Deep Learning Optimization with MNIST and AutoEncoder Data sets

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ABSTRACT: *Optimization algorithms are extensively used in machine learning where optimization techniques are deployed. With the use of deep learning, optimization approaches are widely used with the development of new features in Stochastic Gradient Descent to convex and non-convex and derivative-free approaches. The deep learning models can able to produce systems with speed and final performance that will improve the convexity principles. Highly enhanced optimizers can increase the complexity of the depth and data sets become larger that require good optimization. Thus, in this paper, we have studied the most used optimizer algorithm in a practical way. We have experimented it in MNIST and AutoEncoder data sets. We tested in a variety of applications that can document the common features and differences and suitability of applications. Further, we have presented new variants of optimizers. Finally, we are able to find the better optimizer with the help of extensive analyses.*

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I. Introduction

Deep Learning (DL) algorithms are indispensable with statistical computations for higher efficiency when the datasets size increases. Interestingly, one on the pillar of DL is mathematical tactics of optimization process which makes decision based on previously unseen data. This is accomplished by parameters carefully chosen (near to optimal solution intuitively) for a given learning problem. *The hyper–parameters are the parameters of a learning algorithm and not of a given mode*l. Evidently, the inspiration is to look forward to the optimizing algorithm which works well and predict accurately [1, 2, 3, 4]. In ML, the study of text classification has been undertaken by many as it provides the fundamental problem of learning from examples. Similarly, speech and image recognition have been dealt with great success and accuracy – yet offers the place for new improvements. In

achieving higher goals, use of various optimizing techniques involving convexity principles are much more cited [5, 6, 7] now a days and using logistic and other regression techniques. Moreover, the Stochastic Gradient Descent (SGD) has been very popular over last many years, but also suffers from ill-conditioning and also taking more time to compute for larger data sets. At times it also requires hyper-parameter tuning and different learning rates adaptively.

1.1. Background and Related work

Deep Learning (DL) has made a strong pathway in all areas of engineering applications and has generated keen interest due to its closeness to human cognition. Machine Learning (ML) has taken deep roots in addressing human life problems including healthcare, sociology, economic, logistics and alike. So now we have intelligent products and services, e.g., speech recognition, computer vision, anomaly detection, game playing and many such areas. The ever changing tools and techniques of ML have a widening impact to diagnose diseases, autonomous drive, animated movie making, smart recommended systems and many more. To background work is owing to "optimizer and regularization methodologies"— ranging from Gradient Descent (GD) to Stochastic GD to Momentum based optimizers [8]. Meanwhile, the convex and non-convex theory of optimization is covered briefly and one can refer more on these topics in cited references [9, 10]. In fact the whole convex optimization methods are relaxed now a day with conditioning and suitable niche algorithms are investigated to find the most probable accuracy results and thus less execution time and more result oriented.

The research study seeks a thoughtful process to answer most of the following questions and encompass the gamut of equilibrant to address the issues and challenges. The pertinent ones are:

- How to measure the optimal performance? What are those attributes which compares them using various techniques in deep learning?

- How non-convex methods could be transformed into convex methods?

- How derivative free algorithms are getting importance while reducing computational time?

- How hyper-parameter tuning is done for optimization techniques?

2. Optimization and Role of Optimizer in DL

To control the output from layers, we assess them against expected. The closeness between them is what is desired as expectation. This is the work of the *loss function* of the network- aka, objective function [1, 3, 4]. It takes the predicted value and compares against the true target, i.e., what we wanted as actual output and calculates the difference, suggestive of our performance on this specific input data set, as depicted in Figure 1.

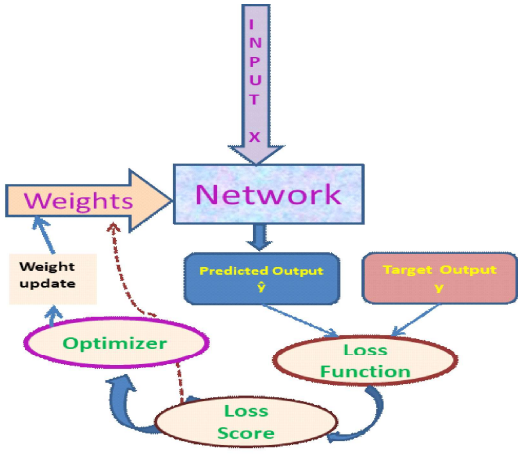

Figure 1. An Optimizer Framework

The basic principle utilizes this scored difference as a pointer to regulate the value of the weights accordingly leading towards lesser loss score. This adjustment is the function performed by '*optimizer*', which augments what is conventionally known as back propagation algorithm [11, 12, 13, 14, 15].

2.1. Optimization Issues

The criticalities of optimization issues in DL are fairly intricate and a pictorial representation is in Fig. 2 with enumeration as:

(i) Making the algorithm starts run and converging to a realistic solution.

(ii) Making the algorithm to converge fast and speed up convergence rate.

(iii) Ensuring convergence with a stumpy value–like global minimum.

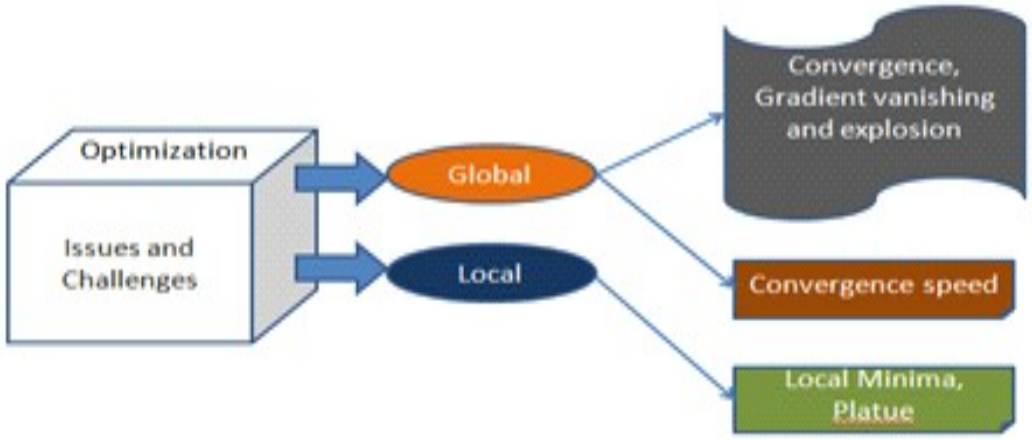

Figure 2. Optimization: Issues and Challenges

2.2. Stochastic GD Optimization

Ironically, SGD closely follow the gradient of a mini-batch selected at random. While training a network, we estimate the gradient using a suitable loss function. At an iteration '**k**', the gradient will be updated accordingly. Hence, the calculation for '**m**' examples input from the training set having $y(i)$ as target, is:

$$\hat{g} \leftarrow \frac{1}{m}\nabla_\theta \sum_i L(f(x^{(i)};\theta), y^{(i)})$$
$$\theta \leftarrow \theta - \eta\,\hat{g} \tag{1}$$

here 'η' (eta) is learning rate. Further, the learning rate is of paramount importance as the magnitude of an update at 'k^{th}' iteration is dictated by this. For instance, if $\eta = 0.01$ (very small to small), then evidently more number of iteration updates will be required for convergence. On the contrary, if $\eta = 0.5$ or more, then in this case the recent updates shall be highly dependent on the recent instance. Eventually, an obvious wise decision is to select (choose) it arbitrary by trial —this is one very important hyper-parameter tuning in DL systems. On the parallel side of it, yet another way could be 'choose *one among several learning rates*' which provide smallest loss value.

2.3. Stochastic Gradient Descent with Momentum

From the preceding section and paragraphs, it is obvious that SGD has a trouble to get towards global optima, and has a tendency to get stuck into local minima, as depicted in Figure 3.

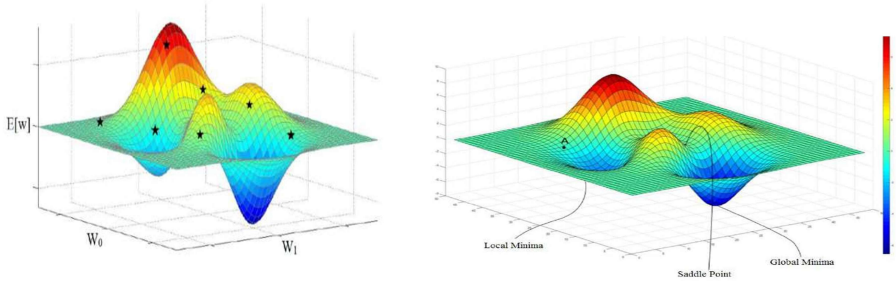

Figure 3. A 3-D Representation with Local and Global Minima (Maxima)

Moreover, smaller values of gradient or noisy ones can create another problem of vanishing gradient issue! To overcome this problem, the method involving momentum (a principle borrowed from physics) is adopted to accelerate the process of learning. The momentum method aims to resolve 2 very important issues:

(i) variance in SGD

(ii) variance when solving Hessian Matrix for poor conditioning

The method takes the brevity of running moving average by incorporating previous update in the recent change as if there is a momentum due to preceding updates.

The momentum based SGD will converge faster with reducing oscillations. To achieve this, we use another hyper-parameter '**v**' known as velocity. This hyper-parameter tells the speed and of course the direction by which its moves in the given parameter space. Usually '**v**' is set as negative of gradient value of exponential decaying average. Moving further on, we would require yet one more hyper-parameter á (alpha): $\alpha \in [0, 1]$, known as momentum parameter and its contribution is to find how fast the previous gradient exponentially decays. The new (updated) values are computes as

$$v \leftarrow \alpha v - \epsilon \frac{1}{m} \nabla_\theta \Big(\sum_{i=1}^{m} L(f(x^{(i)}; \theta), y^{(i)}) \Big) \qquad (2)$$

$$\theta \leftarrow \theta + v$$

From equation (2) it is obvious that the velocity vector '**v**' keeps on adding the gradient values. Moreover, for a bigger value of α (alpha) relative to ε, the gradient affects the current direction more from previous iteration. The commonly used values of α: from 0.5 to 0.99. Despite being so intuitive and nice technique, the limitation of this algorithm is additional parameter inclusion and extra calculations involved.

3. Various Optimizers in DL Practitioner Scenario

The DL practice is more of the optimization and regularization process. The currently available optimizers with their process framework are briefly described with their relative merits and limitations. Each one has some peculiarities or the other and an insight of those will an exemplary thought progression.

3.1. AdaGrad Optimizer

The simplest of optimizing algorithms to begin with is **AdaGrad**, where the algorithm's names itself suggest, the algorithm adapts, i.e., dynamically changes the learning rate with model's parameters Here, for parameters whose partial derivative are higher (larger) for them decrease their corresponding learning rate substantially. Contrary to this intuition, the algorithm takes inversely to where derivatives are lower. A natural question to ask is '*why one needs different learning rates'?* So to accomplish these characteristics, **AdaGrad** employs square value of the gradient vector using a variable '**r**' for gradient accumulation, as stated in following equation(3).

$$g \leftarrow \frac{1}{m}\nabla_\theta \sum_i L(f(x^{(i)};\theta), y^{(i)}) \tag{3}$$

Using this equation (6) the square of gradient is collected and subsequently the update of parameters is computed by a scaling factor $\boldsymbol{\delta} + \sqrt{\mathbf{r}}$, where δ is a very low value constant for numeric stability. The update applied as per the following equation (4) now:

$$r \leftarrow r + g \odot g$$

$$\Delta\theta \leftarrow -\frac{\epsilon}{\delta + \sqrt{r}} \odot g \tag{4}$$

$$\theta \leftarrow \theta + \Delta\theta$$

Here $\odot$ operator implies element-wise multiplication of vectors. As can be inferred from above equations, when '**r**' is close to a 'near-zero' value, the term in the denominator should not be evaluated as 'NaN = Not A Number' and thus the term ä helps to avoid this to happen. Also, the term '**ε**'stands for global learning rate.

3.2. RMSProp

The modified version of **AdaGrad** is **RMSProp** – **R**oot **M**ean **S**quare **Prop**ortional [16]. In order to alleviate the problems of **AdaGrad,** here we recursively define a decaying average of all past gradients. By doing so, the running exponential moving average at each time step depends only on the average of previous and current gradients. It performs better in the non-convex setting as well with same characteristics features. Comparison wise, **AdaGrad** contracts the learning rate according to the entire history of the squared gradient whereas **RMSProp** exploits an exponentially decaying average to discard history from the extreme past such that it can converge quickly after finding a convex bowl. The equation to implement is:

$$r \leftarrow \rho r + (1-\rho) g \odot g \tag{5}$$

here ρ is the decay rate. Then parameter update is computed and applied as follows:

$$\Delta\theta = -\frac{\epsilon}{\sqrt{\delta + r}} \odot g$$

$$\theta \leftarrow \theta + \Delta\theta \tag{6}$$

3.3. Adam

Adam (**Ada**ptive **M**omentum) one majorly used optimization algorithms in DL and joins the heuristic of both the momentum and **RMSProp** and interestingly been designed for deep neural nets [17]. This algorithmic technique has the squared gradient feature of **AdaGrad** and to scale the learning rate analogous to RMSProp and feature of momentum using moving average. The fine algorithm calculates individual learning rate for each parameter using a term called '*first moment*' (similar to velocity vector) and '*second moment*' (similar to acceleration vector). A few salient features are :

— Momentum term is in-built as an estimate of first-order moment

— In-built bias correction while estimating for first and second order moments - sometime called as initialization at origin (start point)

— Update moving exponential averages of gradient '$\mathbf{m}_t$' and square gradient '$\mathbf{u}_t$' with hyper-parameters ρ1 and ρ2 (in original paper by the authors, they are denoted by β1 and β2, respectively) as these control the cited decaying rates.

These moving averages are estimation of mean (first moment) and uncentered variance (second moment) of gradient. Moving ahead in the pipeline process, at time step '*t*', the various estimates are as follows:

$$m_t \leftarrow \rho_1 m_{t-1} + (1-\rho_1) g_t$$

$$u_t \leftarrow \rho_2 u_{t-1} + (1-\rho_2) g \odot g \tag{7}$$

Then, bias is corrected in first and second moments. By using the corrected moment estimates parameter updates are calculated and applied:

$$\hat{m}_t \leftarrow \frac{m_t}{1-\rho_1^t} \qquad \hat{u}_t \leftarrow \frac{u_t}{1-\rho_2^t}$$

$$\Delta\theta = -\epsilon \frac{\hat{m}_t}{\sqrt{\hat{u}_t} + \delta} \tag{8}$$

$$\theta_t \leftarrow \theta_{t-1} + \Delta\theta$$

From [28], the default values are: **ρ1 (β1) = 0.9** and **ρ2 (β2) = 0.999** and **δ = 10 $^{-8}$**. Adam works quite well in deep learning scenarios and is one of the most favored adaptive learning-method algorithms.

3.4. Yogi Optimizer

One of the problems of Adam is that it can fail to converge even in convex settings when the second moment estimate blows up. As a fix [xxx] proposed a refined update optimizer called "Yogi" whose analysis is shown with respect to other optimizers [18].

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4. Experimental Analysis

Simple MNIST ConvNet (CONVolutional neural NETwork) Handwritten Digit Classification (keras.io)

The MNIST dataset contains 60,000 small square 28×28 pixel grayscale images of handwritten single digits between 0 and 9 and to classify them into one of 10 classes representing integer values from 0 to 9.

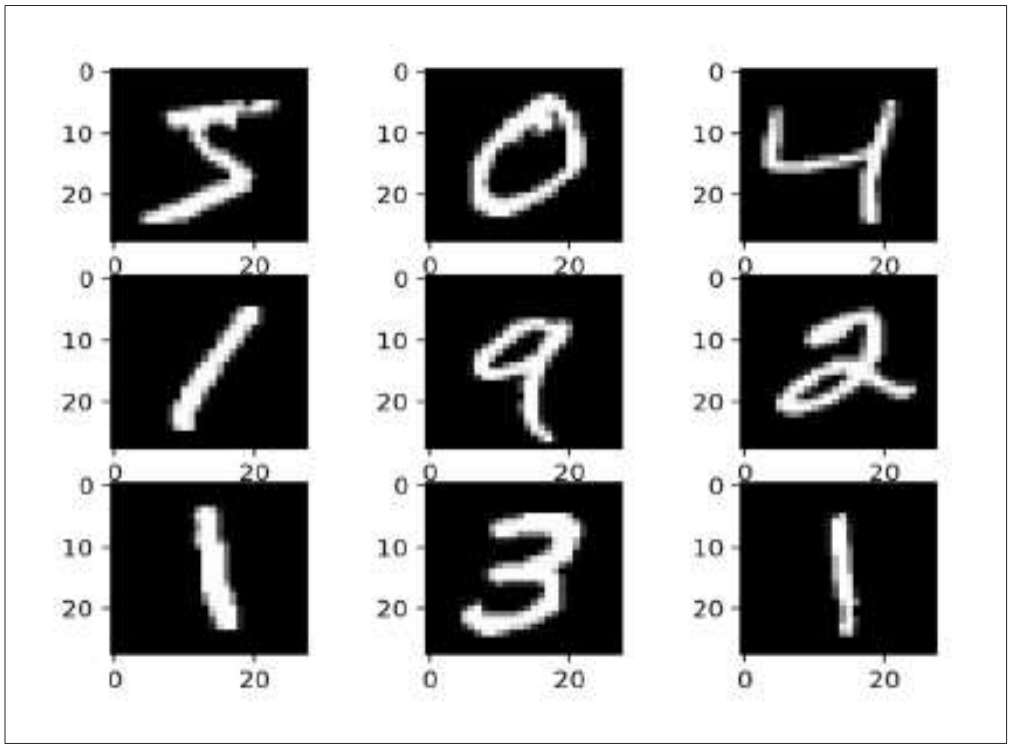

Figure 4. MNIST Hand-writing data

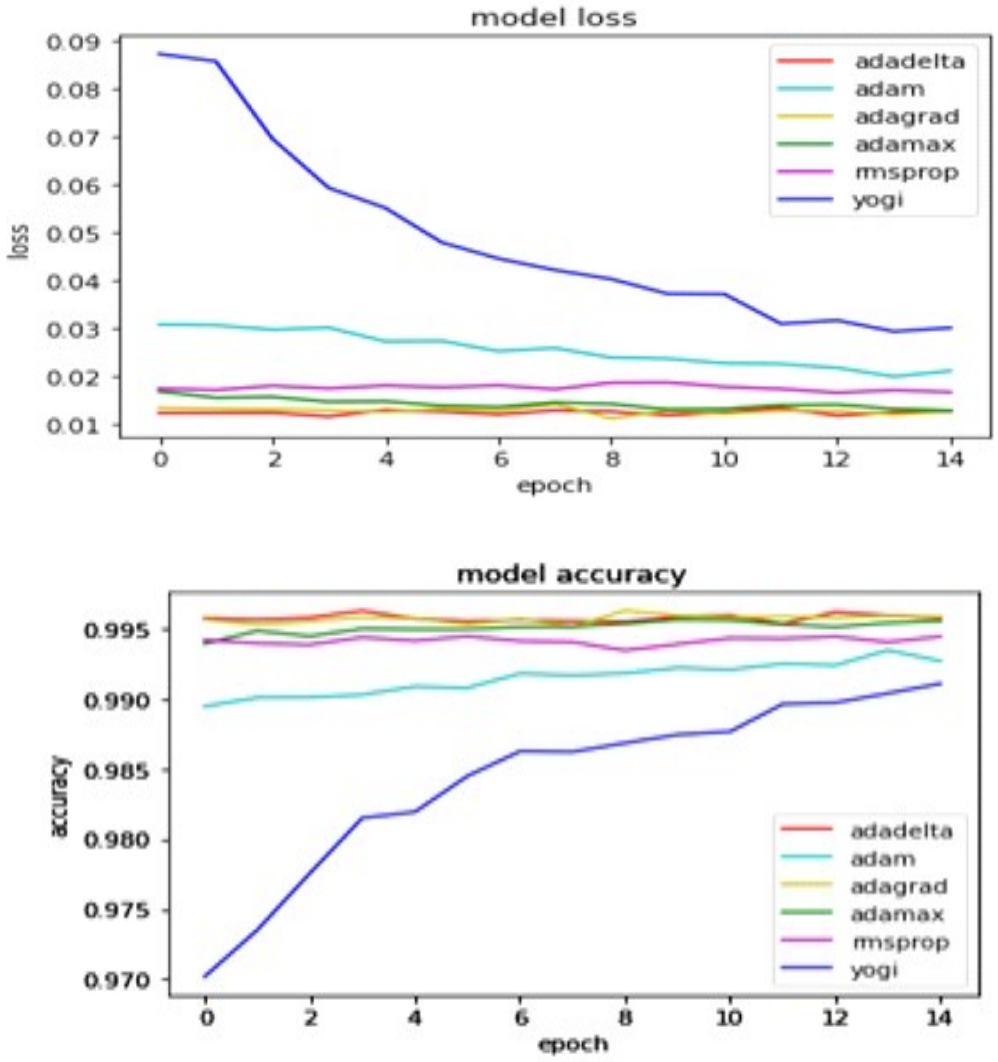

Figure 5. A comparative graph depicting various optimizers

5. Discussion and Conclusion

As can be visually analysed from various graphs from the experimental analysis, some salient points emerges:

— A good start is to have technique that usually works well with ReLU and tuning of a few hyperparameters

— Start with a simple dataset having fewer (easier) samples with a simple network, and after obtaining promising results gradually increase the complexity of both data and network while tuning hyperparameters and trying regularization methods.

— Training with random initialization is first choice and later one can go with pre-trained model to speed up

— Begin with well-known optimizer to get results and trying out with each will provide a reconnaissance and might lead to improvement. The correctly chosen optimizer with learning rate and other parameters makes a good deal.

— *Selecting a Particular Optimizer*: Choose a well-understood optimizer with default learning rates and other parameters settings. Try to change these parameters with iterations (epochs) and see the loss (accuracy) results. Subsequently, shift towards

other similar-featured optimizer and observe the changes. *This is indeed an exhaustive process*! Nevertheless, keep experimenting by increasing towards momentum based and other current blends of optimizers (like Adam, AMSGrad or RADam, etc.).

We have provided with an understanding of reasons for a particular optimizer for a given data set and its give s foundation for the pros and cons of their suitability. The most popular in DL research community are Adam and RMSProp and the results were shown as promising ones. However, this imperativeness provides an insightful for making a visionary choice of optimizers. Also, getting an overview of their criticalities and understanding the reasons for choice makes a footing platform in DL [37,38,39].

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