

Generalized Scalar Systems using Objective Programming

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ABSTRACT: In the objective programming, the issues are normally solved using many interactive models which uses the Generalized Scalar Method. The issues relating to the scalar are solved by using General Scalar Interactive Method. We have used the customized environment for implementing the methods. It is possible to modify the multi-objective issues using the interactive methods to get solutions. We also use the DM preference, using implicit ways.

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1. Introduction

In many practical problems the decision making is based on more than one objective. Unfortunately these objectives are conflicting in nature, i.e. there does not exist a solution optimizing all the objectives simultaneously. The Decision Maker (DM) must make a decision by compromise among a set of equally good solutions.

We consider the following formulation of multiple objective optimization problem:

$$\min \{f_1(x), \dots, f_k(x)\} \quad (1)$$

$$\text{subject to: } g_i(x) \leq 0; i = 1, \dots, m; \quad (2)$$

$$x \in R^n, \quad (3)$$

where x is an n -dimensional vector of variables $x_j, j = 1, \dots, n$ which accept continuous and/or discrete values. The objective functions $f_i(x), i = 1, \dots, k$ are convex. The constraint functions $g_i(x), i = 1, \dots, m$ are also convex.

The solutions of problem (1) - (3) are the so called Pareto optimal solutions [1, 2]:

Definition A solution $x^{(1)}$ is said to be Pareto optimal if and only if there does not exist another solution $x^{(2)}$ such that $f_j(x^{(2)}) \leq f_j(x^{(1)})$ for $j = 1, \dots, k$ and strict inequality holds for at least one index.

The aim of methods for solving Multiple Objective Programming Problems (MOPPs) is to assist the DM to choose one compromise Pareto optimal solution which satisfies to the greatest extent his preferences related to the values of each of the criteria. Different approaches and methods are suggested in literature [1-4]. They differ in the way the DM sets his/her preferences and in the way they scan the set of Pareto solutions. One of the most developed and widespread methods for solving MOPPs are the interactive methods [28]. This is due to their basic advantages – a small part of the Pareto optimal solutions is generated and evaluated by the DM; in the process of solving the multicriteria problem, the DM is able to learn regarding the problem; the DM can change his/her preferences in the process of problem solution.

The choice of MCDM method is not trivial and depends on a number of factors such that the type of solved problem, the DM preferences, on the logic of the solution process. Sometimes it is desirable to change the strategies for solving. In other words to solve the problem with other method(s). Therefore it is convenient to create platforms allowing that. One approach is to create generalized methods unifying several different methods - [24,25, 18]. Development of software environments based on such generalized formulations can be very useful for real decision processes.

An interactive environment developed on the base of generalized interactive method GENS-IM is presented in the paper.

2. The Generalized Method GENS-IM

The scalarising problem GENS [26] unifies the following scalarising problems: scalarising problem of weighted sum [1], Tchebycheff scalarising problem [13], scalarising problem of ε -constraints [19], scalarising problem of STEM (STEP method) [12], scalarising problem of STOM (Satisfying Trade-Off Method) [14], scalarising problem of GUESS method [15], achievement scalarising problem (Reference Point method) [4, 16], scalarising problem MRP (Modified Reference Point) [29], VIG scalarising problem [6,7], scalarising problem of reference directions RD [8, 27], RD2 scalarising problem [9], NIMBUS scalarising problem [20], DALDI scalarising problem [21, 22].

We add also NRNP - new reference-neighborhood scalarising problem [23].

The generalized scalarizing problem GENS is defined as: Minimize $T(x)$

$$T(x) = \max \left(\max_{k \in K^{\geq}} (F_k^1 - f_k(x)) G_k^1 R_1, \max_{k \in K^{\leq}} (F_k^2 - f_k(x)) G_k^2 R_2, \right. \\ \left. \max_{k \in K^<} (F_k^3 - f_k(x)) G_k^3 R_3, \max_{k \in K^>} (F_k^4 - f_k(x)) G_k^4 \right) + \sum_{k \in K^0} (F_k^5 - f_k(x)) G_k^5 + \rho \left(\sum_{k \in K^{\geq}} (F_k^1 - f_k(x)) G_k^1 + \sum_{k \in K^{\leq}} (F_k^2 - f_k(x)) G_k^2 + \right. \\ \left. \sum_{k \in K^<} (F_k^3 - f_k(x)) G_k^3 + \sum_{k \in K^>} (F_k^4 - f_k(x)) G_k^4 - \sum_{k \in K^{\sim} \cup K^{><}} f_k(x) G_k^5 \right) \quad (4)$$

subject to:

$$f_k(x) \geq f_k, \quad k \in K^> \cup K^= \quad (5)$$

$$f_k(x) \geq f_k - D_k, \quad k \in K^{\leq} \quad (6)$$

$$f_k(x) \geq f_k - t_k^-, \quad k \in K^{>>} \quad (7)$$

$$f_k(x) \leq f_k + t_k^+, \quad k \in K^{>>} \quad (8)$$

$$x \in X \quad (9)$$

where

- ρ is a small positive number;
- K is the set of objectives' indices;
- $G_k^1, G_k^2, G_k^3, G_k^4, G_k^5$ are scalarizing coefficients;
- $F_k^1, F_k^2, F_k^3, F_k^4, F_k^5$ are parameters, concerning aspiration, current or other levels of objectives' values;
- $R_1, R_2, R_3 \in \{ "+", " ", "\}$
- D_κ is the amount by which the DM agrees to deteriorate the objective value f_k ($0 < D_\kappa < \infty$);
- f_k - the current objective value of $f_k(x)$, $k \in K$;
- K^{\geq} is the set of objectives indices for which the DM wants to improve the corresponding objective values till to the given aspiration levels F_k^1 ;
- $K^{>}$ is the set of objective indices for which the DM wants to improve the corresponding objective values;
- K^{\leq} is the set of objective indices for which the DM agrees to deteriorate the corresponding objective values till to the given aspiration levels F_k^2 , i.e. $F_k^2 \geq f_k - D_k$;
- $K^{<}$ is the set of objective indices for which the DM agrees to deteriorate the corresponding objective values;
- $K^{=}$ is the set of objective indices for which the DM wants their current values not to be deteriorated;
- $K^{>>}$ is the set of objective indices for which the DM wants to change the corresponding objective values in some intervals;
- t_k^- and t_k^+ are the lower and upper bounds for the objective values of the objective $f_k(x)$ with index $k \in K^{>>}$;
- K^0 is the set of objective indices for which the DM does not set explicitly aspirations to change the corresponding objective values.

Here we assume that $K^{\geq} \neq \emptyset$ and/or $K^{>} \neq \emptyset$ and $K^0 \neq K$ without loss of generality.

By setting appropriate values to the parameters of GENS we generate the scalarizing problem of the one of the above mentioned methods.

3. The Interactive Environment

The interactive environment GENS-IM consists of the following main parts: interface module, parser module and solver module.

The interface module realizes the communication with the DM in the solution process.

The parser module is used to analyze the input data according to the developed extension of LP format in multiple objectives' case. LP file format is the native LP solve format for providing LP models via an ASCII file to the solver.

The problem is solved with solver module. The generalized scalarizing method GENS-IM is realized in the solver.

Via interface module the DM enters the problem solved or edits it later – see Figure 1. Input data format is very similar to the mathematical notation.

Also the DM sets information about the type of desired solution – integer or continuous, weak or strong Pareto optimal.

The interface module organizes also the communication with the DM to set his/her preferences in the most convenient and understandable form for him/her. He/she can choose among a set of four possible types of preferences: criteria weights, optimize one object vs. rest of others, reference points in the objective space, number of solution along a specified direction and classification of the objectives.

The initial solution could be generated automatically or the DM sets the desirable objective values. The DM could set also the assessment of the relative importance of objectives or for the desired changes of their values.

After analysis of entered information a set of scalarizing problems is suggested for generating the next (weak) Pareto optimal

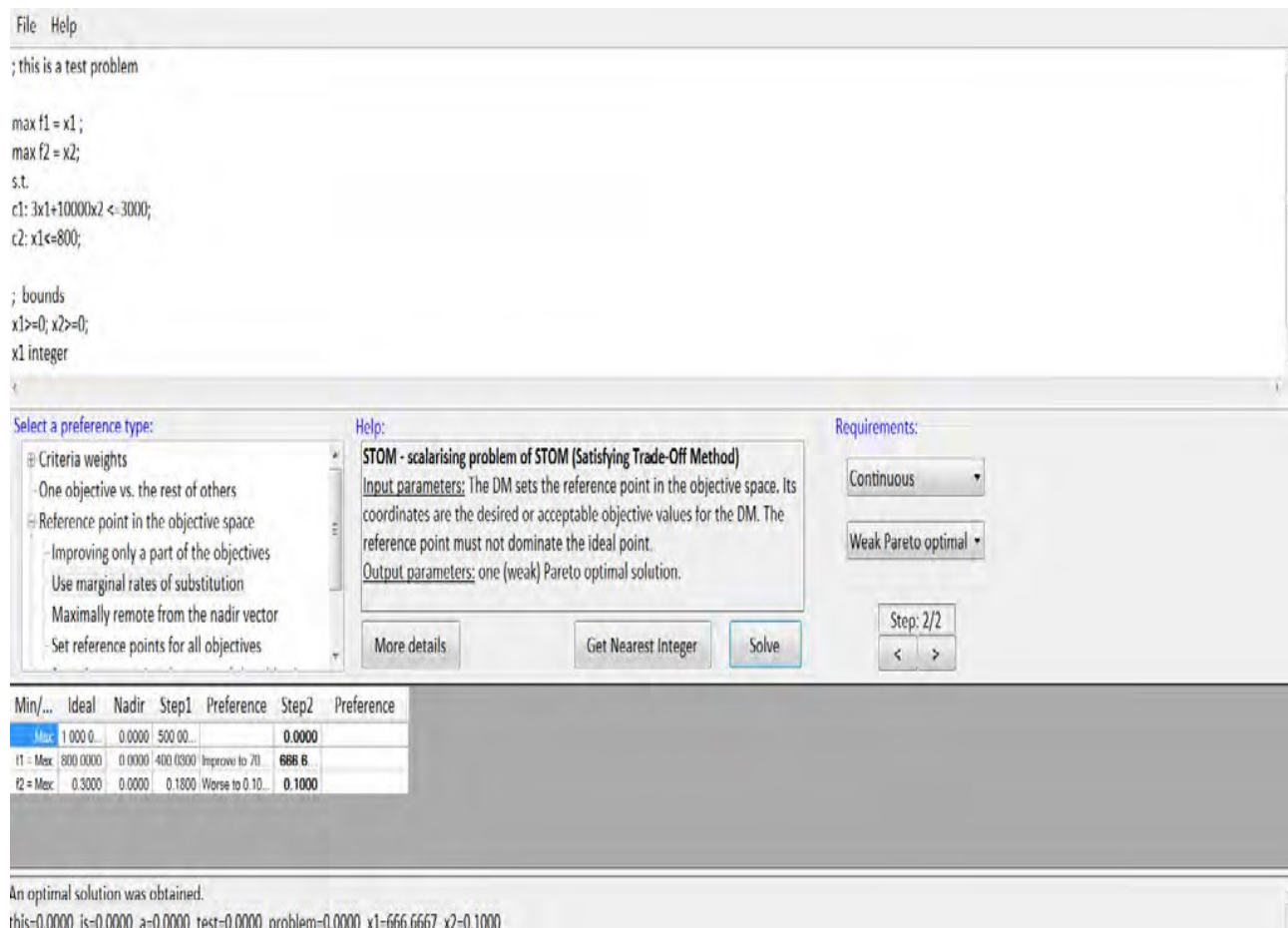


Figure 1. The interface of the interactive environment

solution (single or a defined number in advance). If the DM is not an expert in the area of multiobjective optimization, he/she can find the solution(s) by the help of the automatically suggested scalarizing problem. Usually this is the most popular scalarizing problem from the corresponding group. For example, if the DM has chosen to work with aspiration levels (reference point) he/she is suggested a set of the following scalarizing problems: Chebyshev's, STEM, STOM, GESS, Wierzbicki's, MRP. In this case the DM is offered by default a scalarizing problem of Wierzbicki's method.

The DM can save the chosen solution and to use it later as an initial solution for new problem solving with another type of preferences or new of the same type. When solving integer problems with high dimensionality the DM can start searching with continuous variables while exploring the possibilities of the problem until obtaining satisfactory objective values. After that, the closest feasible integer solution can be found or to start a new search with integer variables.

The input data and generated solutions are stored at each iteration in a XML file.

This interactive environment is written on Dot.Net. It works under Windows environment. At present it is designed to solve linear integer or continuous MOPPs.

4. Conclusion

The software environment GENS-IM is designed for users with different levels of knowledge in the area of multiple objective optimization. It has user friendly interface. The system can be used for research studies, educational training, practical implementations.

The DM can model, solve and study the properties of the problem. He/she has the possibility to use the history of the problem's solutions and this way being more confident in the final choice. The built in method GENS-IM enables switching between different methods, using different types of preferences.

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