha}_{min}$  [dB] given in Table 2 are higher than the attenuations of classical CIC filter in the stopband area  $\alpha_{min}$  [dB] given in Table 1. Filter functions are compared under the fair conditions: the same number of constant group delay and the same number of cascaded sections  $K$ .

The relative difference of passband cut-off frequencies in [%] is defined as ratio

$$\Delta f_{cp} = [(f_{cp\_CIC} - f_{cp\_novel\_class}) / f_{cp\_CIC}] \cdot 100 \quad (11)$$

and obtained to judge the improvement of the frequency cut-off frequencies are  $\Delta f_{cp} = 5.25\%$  for  $L=1$ , i.e.  $K=7$ ,  $\Delta f_{cp} = 5.51\%$  for  $L=2$ , i.e.  $K=14$ , and  $\Delta f_{cp} = 5.52\%$  for  $L=3$ , i.e.  $K=21$ .

<b>K</b>	$f_{cp}$	$\alpha_{max}$	$f_{cs}$	$\alpha_{min}$	$(f)$
7	0.00894	0.28	0.13354	86.979	17.5
14	0.00632	0.28	0.13354	173.958	35.0
21	0.00516	0.28	0.13354	260.937	52.5

Table 1. Cut-off frequencies in passband and stopband, constant group delay and stopband attenuation of classical CIC filter for  $K \in \{7, 14, 21\}$  and  $N=6$

$K$	$f_{cp}$	$f_{cs}$	$\alpha_{min}$	$\hat{f}_{cs}$	$\hat{\alpha}_{min}$	$\tau(f)$
7	0.00847	0.10956	80.855	0.13727	91.758	17.5
14	0.00599	0.10956	161.709	0.13727	183.516	35.0
21	0.00489	0.10956	242.564	0.13277	275.274	52.5

Table 2. Cut-off frequencies in passband and stopband, constant group Delay and stopband attenuation of novel CIC filters for  $K \in \{7, 14, 21\}$  (obtained for  $L \in \{1, 2, 3\}$ ),  $N=6$  and  $\alpha_{max} = 0.28\text{dB}$

In order to illustrate clearly the achieved improvements of the new class, the normalized magnitude response characteristics in dB of the new functions and classical CIC filters are summarized in Figure 1. It can be concluded that the attenuation in the stopband region is closely related to the parameter  $L$ . By increasing  $L$  for the constant value of  $N$ , the higher stopband

attenuation is achieved.

## 6. Comparison Among Filter Functions

This kind of modified CIC filters has been widely studied in the literature in [2]-[8]. A detailed comparison of novel filter functions with the selected modified CIC filters from literature is carried out. The CIC FIR filter functions which closed-form design

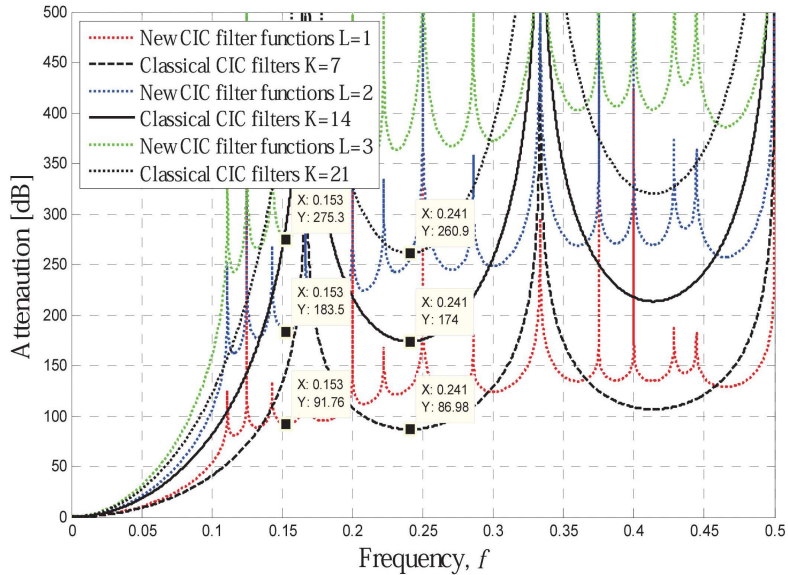


Figure 1. Magnitude response characteristics in dB versus normalized frequency  $f = \omega / (2\pi)$  of novel CIC FIR filter functions and those functions of classical CIC filters for  $N = 6$ , and different values of parameter  $L \in \{1, 2, 3\}$ .

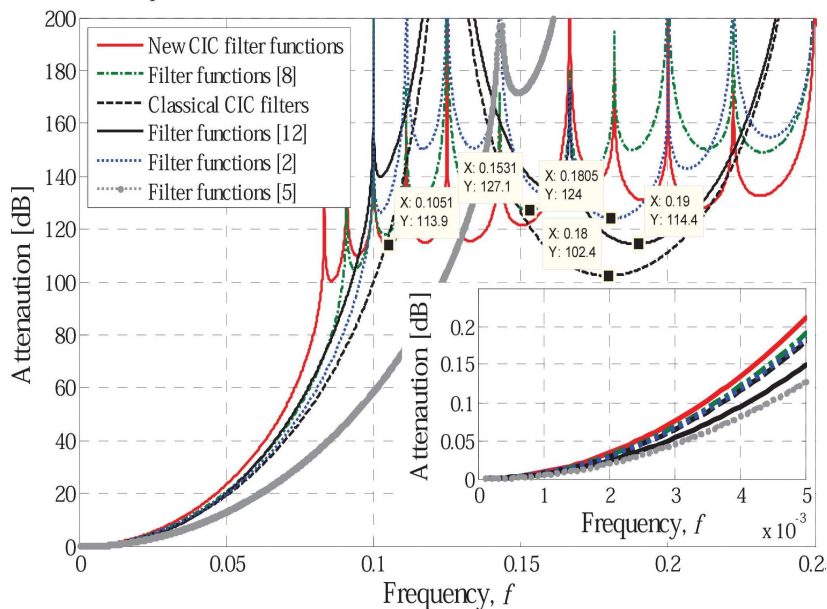


Figure 2. Normalized magnitude response characteristics in dB (dashed black lines - classical CIC filter for  $K = 8, N = 8$ ; solid red lines - novel filter functions for  $N = 9, K = 7, L = 1$ ; green line - filter functions given in [8] for  $N = 8, L = 1, K = 8$ ; black lines - filter functions given in [12] for  $M = 8, K = 8, N_1 = 3, N_2 = 5$  and  $b = 0$ ; dashed blue line - filter functions given in [2] for  $N = 8, L = 1, K = 8$ ; dashed gray line - filter functions given in [5] for  $N = 5, L = 2, K = 14$ )

equations are presented in [2], [5], [8] and [12] are designed to compare feature of that class with the new CIC FIR filter functions presented in this paper. Each class is designed for specified group delay  $\tau = 28s$ . Comparison is summarized in Figure 2.

Technique proposed in [12] includes passband droop compensator and stopband improvement filters. The new filter functions show higher selectivity in the transition area. The proposed filters have bigger attenuation in the stopband area without added additional filter for improvements versus solution given in [12] where stopband improvement filter is included.

In order to implement the design forms the following steps are to be followed: 1) choose constant group delay value, 2) identify free filter parameters for the chosen group delay, and 3) based on the form (3) or (4), design new filter class with the chosen parameters.

### 7. Influence of Added Filter Blocks

Influence of cosine prefilter and compensator blocks to frequency response characteristics of novel CIC FIR filter functions is investigated here.

#### 7.1. Compensator Influence

Like classical CIC filters, new designed CIC FIR filter functions exhibit a significant passband droop in the passband of interest, as can be seen from the passband detail given in Figure 2, what is usually undesirable in many applications. Hence, it is of great interest to get a flat passband in some way, i.e. by connecting CIC compensator [12] in cascade with the original filter.

#### 7.2. Compensator and Prefilter Influence

The proposed filter structure is the cascade of the cosine prefilter structure (3), the new filter functions (6), and the compensator filter(4):

$$H_p(z) = H_{\cos}(z^N) \cdot H(N, K, L, z) \cdot G(z^N) \tag{12}$$

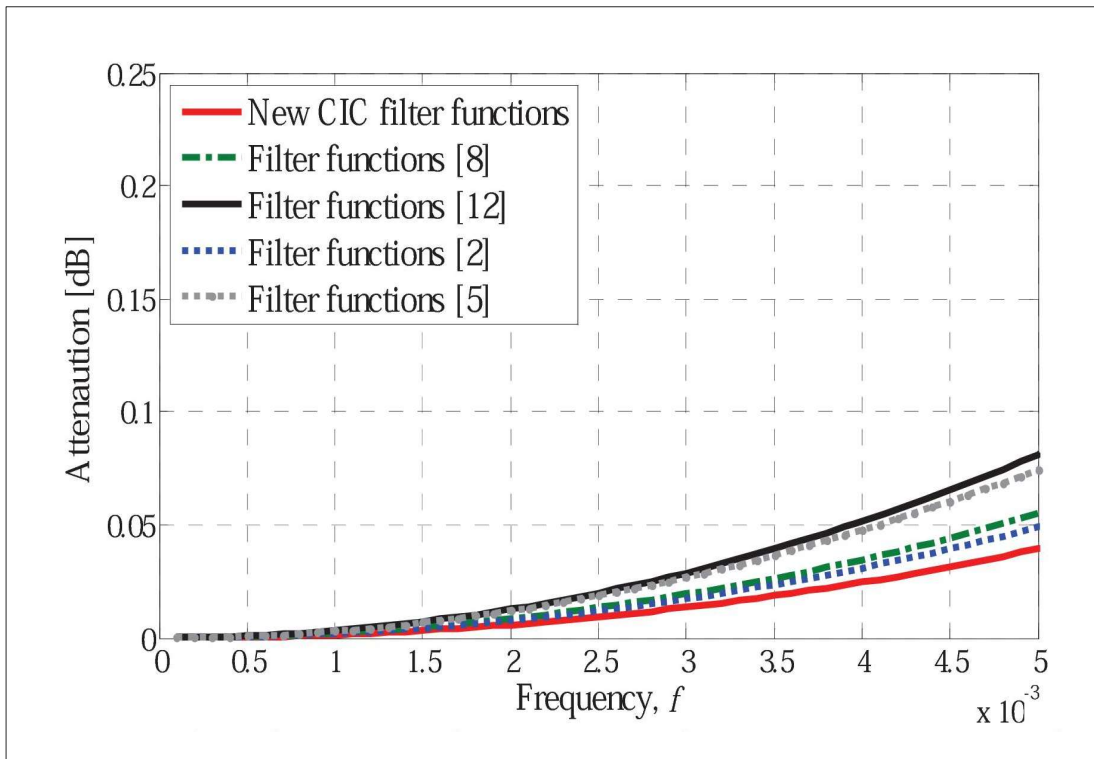


Figure 3. Normalized magnitude response characteristics in passband in case with added compensator  $b = 0$

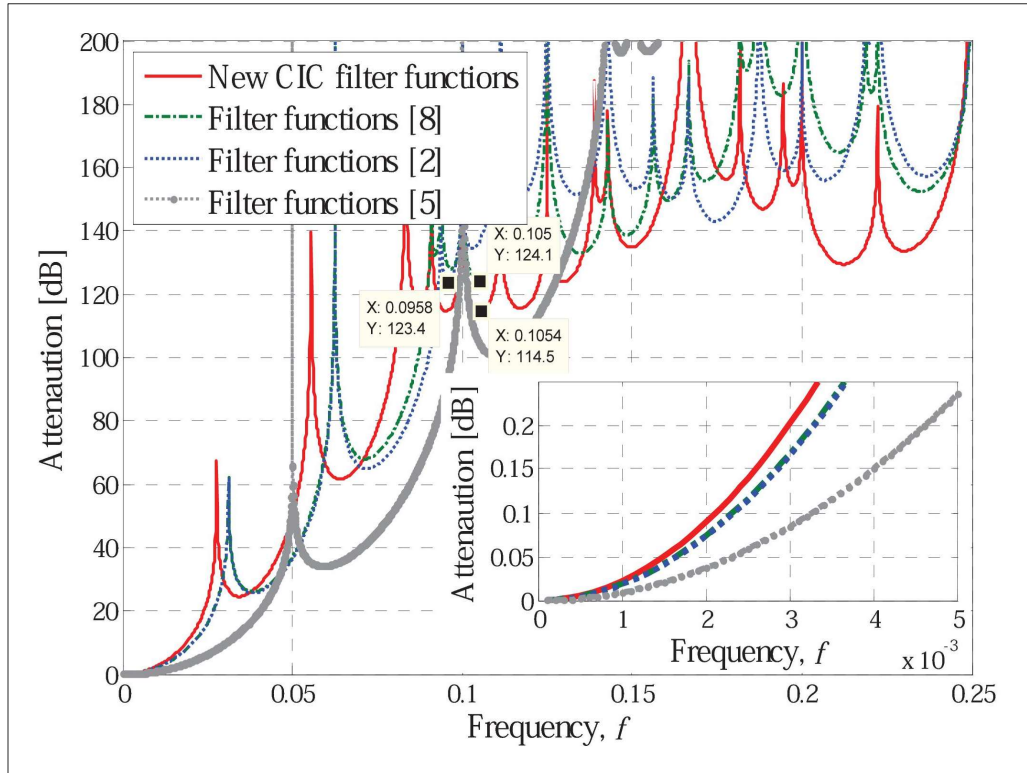


Figure 4. Filter functions with added cosine prefilter and compensator and their normalized magnitude response characteristics in dB (solid red lines - novel filter functions for  $N = 9$ ,  $K = 7$ ,  $L = 1$ ; green line - filter functions given in [8] for  $N = 8$ ,  $L = 1$ ,  $K = 8$ ; dashed blue line - filter functions given in [2] for  $N = 8$ ,  $L = 1$ ,  $K = 8$ ; dashed gray line - filter functions given in [5] for  $N = 5$ ,  $L = 2$ ,  $K = 14$ ), and  $b = 0$ .

Figure 4 demonstrates the improvement in the stopband and the increased passband droop of the resulting filter. The proposed filter has a very little increased attenuation in the folding bands. However, the passband exhibits a high passband droop. The passband characteristic of the proposed filter structure is inferior to that of novel CIC filter functions.

### 7.3. General Conclusion

In this kind of modified CIC filter functions, the cosine prefilter is not required in order to improve the magnitude characteristic. Novel CIC filter functions exhibit high stopband attenuation of themselves. Compensator block can be used if better passband characteristic is required.

## 8. Concluding Remarks

This paper briefly introduced the history and classical CIC filters. Also, the paper investigates and proposes a new CIC (cascade of integrator and comb filters) digital filter with presumably advantages such as higher stopband attenuation, smaller impulse-response coefficients and better passband characteristics.

On the basis of the main idea of CIC filter, in this paper, a modified CIC linear phase FIR filters are designed. For the novel filters, filter coefficients are computed by some closedform equations, and the frequency response characteristics are also studied. Some performance comparison results are presented. Design examples for proposed filter are given and performance comparisons are given as well. These new designs can be useful, for example, in digital communications and sigma delta converters.

The phase response doesn't change by cascading CIC filter sections of different lengths. Group delay is a constant value. These cascade connections of non-identical CIC sections are therefore very useful for modifying the magnitude frequency response characteristics of filters without changing the group delay of classical CIC filters.

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