

# Solver Optimization Tools for Water Overflow Control



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**ABSTRACT:** *Hydro power plants have cascade which are linked using the optimal operation process. The storage capacity of the hydro power plants needs organized actions that consider the level of the reservoirs which ensure to limit the overflow. We have deployed the solver optimization tool for realization. We in this paper have introduced the model for cascade operation. This model is elegant which is tested in live environment.*

**Keywords:** Optimization, Cascade Hydro Power Plants, Energy Production

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## 1. Introduction

The paper presents an approach of the daily optimal operation of three cascade connected hydro power plants. Because of the specific technical characteristics of the overall hydro system, the mode of operation of the hydro power plants should be mutually coordinated. The paper includes various modes of operation of the analyzed hydro power system which of course is dictated by the first hydro power plant, but with coordination of the reservoirs state of the both downstream hydro power plants. Figure 1 gives the overview scheme of the whole hydro energy system.

The first HPP1 has large storage capacity of 260 million m<sup>3</sup> (MCM), which can accept the water inflow for the whole year. The downstream hydro power plants HPP2 and HPP3 have small storage capacities of around 1.1 MCM. According technical characteristics, HPP1 is seasonal hydro power plant which operates in peak hours of the day, while HPP2 and HPP3 are daily hydro power plants which operate in whole day and night (24 hours) with almost full installed capacity. In order to avoid overflows and to avoid losing water coming from HPP1, downstream power plants HPP2 and HPP3 should operate during 24 hours similar as run of river power plants.

Table 1 presents the basic technical parameters for the three cascaded hydro power plants and their reservoirs.

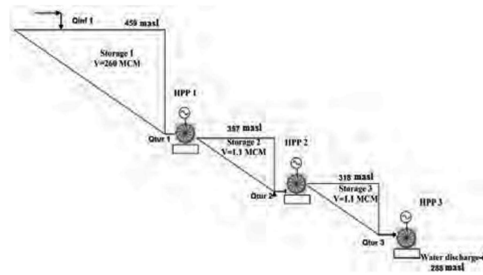


Figure 1. Hydro energy system consisted of 3 cascade power plants and storages

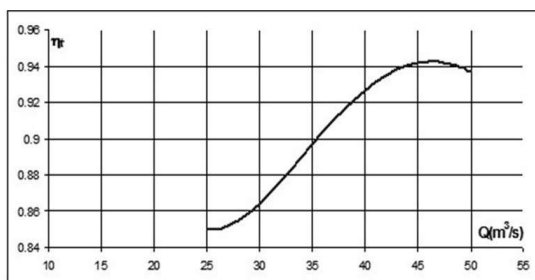
		HPP1	HPP2	HPP3
Number of units		2	2	2
Total Installed flow ( $Q_{inst}$ )	m <sup>3</sup>	100	100	40
Total Installed power ( $P_{inst}$ )	MW	85	36.4	9.6
Maximum upper level ( $H_{k,max}$ )	m.a.s.l	459	357.3	318
Minimum upper level ( $H_{k,min}$ )	m.a.s.l	432	355.0	312.0
Turbine lower level ( $H_{dv}$ )	m.a.s.l	355	313.5	287.5
Maximum gross height ( $H_{b,max}$ )	(m)	104	43.8	30.5
Minimum gross height ( $H_{b,min}$ )	(m)	77	41.5	24.5
Storage capacity (Vk)	Mil. m <sup>3</sup>	260.0	1.1	1.1

Table 1. Basic technical characteristics of three cascaded hydro power plants and their reservoirs

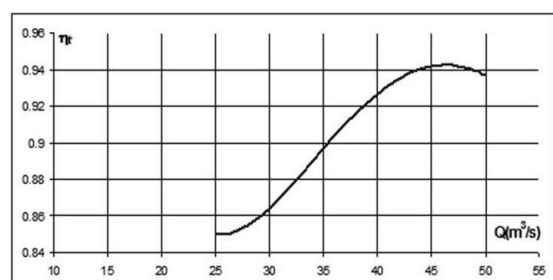
The turbine characteristic of each hydro power plant is approximated with fourth order polynomial function:

$$\eta_i(Q) = a \cdot Q^4 + b \cdot Q^3 + c \cdot Q^2 + d \cdot Q + e \quad (1)$$

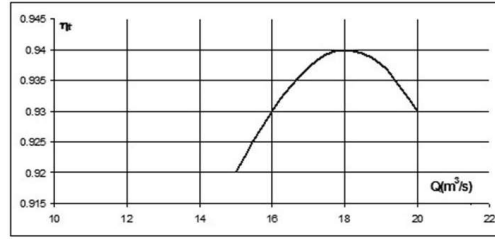
The graphical representatives for the turbine characteristic of each hydro power plant are shown on Figure 2.



HPP1



HPP2



## HPP3

Figure 2. Turbine characteristic  $\eta_t(Q)$  of the three hydro power plants

## 2. Mathematical Model of Daily Operation of the Cascade Hydro Power Plants

In order to find the optimal solution of operating regime for the whole cascade hydro system, a mathematical model has been developed for the system. The input parameters in the model are the following ones: installed flow ( $Q_{inst}$ ), hydraulically losses, minimum and maximum turbine flow, starting upper level of the reservoirs ( $i = 1, 2, 3$ ), minimum ( $H_{k,min}$ ) and maximum ( $H_{k,max}$ ) upper level of the reservoirs, altitude of the turbines ( $H_{dv}$ ), storage capacity (useful volume of water) of the reservoirs ( $V_k$ ), turbine flow of upper HPP1 ( $Q_{tur,1}$ ), characteristic  $\eta_t(Q)$  of the turbines in the hydro power plants, period of high and low tariff ( $T_{high}+T_{low}=24$ ) and corresponding price.

For the calculations the following assumptions are taken: operating of HPP1 in peak hours of the day with high tariff and avoiding overflow of the reservoirs of HPP2 and HPP3.

The program gives the following output results for each hour of a daily operation ( $t=1, \dots, 24$ ): operating regimes for HPP2 and HPP3, turbine flow ( $Q_{tur,2}(t), Q_{tur,3}(t)$ ) for HPP2 and HPP3, storage level ( $H_{gk}(t)$ ) and water volume for the reservoirs of HPP2 and HPP3, power output of all HPPs, electrical energy generation in high and low tariff and financial benefits of operating regime.

Hydraulically connection can be taken as the relation:

$$Q^{i+1}_{RESin}(t) = Q^i_{HPPtur}(t) \quad (2)$$

$$V^i_{RES} = V^i_{RES0} + \sum_{T=24h} Q^i_{RESin}(t) \cdot \Delta t - \sum_{T=24h} Q^i_{HPPtur}(t) \cdot \Delta t \quad (3)$$

where:

$Q^i_{RESin}(t)$  is the water inflow for the reservoir ( $i$ ) in time interval ( $t$ ),

$Q^i_{HPPtur}(t)$  is the turbine flow of the HPP ( $i$ ) in time interval ( $t$ ),

$V^i_{RES}(t)$  is the storage capacity for the reservoir ( $i$ ) in time interval ( $t$ ),

The power output of each hydro power plant HPP ( $i$ ) can be expressed as:

$$P_g(t) = 9.81 \cdot \eta_g \cdot \eta_t(Q(t)) \cdot Q(t) \cdot (H_{gk}(t) - H_{dv} - \alpha \cdot Q(t)^2) \quad (4)$$

where  $H_{gk}(t)$  is upper level of the reservoir in time interval ( $t$ ).

The electrical energy production for whole 24 hours period of each hydro power plant, HPP ( $i$ ), is given with the following formula:

$$W = \int_{T=24} P_g(t) \cdot dt = 9.81 \eta_g \cdot \int_{T=24} \eta_t(Q(t)) \cdot Q(t) \cdot (H_{gk}(t) - H_{dv} - \alpha \cdot Q(t)^2) \cdot dt \quad (5)$$

The optimization function is representing by maximum electrical energy generation of the HPP2 and HPP3:

$$MAX \text{ function} \rightarrow W_{total} = W_{HPP2} + W_{HPP3} \quad (6)$$

Based on energy calculations, a financial analysis is made for day-night operating regimes of operation taking into account the high and low tariff price of electrical energy generation:

- High tariff price of 60 Euro/MWh for the period from 8 until 24 hour in a day,
- Low tariff price of 40 Euro/MWh for the period from 1 until 7 hour in a day.

### 3. Daily Operating Regimes for the Cascade Hydro System

Accordinging mathematical model and the technical parameters for the hydro energy system, three cases of 24 hours operating regime are analyzed:

- **Case 1** - HPP1 operates with maximum turbine flow of 100 m<sup>3</sup>/s in two separated intervals,
- **Case 2** – HPP1 operates with turbine flow less than its maximum (<100 m<sup>3</sup>/s),
- **Case 3** - HPP2 operates with same turbine flow as HPP1.

The calculation includes the bio minimum flow of hydro power plant HPP3 which is 5.6 m<sup>3</sup>/s.

#### 3.1. Case 1- HPP1 operates with maximum turbine flow of 100 m<sup>3</sup>/s

In this case HPP1 operates with maximum turbine flow of 100 m<sup>3</sup>/s in two separated intervals of the day. Figure 3 presents the turbine flow during the day of each hydro power plant.

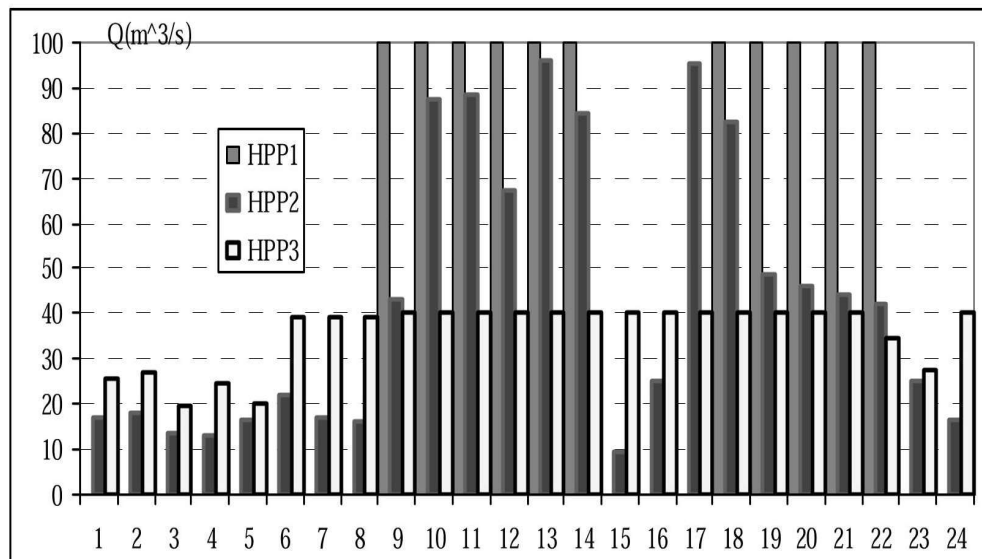


Figure 3. Turbine flow during the day of each hydro power plant, case 1

HPP1 operates with maximum turbine flow, with starting upper level of the reservoir of 455 m.a.s.l, from 9h to 14h (6 hours) and from 18h to 22h (5 hours). The starting upper level of the reservoir of HPP2 and HPP3 is 356 m.a.s.l and 314 m.a.s.l respectively. From 1h to 8h the turbine flow of HPP2 is up to 22 m<sup>3</sup>/s, then it is larger and has maximum at 13h, at 15h it is minimal (9.5 m<sup>3</sup>/s). From 18h to 24h the turbine flow of HPP2 begins to decline. From the results can be seen that from 9h to 21h the turbine flow of HPP3 is constant and equal to its total installed flow (40 m<sup>3</sup>/s). Figure 4 presents changes in upper level of the reservoir during the day of HPP2 and HPP3.

From the figure 4 it is obvious that variations of upper level of the reservoir during the day of HPP2 and HPP3 is a major, because of their small storage capacities, and practically it varies from their minimum and maximum upper level of the reservoirs. Figure 5 presents the power production of each HPP and cumulative of whole hydro system. It is clear that the largest contribution has HPP1 in both time intervals when it operates.

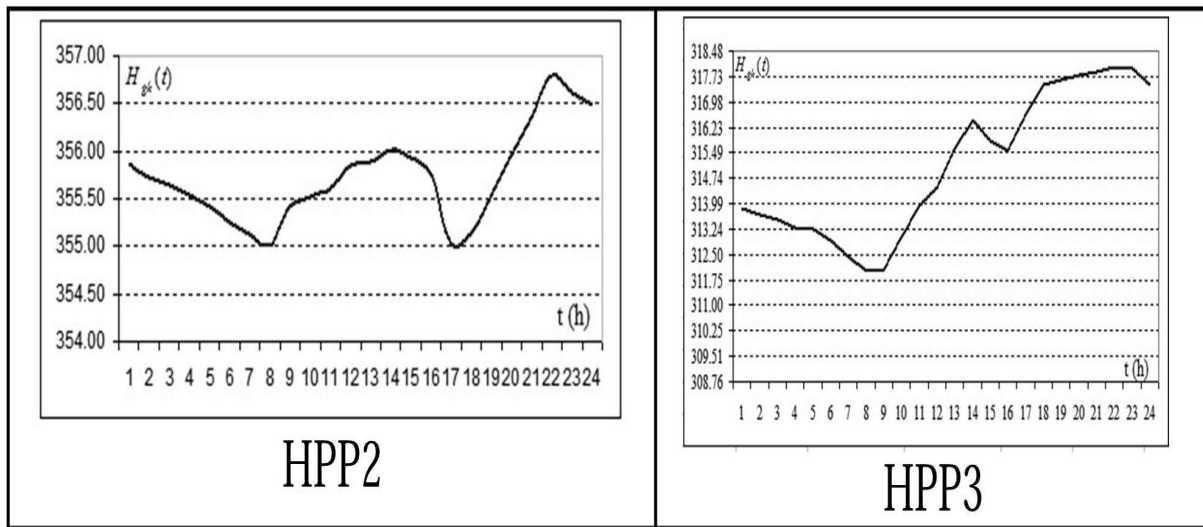


Figure 4. Changes in upper level of the reservoir during the day of HPP2 and HPP3, case 1

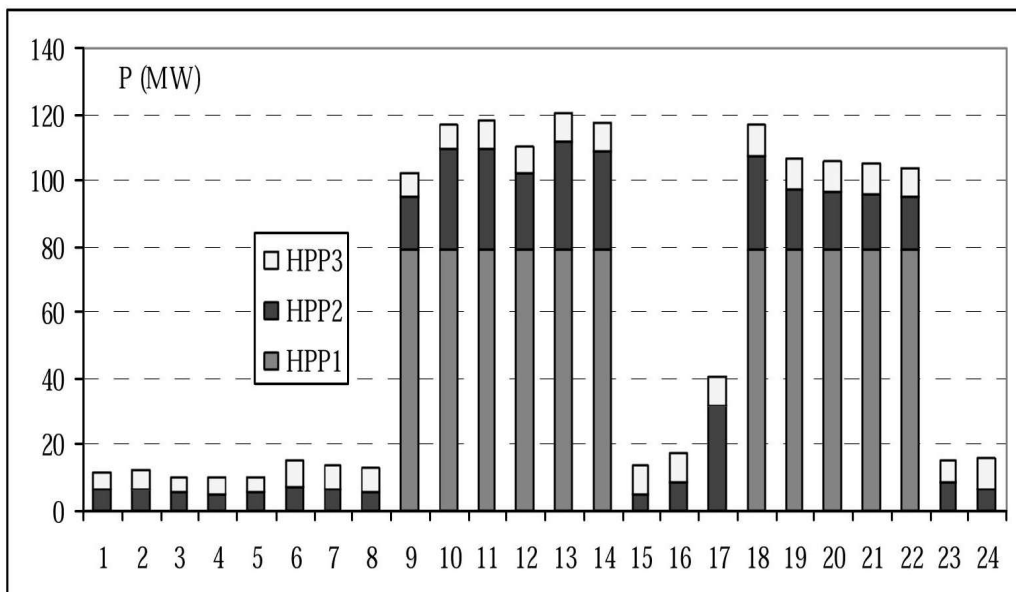


Figure 5. Power production of each HPP and cumulative of whole hydro system, case 1

**3.2. Case 2-HPP1 operates with turbine flow less than its maximum**

In this case HPP1 operates with turbine flow less than its maximum, but in a wider range of the day. HPP1 operate in one interval of the high tariff period from 9h to 22h. In order to avoid the overflow of HPP3 it operates with its maximum turbine flow from 9h to 24h. HPP2 operates with turbine flow higher than 50 m<sup>3</sup>/s from 9h to 14h and at t = 18, 24h. Figure 6 presents the turbine flow during the day of each hydro power plant in this case.

Storage level of the reservoirs of HPP2 and HPP3 is given in Figure 7. In order to provide adequate space volume of the reservoirs and to avoid overflow, first in the early hours of low tariff period to 8 h, they need to be emptied. This is in order to accept the turbine flow of HPP1 during high tariff period from 9h to 22h, where accumulations of HPP2 and HPP3 are filled near to maximum upper level of the reservoirs, and yet to avoid overflow.

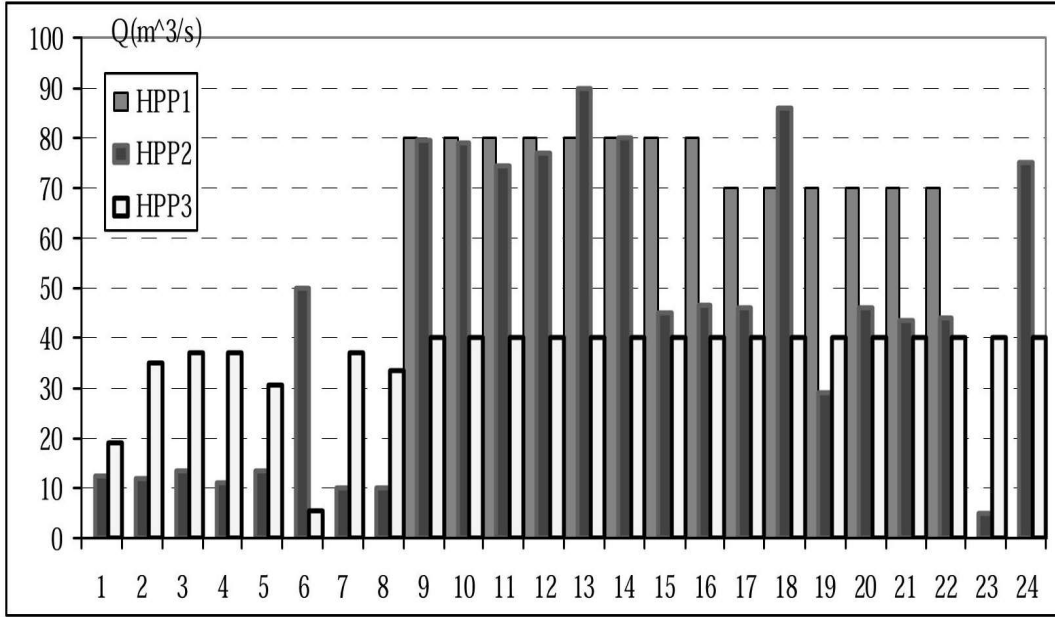
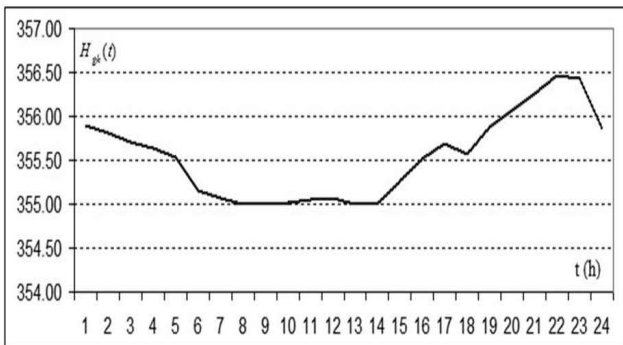


Figure 6. Turbine flow during the day of each hydro power plant, case 2



HPP2



HPP3

Figure 7. Changes in upper level of the reservoir during the day of HPP2 and HPP3, case 2

In Figure 8 the power production of each HPP and cumulative of whole hydro system during the day is given. It is clear that the contribution of HPP1 to cover the variable part of the consumption is in a wider range and up to 14 hours (from 9h to 22h).

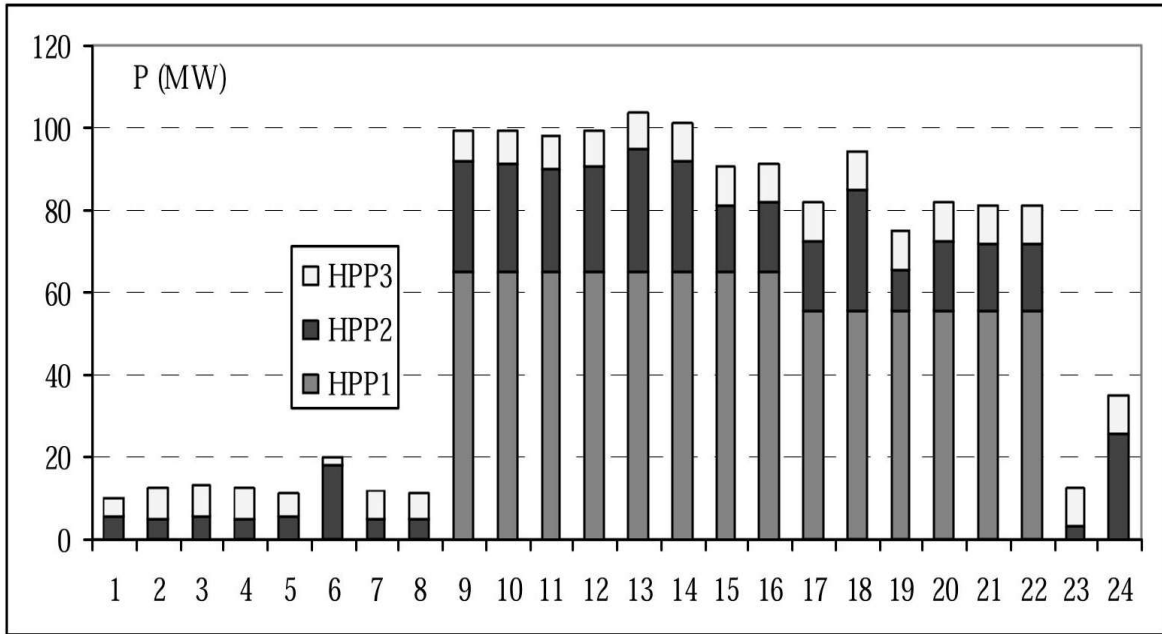


Figure 8. Power production of each HPP and cumulative of whole hydro system, case 2

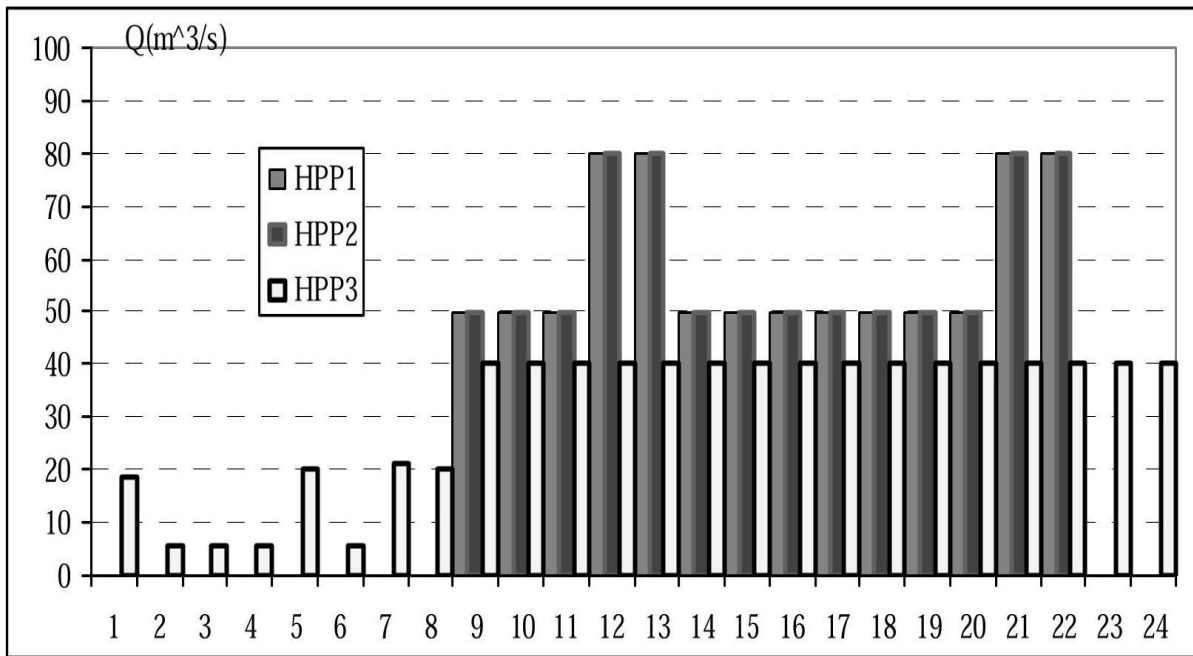


Figure 9. Turbine flow during the day of each hydro power plant, case 3

#### 4. Comparing the Results

This section give a comparison of the results and financial indicators, i.e. the profit from operating regime of the whole hydro system for all three cases. In table 2 a comparison of the electrical energy production of all three plants separately and from the whole hydro system during 24-hour operation is presented.

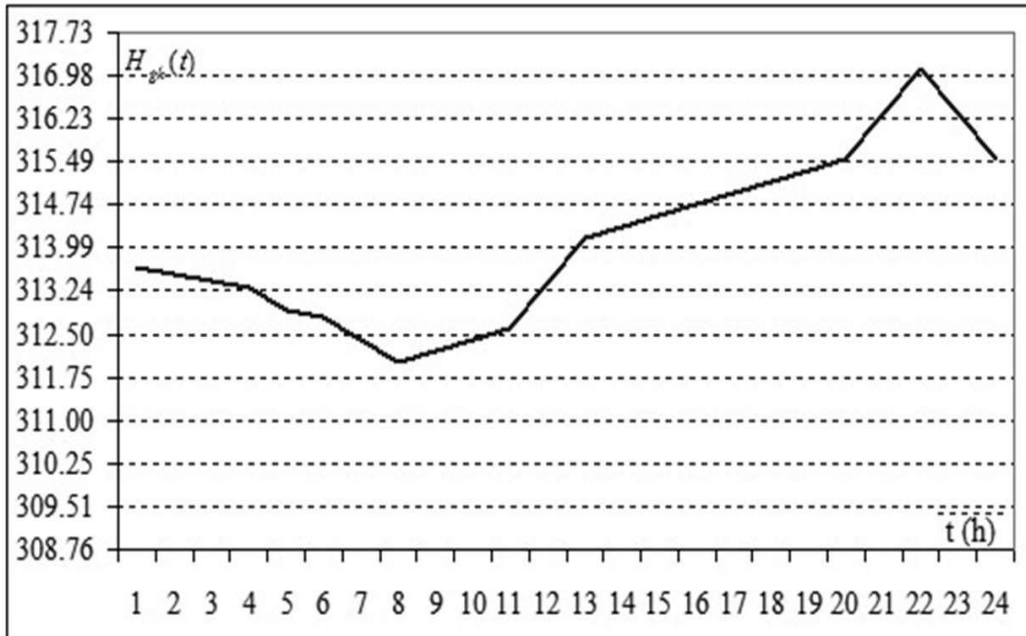


Figure 10. Changes in upper level of the reservoir during the day of HPP3, case 3

**Case 3- HPP2 operates with same turbine flow as HPP1**

In this case run of river operating regime of HPP2 is analyzed, i.e. HPP2 operates with the same turbine flow as HPP1. Figure 9 provides a graphical overview of turbine flow during the day of each hydro power plant. HPP1 and HPP2 operate from 9h to 22h. From the results it can be seen that HPP3 operates with turbine flow up to 21 m<sup>3</sup>/s in the early hours of low tariff period to 8 h and with its maximum turbine flow 40 m<sup>3</sup>/s from 9h to 24h.

Storage level of the reservoirs of HPP3 is given in Figure 10. The condition of the reservoir of HPP2 is flat. This operating regime does not allow the overflow of HPP2 and HPP3. Figure 11 presents the power production of each HPP and cumulative of whole hydro system during the day.

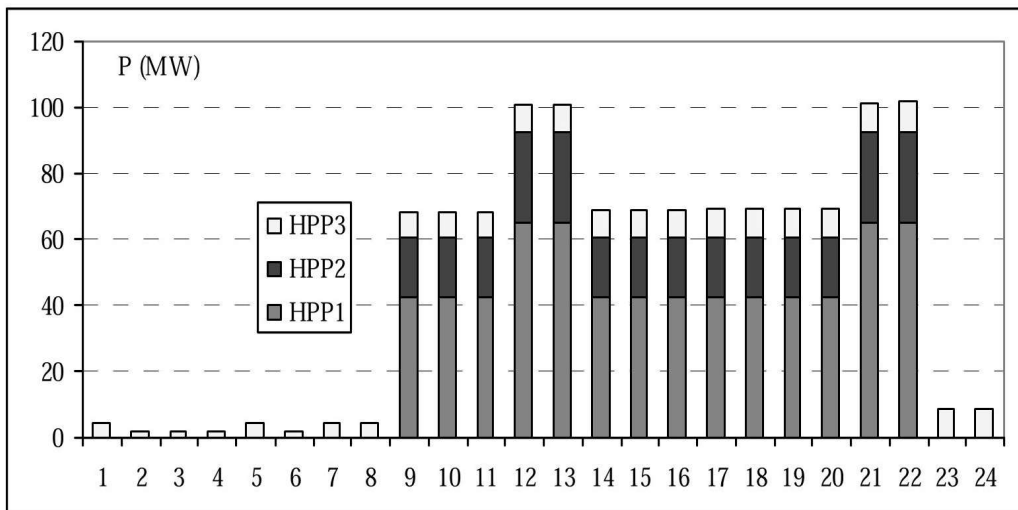


Figure 11. Power production of each HPP and cumulative of whole hydro system, case 3



W(MWh)	HPP1	HPP2	HPP3	Total
Case 1	873.23	364.68	186.55	1424.45
Case 2	852.54	383.90	192.81	1429.26
Case 3	682.82	293.36	159.18	1135.36

Table 2. Electrical Energy Production

From the results it can be seen that the greatest electrical energy production is obtained in the case 2 where HPP1 operates with turbine flow less than its maximum, but in a wider range of the day. Table 3 presents the financial benefit from electrical energy production during 24-hour operation for all three cases.

Profit (Euro)	HPP1	HPP2	HPP3	Total
Case 1	52393.54	21009.80	10390.29	83793.63
Case 2	51152.56	22042.93	10739.25	83934.74
Case 3	40969.05	17601.82	9144.26	67715.13

Table 3. Financial Profit From Electrical Energy Production

It is clear that according to the greatest electrical energy production for case 2 follows and greatest financial effect of about 83935 Euro per day.

## 5. Conclusion

The paper presents a model for daily optimal operation of hydro power system, composed of cascade connected hydro power plants with specific technical parameters, and thereby get the maximum cumulative power generation of HPP2 and HPP3 and to avoid situations of overflow of the hydro power plants. The model can be implemented on various variants of production of all three power plants. This system is also characterized by the ability to overflow occurs in the lower reservoirs of HPP2 and HPP3 when the frontal accumulation of HPP1 is full and with high water inflows, which is inevitably continuous operation of HPP1. The basic assumption for the daily operating regime of operation is to avoid overflow of the reservoirs of HPP2 and HPP3. This entails at the beginning of the daily operating regime, the reservoirs of HPP2 and HPP3 to be at their minimum upper level, or empty that would be willing to accept water from HPP1 during the high tariff period from 8h to 24h.

If HPP2 works in run of river operating regime, i.e. with the same turbine flow as HPP1, then there is no change in its upper level of the reservoir during the day. Then that would be the same operating regime for HPP3 as without HPP2. In the season of large water inflows the power operation of the system, without overflow, can be increased with relatively small losses when HPP3 operates in run of river regime with its installed flow of 40  $m^3/s$ , or spent 3.46 million  $m^3$  water per day.

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