Measuring the Capacitance per Unit Length of Rectangular Coaxial Transmission Lines using the Fem Formulation

Vladimir V. Petrovic[1] and Žaklina J. Mancic[2]
[1]Vladimir V. Petrovic is with the Robert Bosch, GmbH
Reutlingen, Germany
vvpetf@rcub.bg.ac.rs

[2]Faculty of Electronic Engineering,
University of Niš, Aleksandra Medvedeva 14
18000 Niš, Serbia
zaklina.mancic@elfak.ni.ac.rs

ABSTRACT: *In this study we have used the FEM formulation to measure the capacitance per unit length of rectangular coaxial transmission lines with offset nonzero thickness inner conductor. We then compared the outcome of the work with that of the experimentation conducted to measure the commercial software FEM, which uses node-based first-order basis function. After concluding the whole exercise, we have drawn correct inferences.*

Keywords: Quasi-static Analysis, Finite Element Method, Strong Fem Formulation, Lines With Rectangular Cross Section, Offset Inner Conductor, Isotropic And Anisotropic Dielectric, Capacitance Per Unit Length

Received: 5 November 2021, Revised 20 January 2021, Accepted 17 February 2022

DOI: 10.6025/jitr/2022/13/2/44-51

1. Introduction

Problem of capacitance per unit length of square or rectangular lines calculation, especially lines with offset inner conductor is topical in theory and practice. The paper [1] gives a review of the literature, dealing with this task and it performs the calculation of capacitance of the rectangular coax line with offset inner conductor by using the weak FEM formulation [2]. This paper deals with calculation of capacitance per unit length of square and rectangular coaxial lines filled with isotropic and anisotropic dielectric by using strong FEM formulation [3-6]. The results are compared with those obtained by weak FEM [1] and by commercial software FEMM [6]. FEM is a very suitable method for the analysis of closed polygonal structures and it can be simply used for analysis of geometries with anisotropic dielectrics, unlike the methods that use Green's function (e.g., MoM or EEM) for which an additional complicated step of anisotropic Green's function determination is needed [7]. Besides classifying FEM into strong and weak formulation, this method can be classified as a node-based [1,6,8,9] and non node-based (with hierarchical basis functions) [2-5, 10-12]. Node-based FEM can be found much more often than non node-based FEM.

However, weak FEM formulation is usually presented in the literature, while strong formulation can rarely be found. In weak FEM formulation, only function's continuity condition is exactly satisfied, whereas in strong FEM formulation, boundary conditions for the both function and its first derivative are satisfied exactly [2-5,10-12]. In this paper are obtained for the third order basis functions ($n = 3$).

2. Brief Description of the Strong FEM Formulation

FEM approach in this paper is based on hierarchical strong basis functions of higher (arbitrary) order that are constructed by using mutual multiplication of 1D strong basis functions [13]. Consider a two-dimensional domain, uniform with respect to z-axis, Figure 1, filled with linear inhomogeneous dielectric without free charges, in which the distribution of electrostatic potential, $V(x, y)$, is the unknown function. Let the problem be of the closed type: on one part of the domain boundary (C_1), boundary conditions of the first kind (given V), and on the rest of the boundary (C_2), boundary conditions of the second kind (given $\partial V / \partial n$), are imposed (Figure 1). (Boundary condition of the second kind here is equivalent to given $D_n = -\varepsilon\, \partial V / \partial n$.) Differential equation for $V(x, y)$ can be defined with:

$$\mathrm{div}_S(\varepsilon\, \mathrm{grad}_S V) = 0\,, \tag{1}$$

In previous equation div_S and $grad_S$ denote surface divergence and gradient, respectively. Calculation domain is divided into M sub-domains (elements) in FEM solution of Eq. (1).

Exact solution $V(x, y)$ is expressed as a linear combination of basis functions with unknown coefficients,

$$V \approx f = \sum_{j=1}^{N} a_j f_j \tag{2}$$

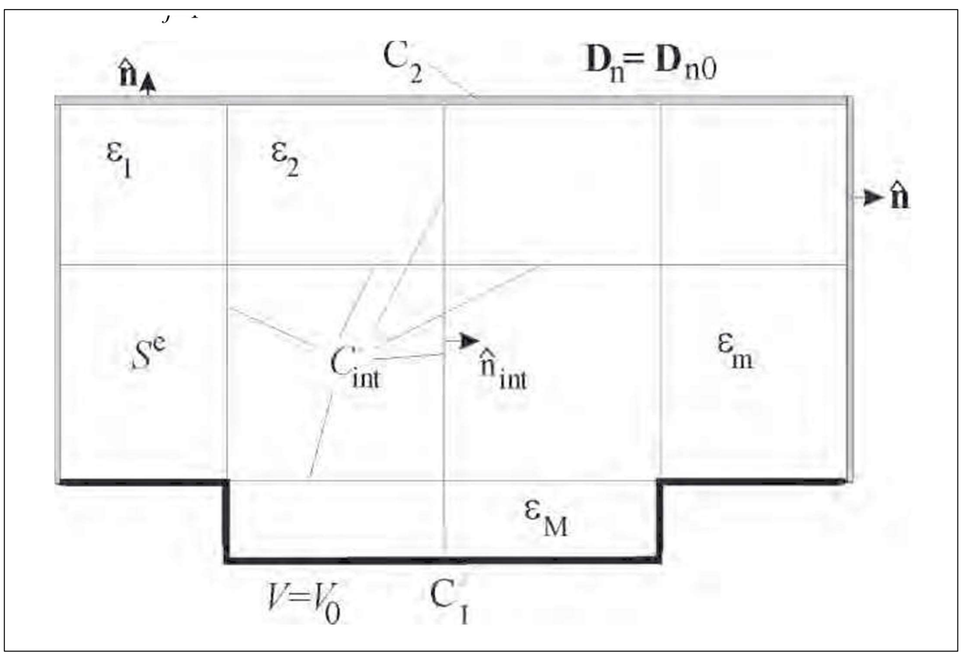

Figure 1. Two-dimensional calculation domain divided into elements

The system of linear algebraic equations for unknown coefficients is obtained by applying the weak Galerkin formulation [14, 15], and it is defined with:

$$[K_{ij}][a_j] = [G_i], \; i, j = 1, \ldots, N, \tag{2}$$

where

$$K_{ij} = \int_S \varepsilon\left(\operatorname{grad} f_i\right)\left(\operatorname{grad} f_j\right) \mathrm{d}S \; , G_i = \int_{C_2} f_i D_{n0} \, \mathrm{d}l \; . \tag{3}$$

In previous equation with D_{n0} is denoted a normal component of vector **D** on the contour C_2, whereas i and j represent global serial numbers of basis functions. Furthermore, S represents the union of all the element's surfaces, defined with $S = \bigcup_{e=1}^{M} S^e$. Next, rectangular elements of arbitrary order are utilized for strong formulation. Strong basis functions automatically satisfy continuity of potential V (C^0 continuity) and continuity of D_n (generalized C^1 continuity) on interelement boundaries (int C in Figure 1). Complete set of strong basis functions for 2-D problems in homogeneous (isotropic or anisotropic) media is presented in [13]. instead of ε, for anisotropic dielectrics it should be used $\varepsilon = [\varepsilon_x, \varepsilon_y]$ in equation (3) .

3. Numerical Examples

3.1. Square Coaxial line with Offset Inner Conductor

For a square coaxial line with offset inner conductor, Figure 2, for $b / a = 4$, results for normalized capacitance per unit length, C'/ε, are presented in Figure 3. When the inner conductor is moved from the center and positioned closer to the outer conductor, the normalized capacitance increases. The results of C'/ε in the case when $b / a = 4$ are compared with the corresponding results obtained by FEMM [6] and results obtained by weak FEM [1]. The results are shown in Figure 3 and an excellent agreement can be observed. In this case, it is not possible to exploit symmetry for the problem solution.

In all the cases the mesh that consists of 288 rectangular elements is used for strong and weak FEM. This resulted in 1152 unknowns for strong FEM and 2448 unknowns for weak FEM formulation. In order to obtain results of the similar accuracy by using FEMM software, the number of nodes (which is equal to the number of unknowns) was between 3980 and 4130 while the number of triangular mesh elements was between 7592 and 7830.

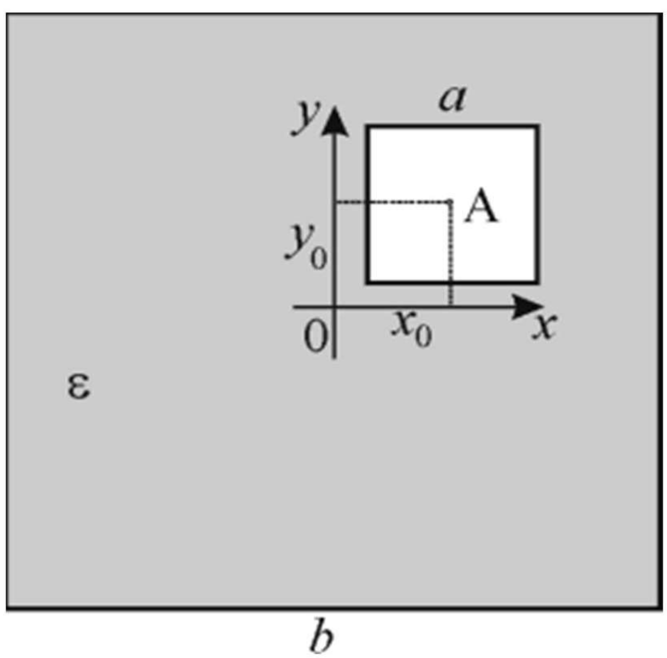

Figure 2. Square coaxial line with offset inner conductor. Coordinate origin is in the center of the outer conductor

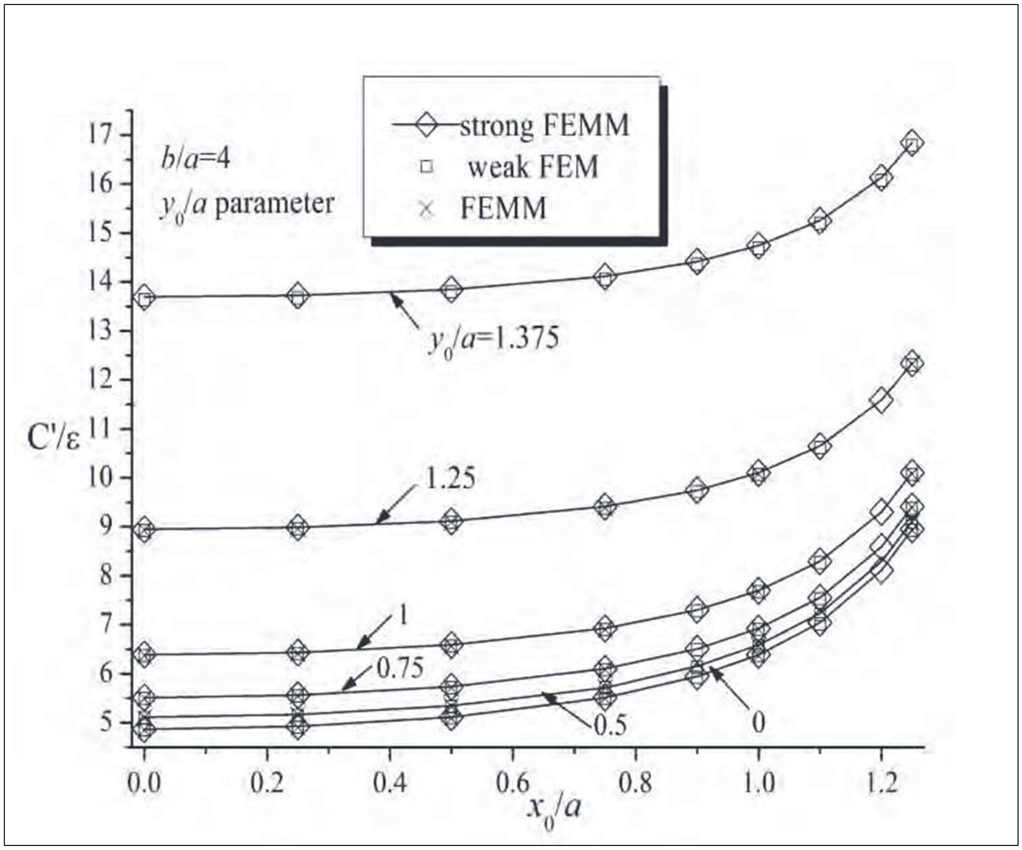

Figure 3. Ratio C' / ε depending on x_0 / a , where y_0 / a is parameter, $b / a = 4$ and dielectric is isotropic

2. Rectangular coaxial line with offset inner conductor

For rectangular coaxial line, Figure 4, C'/ ε dependance of x_0 / a is shown in Figure 5.

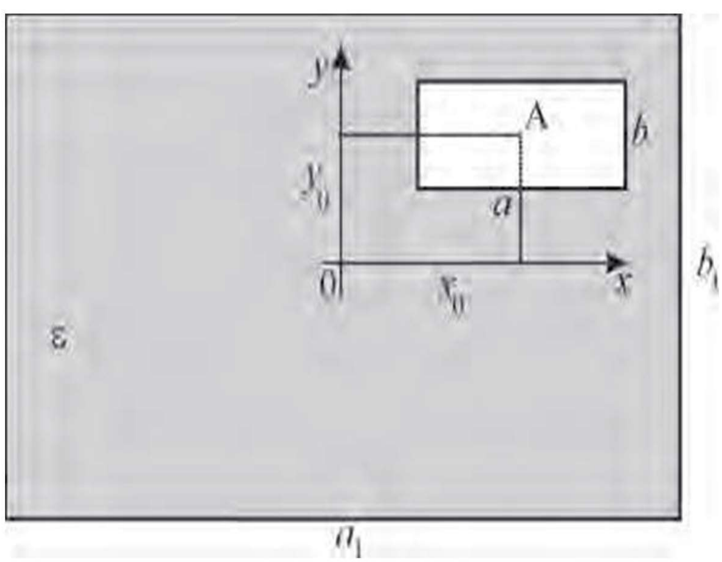

Figure 4. Rectangular coaxial line with offset inner conductor

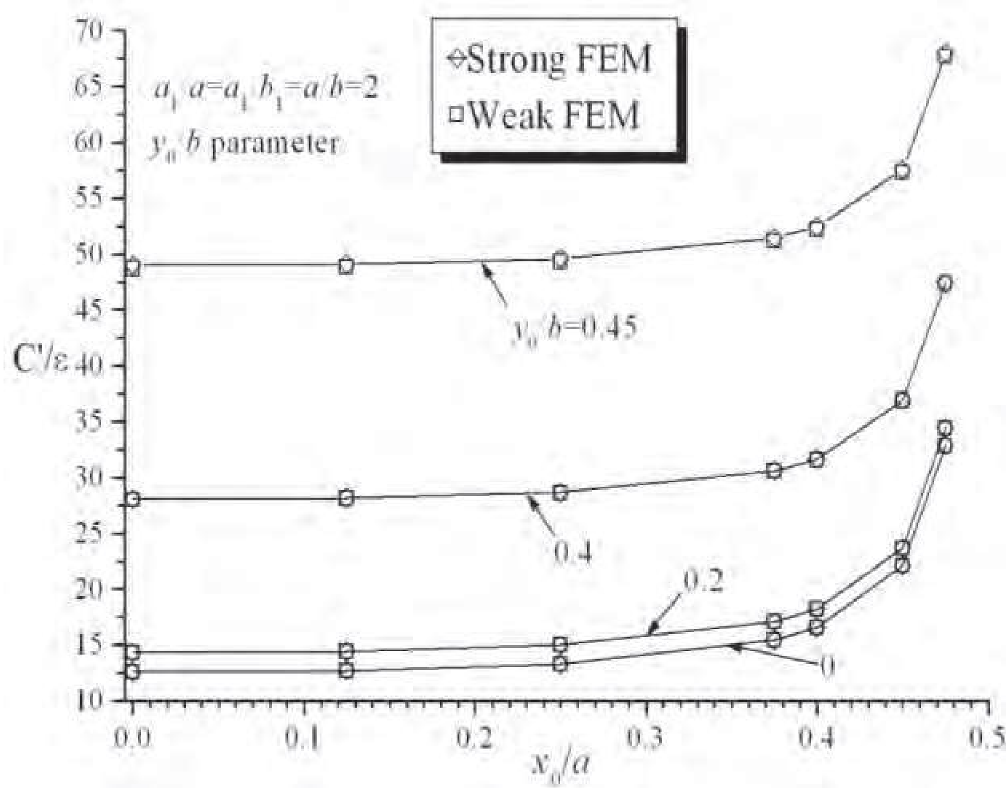

Figure 5. Ratio C' / ε depending on x_0 / a , where y / b_0 is a parameter, $a / b = 2$ and dielectric is isotropic, Figure 4

3. Rectangular coaxial line with offset inner conductor and multilayered dielectric

Figure 6 shows the structure with layered isotropic dielectric in which the inner conductor was moved in direction *t*.

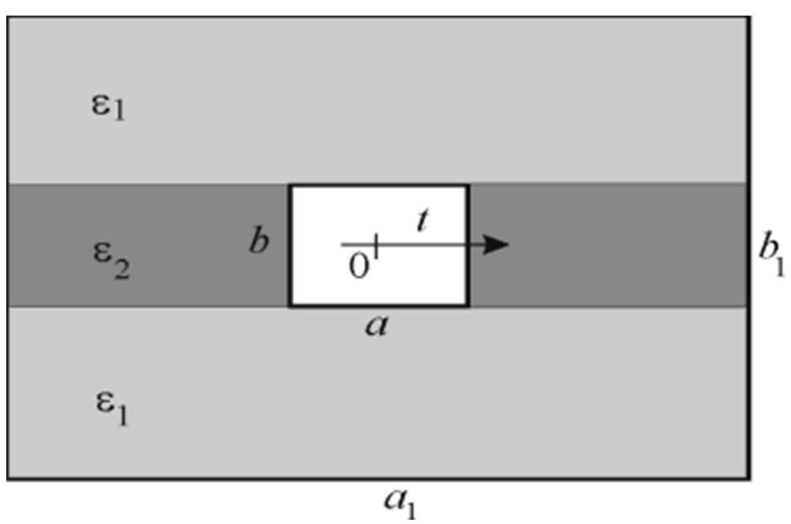

Figure 6. Rectangular coaxial line with offset inner conductor and multilayer isotropic dielectric

In Figure 7 dependence of the normalized effective permittivity $\varepsilon_e / \varepsilon_1$ on $\varepsilon_1 / \varepsilon_2$ for two different values of *t/b* for a square coaxial line from Figure 1 is shown, where $\alpha_1 / \alpha = \alpha_1 / b_1 = \alpha / b = 2$.

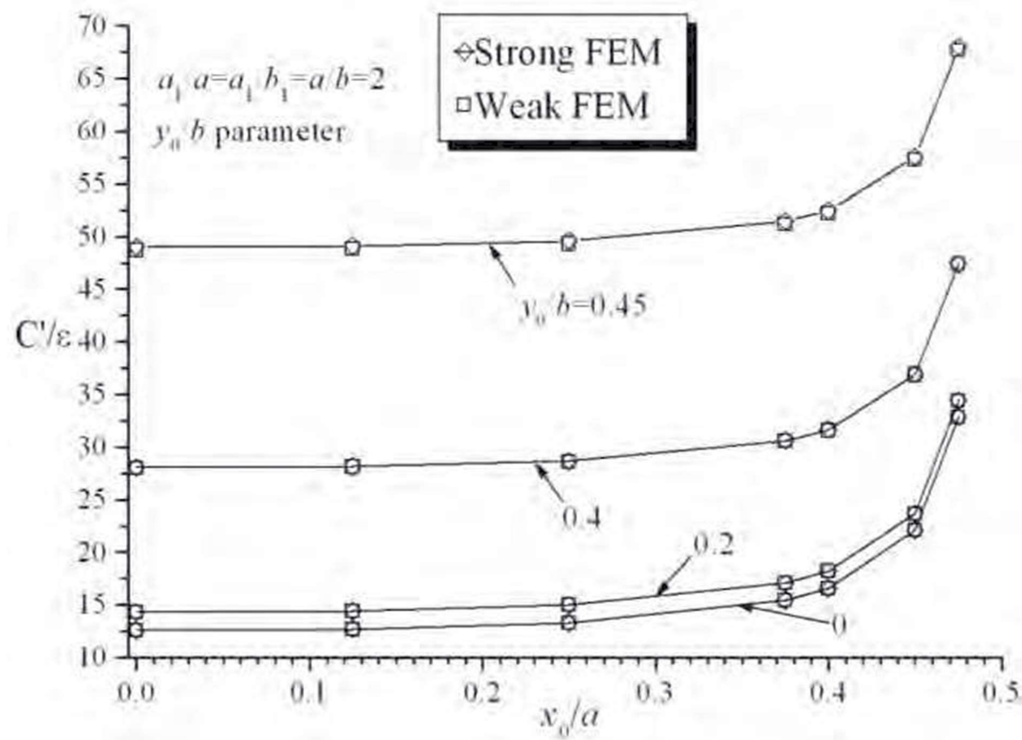

Figure 7. Normalized effective permittivity $\varepsilon_e / \varepsilon_1$ of a rectangular coaxial line with offset inner conductor and multilayered isotropic dielectric, Figure 6, for two different values of ratio *t / b*

6. Square Coaxial Line with Offset Inner Conductor and Anisotropic Dielectric

For a square coaxial line with offset inner conductor, Figure 1, for $b / a = 4$, filled with anisotropic dielectric Sapphire, where $\varepsilon = [\varepsilon_x \, \varepsilon_y]$, results for relative permittivity ε_{re}, are presented in Figure 8, for the following cases: a) $\varepsilon_x = 9.4$, $\varepsilon_y = 11.6$ and *b*). The required number of unknowns for strong FEM formulation is 1152 and for weak FEM formulation is 2448, whereas the number of rectangular elements is 288. On the other hand, FEMMrequires the number of unknowns between 3964 and 4088, whereas the number of triangular elements is between 7559 and 7804. From Figure 8 both effects of the proximity and anisotropy can be noticed, as described in detail in [2, 4, 5].

Moreover, an excellent agreement with FEMM results can be noticed, which proves that the strong FEM can be successfully applied for an accurate and efficient calculation of rectangular coaxial line with offset anisotropic dielectric.

Conclusion

Based on numerical examples shown in section III it can be concluded that the strong FEM formulation of the higher order and hierarchical basis functions can successfully be applied for accurate and efficient analysis of transmission lines with offset inner conductor of finite thickness in the case of isotropic and anisotropic dielectrics. Excellent agreement of obtained results and those obtained by weak FEM and commercial software FEMM has been observed. The advantage of strong FEM formulation compared to weak FEM is approximately one half of the number of unknowns. The advantage of both strong and weak FEM, is more than 25 times smaller number of required finite elements with respect to FEMM.

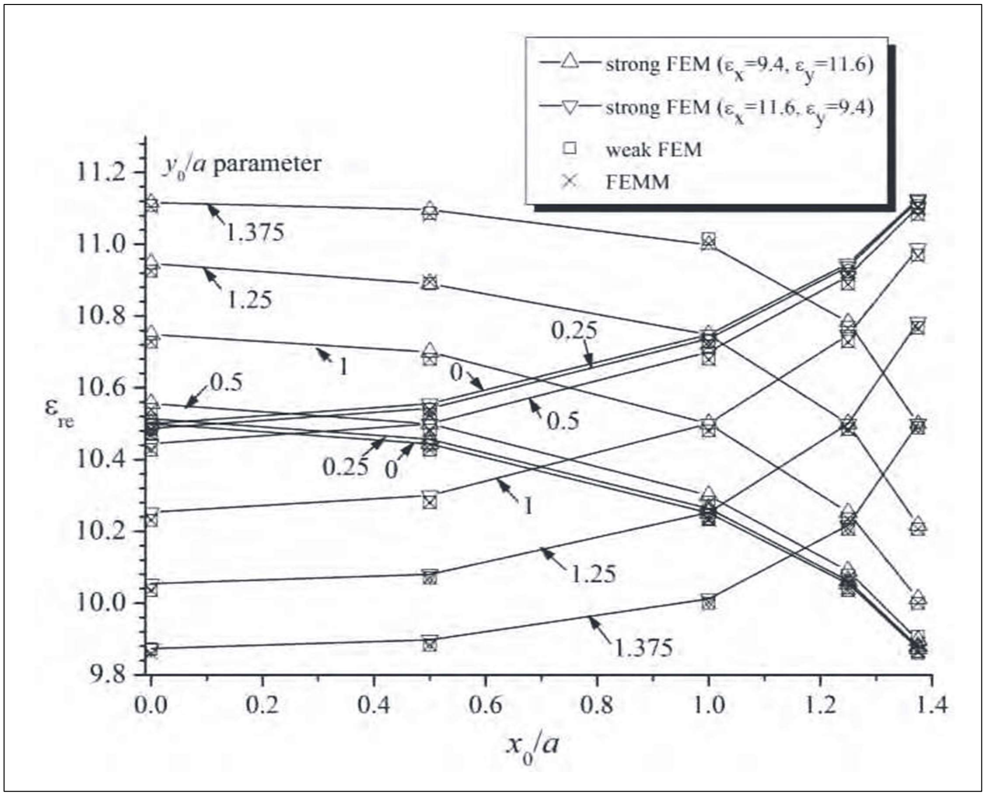

Figure 8. Effective relative permittivity ε_{re}, of a rectangular coaxial line with offset inner conductor and anisotropic dielectric Sapphire, Figure 2, for different ratios y_0 / a

Acknowledgement

This research is supported by Serbian Ministry of Education, Science and Technological Development (Project TR-32052 MNTR)

References

[1] Petrovic, V., Mancic, Žaklina J. (2015). Calculation of Capacitance of Rectangular Coaxial Line with Offset Inner Conductor bu Using Weak FEM Formulation, Telecommunication in modern Satellite, Cable and Broadcasting Services (TELSIKS), 2015, Pages 342- 345, DOI:10.1109/TELSKS.2015.7357803 51

[2] Mancic, Ž. J., Petrovi, V. V. (2011). Strong FEM Calculation of the Influence of the Conductor's Position on Quasi- Static Parameters of the Shielded Stripline With Anisotropic Dielectric, *In*: Proceedings of the ICEST conference, Niš, 2011, 191-194, (ISBN 978-86- 6125031-6).

[3] Mancic, Ž. J., Petrovic, V. V. (2011). Strong and Weak FEM Formulations of Higher Order for Quasi-Static Analysis of Shielded Planar Transmission Lines, Microwave and Optical Technology Letters (MOTL), 53 (5), 1114-1119, May 2011. (DOI 10.1002/mop.25917, online ISSN 1098-2760.

[4] Mancic, Žaklina J., Petrovic, Vladimir, V. (2015). Analysis of a square coaxial line with anisotropic substrates by strong FEM formulation, *Facta universitatis - series: Electronics and Energetics*, 28 (4) 625-636, 2015.

[5]Mancic, Ž. J., Petrovic, V. V. (2012). Strong FEM Formulation for Quasi-Static Analysis of Shielded striplines in Anisotropic Homogeneous Dielectric, *Microwave and Optical Technology Letters (MOTL)*, 54 (4) 1001-1006, April 2012. (DOI 10.1002/mop.26676.

[6] http://www.femm.info/Archives/bin/femm42bin_x64.exe

[7] Milovanovi, A., Koprivica, B. (2012). Calculation of Characteristic Impedance of Eccentric Rectangular Coaxial Lines", PRZEGLD ELEKTROTECHNICZNY (Electrical Review), ISSN 0033-2097, R. 88 NR 10a/2012. (http://pe.org.pl/articles/2012/10a/54.pdf)

[8] Pantic, Z., Mittra, R. (1986). Quasi-TEM analysis of microwave transmission lines by the finite-element method, *IEEE Trans MTT,* 34 (1986), 1096–1103.

[9] COMSOL Multiphysics Modeling Software, (www.comsol.com)

[10] Manic, A. B., Mancic, S. B., Ilic, M. M., Notaroš, B. M. (2012). Large anisotropic inhomogeneous higher order hierarchical generalized hexahedral finite elements for 3-D electromagnetic modeling of scattering and waveguide structures," Microwave and Optical Technology Letters (MOTL), 54 (7) 1644–1649.

[11] Ilic, M. M., Ilic, A. Ž., Notaroš, B. M. (2005). Efficient Large-Domain 2-D FEM Solution of Arbitrary Waveguides Using p-Refinement on Generalized Quadrilaterals, IEEE Transactions on Microwave Theory and Techniques, 53 (4) April 2005, 1377-1383.

[12] Mancic, Ž. J., Petrovic, V. V. (2014). Strong FEM formulation for 2D quasi-static problems and application to transmission lines, Invited Paper, 22nd T5l5 communictions Forum, TELFOR 2014, Belgrade, 25-27.11.

[13] Petrovi, V., Popovi, B.D. (2001). Optimal FEM solution for onedimensional EM problems, *Int. J. of Numerical Modelling* 14 (1) 49-68, Jan-Feb.

[14] Jin, J. (1993). The Finite Element Method in Electromagnetics, New York: Wiley.

[15] Harrington, R. F. (1968). Field computation by moment method, Macmillan, New York.