A System to Measure the Distribution of the Voltage Measures

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ABSTRACT: *During conducted measurements the reflection is one of the most varying components of the measurement uncertainty. For studying the distribution of voltage during cable reflections, in this paper we make a simple computational odel and statistical analysis of the electric field to determine the distribution of uncertainty for the Voltage (Power) measurement. We also show, that the usually suggested U distribution does not always describe the behavior of the measured values, a beta-distribution is more proper for this purpose.*

Keywords: Reflection, EM Field, Beta Distribution, Measurement, Measurement Uncertainty

Received: 11 June 2022, Revised 1 September 2022, Accepted 13 September 2022

DOI: 10.6025/jio/2022/12/4/81-88

1. Introduction

As the RLAN devices are getting more and more widespread, their ERM measurements have to be carried out more precisely according to harmonized EN standards, e.g., [300 328]. The estimation of measurement uncertainty is an essential requirement for the test laboratories. More generally it can be said, that the standards to be applied operation of the laboratories imply that the measurement uncertainties – as a quantity qualifying the liability of the measured data – should be indicated in the test report, beside the measured values. During radio frequency measurements the measurement uncertainty components caused by reflections (mismatch) have peculiar properties compared to other, more general components.

The uncertainty contribution values attributed to reflections are more likely to be near the maximal or minimal value, than to be small value. According to ETSI ERM standards, the estimation of the uncertainty have to use the Technical report [TR 100 028-1]. According to page 27 of this Technical report, “mismatch uncertainties have the "U" distribution”, which is presented in Figure 1.

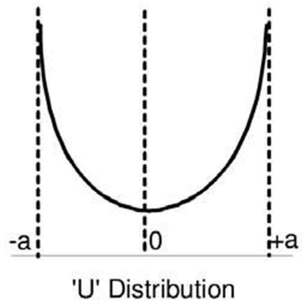

Figure 1. The effect of the reflections according to Figure 5. [TR 100 028-1 p. 27]

2. The Effect of the Reflections in Practise

As we have sown in [1], multiple reflections at a given frequency basically do not modify the field distribution in the waveguides, it causes at most of the cases a minimal amplitude and phase shift only.

Compared to the single reflected wave case the secondary and higher reflection component (Figure 2) add only an additional components to A and B with a certain phase and amplitude. (Figure 3)

Figure 2. Reflection places at non homogeneous waveguide

Using this line of thoughts we can introduce a modified reflection coefficient " for the all studied waveguide part.

$$\Gamma = \frac{B}{A} \cdot \tag{1}$$

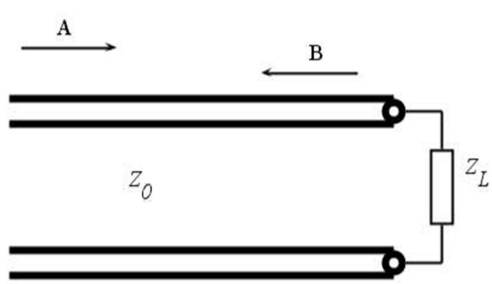

Figure 3. Reflection at waveguide

The effect resulting from the reflections at a given measuring frequency depends on the applied waveguide lengths and frequency. The frequency dependence for a given waveguide with multiple reflections can be seen in Figure 4.

Practically we can suppose that measuring frequency and the applied cable length are independent random variables during a measurement [2]. Therefore the attenuation gives a function of random variables.

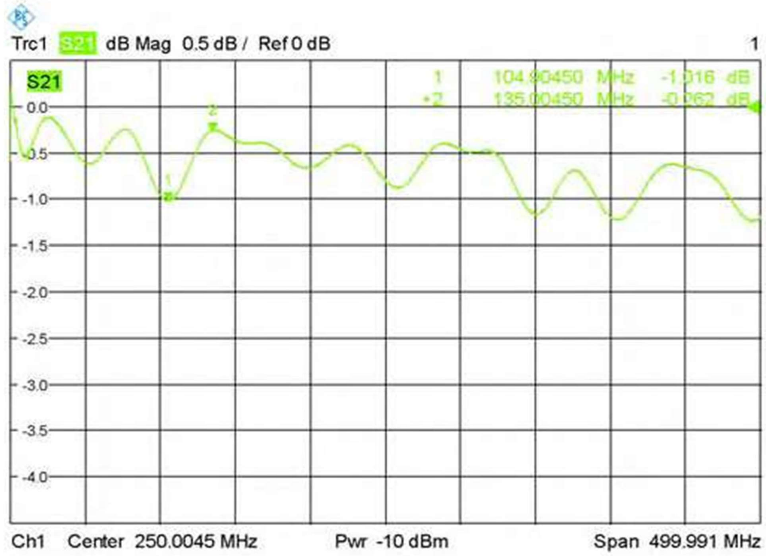

Figure 4. Attenuation of a given coaxial waveguide with multiple reflections as a function of frequency

3. Computational Model

The equation for the propagating waves are

$$A(x,t) = A_0 \cdot e^{-\alpha \cdot x} \cdot e^{-j \cdot \beta \cdot x} \cdot e^{j \cdot \omega \cdot t}, \tag{2}$$

$$B(x,t) = B_0 \cdot e^{\alpha \cdot (x-L)} \cdot e^{j \cdot \beta \cdot (x-L)} \cdot e^{j \cdot \omega \cdot t}, \tag{3}$$

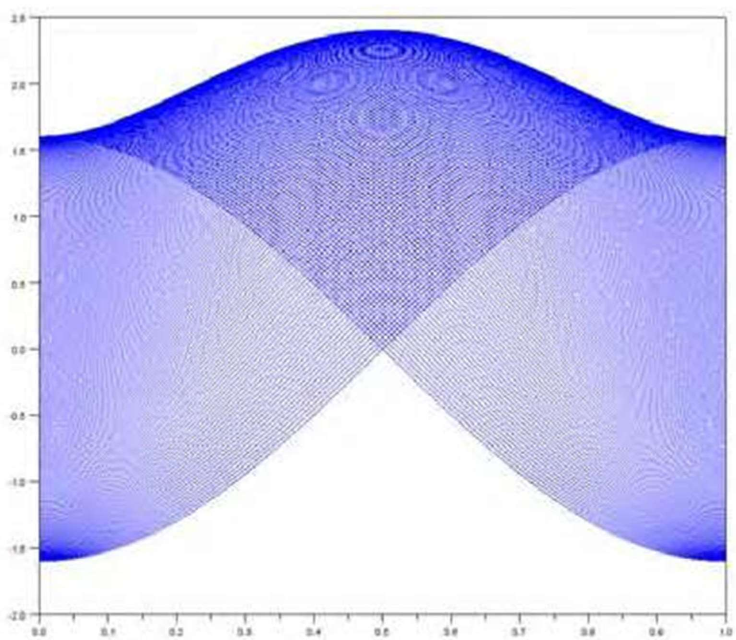

Figure 5. Only positive peak values calculated during simulation

During the simulation the voltage function for both A and B have to be calculated as a function of time and place. A finite difference model is used for this purpose. We have found, that it is really sufficient to model the distribution of voltage on half wavelength (?g/2) long part of the waveguide. In a measurements, the test receivers measure, e.g., the peak of the measured signal. According to this fact in the simulation it is enough to calculate the positive peak values on the waveguide. We present the positive peaks of our finite difference study in Figure 5. The lower peaks are missing from the simulation, as they carry no extra information for our purposes. The model was developed and run in scilab environment [4].

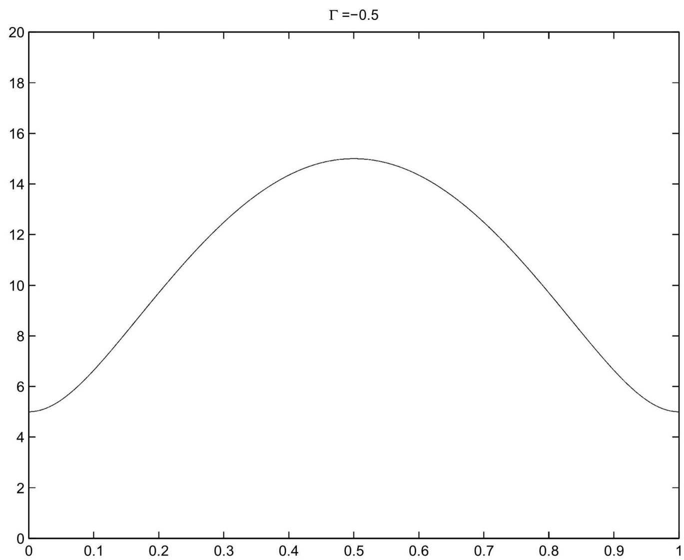

Figure 6. Distribution of U_{peak} at $|\Gamma| = 0.5$ as a function of position at a half-wavelength long waveguide

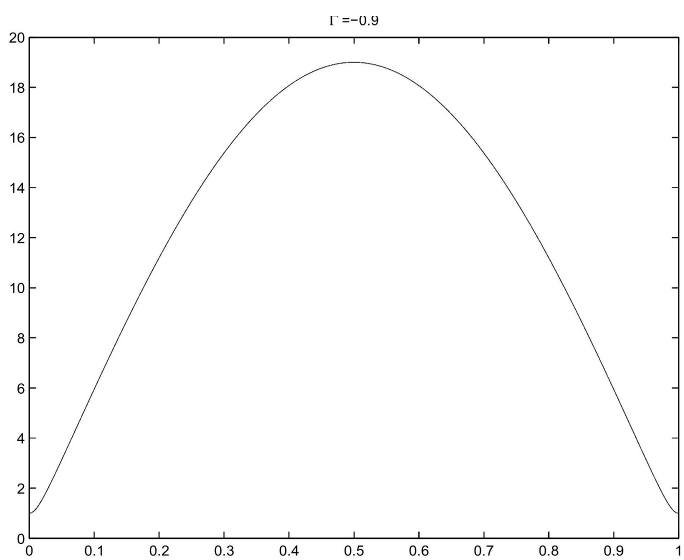

Figure 7. Distribution of U_{peak} at $|\Gamma| = 0.9$ as a function of the position

As Figure 6 and Figure 7 show, the shape of the distribution of the voltage depends on the reflection coefficient Γ.

The empirical cumulative distribution function - calculated by the Statistical toolbox of Matlab [5] - of the normalized voltages are shown in Figs. 8-10. Three different reflection coefficient values are presented, a small, a medium and a large one. The differences are visible, but the tendencies are similar for all three cases.

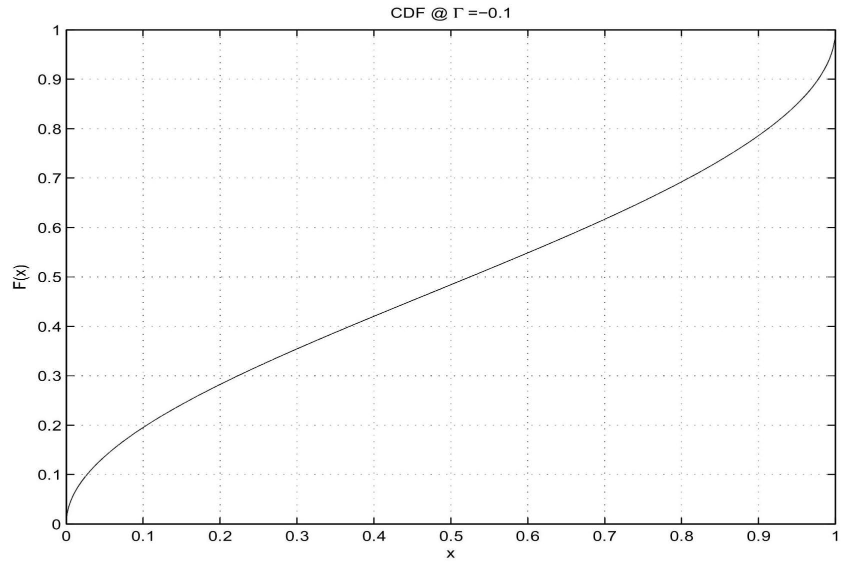

Figure 8. Empirical CDF of U_{peak} at $|\Gamma| = 0.1$.

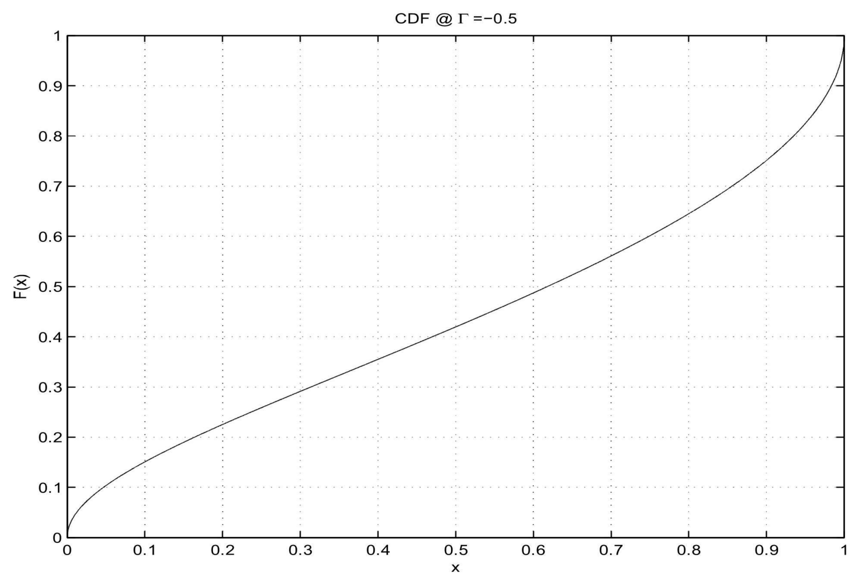

Figure 9. Empirical CDF of U_{peak} at $|\Gamma| = 0.5$.

We have tried to fit several types of distributions - from normal through exponential and Weibull to Rayleigh - to the data. The results can be fitted the best by beta distribution functions. The Bata probability density function is

$$f(X) = \frac{X^{\alpha-1} \cdot (1-X)^{\beta-1}}{B(\alpha,\beta)}, \tag{4}$$

where B is the beta function with parameter α and β

$$B(\alpha,\beta) = \int_0^1 X^{\alpha-1} \cdot (1-X)^{\beta-1} dX, \tag{5}$$

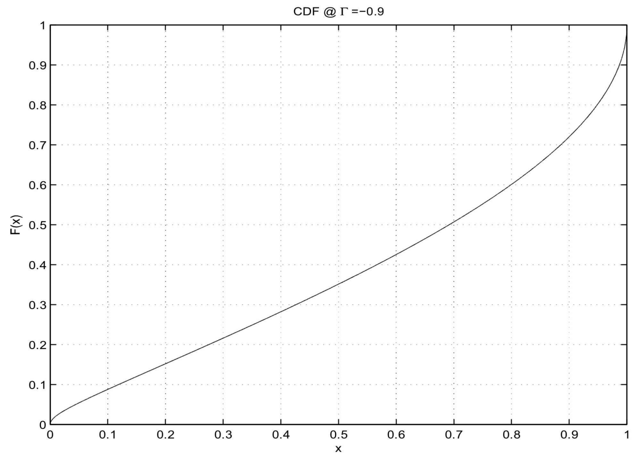

Figure 10. Empirical CDF of U_{peak} at $|\Gamma| = 0.9$.

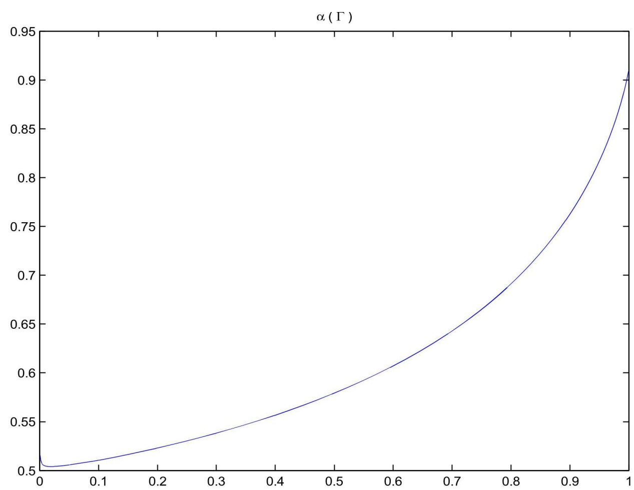

Figure 11. Parameter α of the beta distribution as a function of $|\Gamma|$.

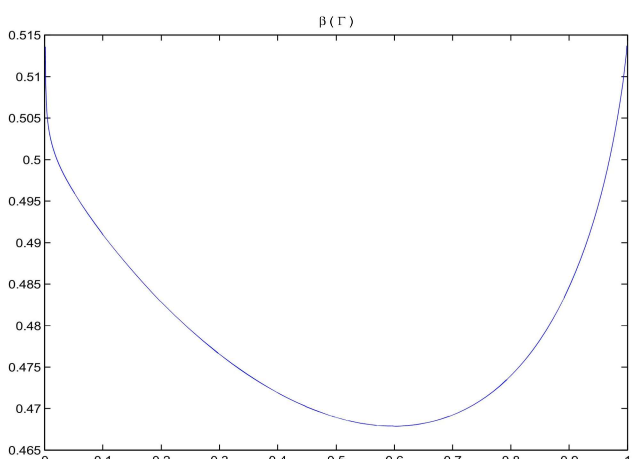

Figure 12. Parameter β of beta distribution as a function of $|\Gamma|$.

and there is no closed form in general for the distribution function [6]. The parameters α and β as a function of the reflection parameter Γ are plotted in Figures 11 and 12.

The two parameters α and β as a function of the reflection coefficient Γ can be seen in the 3D plot of Figure 13. A continuous line can be seen. In case of measurement results, we are not expecting the points to be on the line, but in a small environment of it.

The mean value estimation of the beta distribution is

$$mv = \frac{\alpha}{\alpha+\beta}, \tag{6}$$

and the variance can be written as

$$\rho^2 = \frac{\alpha+\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}, \tag{7}$$

Using these formulae, the confidence range of the distribution can be calculated. The confidence range of the two parameters α and β can be seen in Figs. 14 and 15.

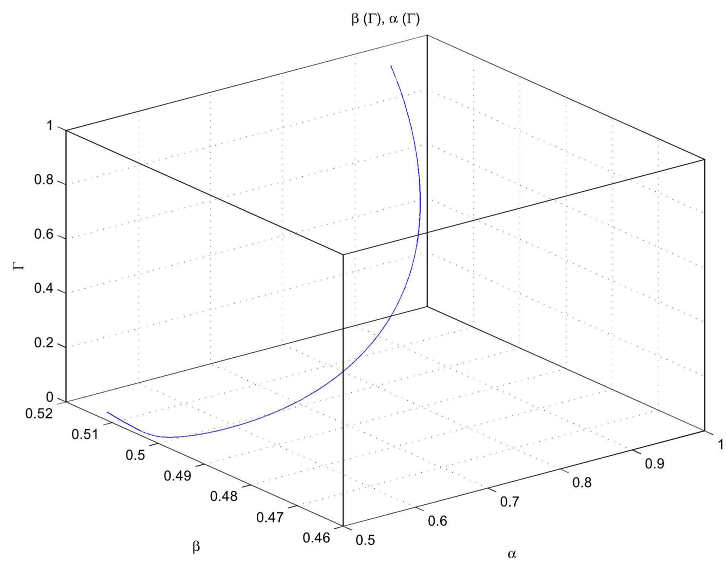

Figure 13. Dependency of α, β, |Γ|

4. Conclusion

The method presented in Section 3 is suitable for characterizing the distribution of voltage on the transmission lines. The distribution of voltage - in contrast with the U distribution suggested by the standards - is rather a Beta distribution, its parameters α and β are clear functions of the effective reflection coefficient $|\Gamma|$. The confidence range is shown in Figures. 14 and 15 (blue line denotes the parameters α and β the red and green lines show the upper and lower limit of their confidence range).

Acknowledgement

This work was supported by the TáMOP 4.2.2/B-10/1- 2010-0010 project of the Széchenyi István University.

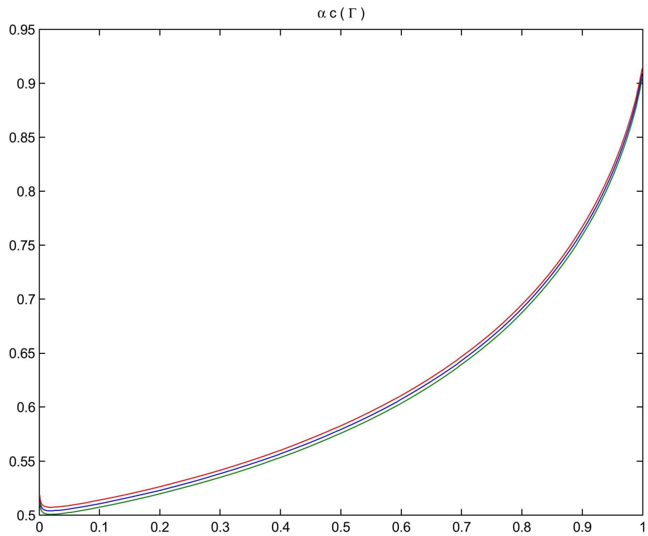

Figure 14. Confidence range of $\alpha(\Gamma)$.

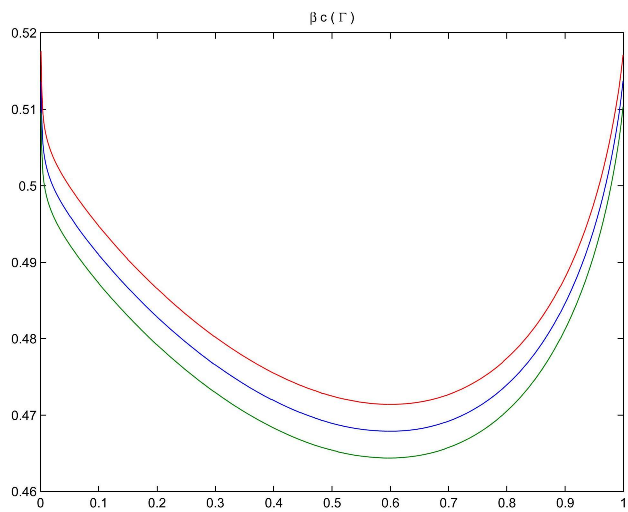

Figure 15. Confidence range of β (Γ).

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