Wavelength Assignment for Conducting Exchanged Folded Hypercube Communication Pattern on Optical Bus

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ABSTRACT: *The (s + t + 1)-dimensional exchanged folded hypercube, denoted by EFH(s,t), is a brand-new interconnection network proposed by Qi et al. Besides, the bus topology is the simplest topology in optical networks, which can be modeled by a linear array graph, denoted by* LA_n*. In this paper, the Routing and Wavelength Assignment (RWA) problem for conducting EFH(s, t) communication pattern on* LA_n *is investigate, where n = s + t + 1. To address this problem, an embedding scheme as well as a wavelength assignment algorithm are proposed. The author also shown that the number of wavelengths required by the wavelength assignment algorithm is* $2^{s+t} + 2^{s+t-2} + [2^t/3]$.

Keywords: Bus Topology, Exchanged Folded Hypercube, Routing And Wavelength Assignment Problem, Wavelength Division Multiplexing

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1. Introduction

It is well known that interconnection networks take important roles in parallel computing systems. The n-dimensional hypercube, deonted by *Q(n)*, has received much attention due to many of its attractive features, e.g., high regularity, symmetry, strong connectivity and bipancyclicity [18]. Some of hypercube's varaints have been discussed, such as extended crossed cubes [1], folded hypercubes [6], Fibonacci cubes [7], exchanged hypercubes [14], extended folded cubes [15], enhanced hypercubes [19].

As a variant of *Q(n)*, the n-dimensional folded hypercube, deonted by *FHC(n)*, is proposed by El-Amawy [6] et al.

FHC(n) is constructed from *Q(n)* by connecting each vertex to the unique vertex farthest from it. Like *Q(n)*, *FHC*(*n*) is a regular graph composed of 2^n vertices, but the diameter of *FHC*(*n*) is only about half of that of *Q(n)* (i.e.,*dn* = 2e). The (*s* + *t* + 1)-dimensional exchanged hypercube, denoted by *EH*(*s,t*), is another famous variant of *Q(n)*, proposed by Loh et al [14]. *EH*(*s,t*) is developed from *Q(n)* by systematically removing edges from it. Hence, *EH*(*s,t*) has lower degree and hardware cost compared to *Q(n)*.

Based on $EH(s, t)$ and $FHC(n)$, a new variant called (s+t+1)-dimensional Exchanged Folded Hypercube, denoted by $EFH(s, t)$, is proposed by *Qi* et al. [17]. $EFH(s,t)$ is obtained by adding some complementary edges to $EH(s, t)$. More specifically, each complementary edge connects a vertex u to a vertex v such that the hamming distance between u and v is maximized. $EFH(s,t)$ combines the advantages of $EH(s,t)$ and $FHC(n)$, and it has some prominent performances, such as smaller diameter, shorter delay and less message density, compared to other varants of Qn [17]. Note that the diameter of $EH(s, t)$ is $s + t + 2$, and the diameter of $EFH(s, t)$ is $ds = 2e + dt = 2e + 2$ (when $s \geq 2$ and $t \geq 2$). Up to now, only few works related to *EFH* have been addressed, e.g., shortest path routing [2] and domination number [8].

Optical network has been widely used as a communication media in many applications, because it possesses many good charateristics. For instance, it has extremely high bandwidth, extremely small communication delay and extremely low power comsumption [24]. Besides, Wavelength Division Multiplexing (WDM) is a common technique that divides the bandwidth of an optical fiber link into multiple communication channels, and each communication channel is represented by a respective wavelength. Then multiple data streams can be transmitted concurrently across the same optical fiber link.

A WDM optical network is an interconnection of optical routing nodes, and the routing nodes are linked through optical fibers. Each link is constructed by a pair of fibers with one fiber supporting connections in the forward direction while the other fiber supports connections in the reverse direction. Due to high cost incurred by optoelectrical conversions, this paper does not consider wavelength converters. Therefore, for each connection, one dedicated end-to-end lightpath needs to be set up.

The bus topology is the simplest topology in optical network, and it involves N communication nodes and a global bus [23]. In this topology, one of the communication nodes arbitrates whether the global bus is free for use so that it can communicate with the other $N - 1$ nodes. The bus topology with N communication nodes can be described by a Linear Array graph, denoted by LAn, where $N = 2^n$. Figure 1 (a) and (b) show the bus topology with N communication nodes and a linear array with N vertices, repectively.

The available wavelength resources in optical networks are limited, consequently, routes and wavelengths must be planned carefully so that the total number of wavelengths used is minimized. This corresponding problem is called the Routing and Wavelength Assignment (RWA) problem [16], [22], in which the optimization goal is to minimize the number of required wavelengths. Also, a suitable lightpath and its corresponding wavelength are chosen for each connection of a given communication pattern, which satisfies the Wavelength Continuity Constraint (WCC for short) and the Distinct Wavelength Constraint (DWC for short). So far, the RWA problem has attracted the interest of numerous researchers [4], [5], [9]–[13], [20], [21], [23], [24]. However, the RWA problem for conducting exchanged folded hypercube communication patterns on optical buses has never been investigated.

To address the RWA problem for conducting $EFH(s, t)$ communication patterns on LA_n, the rest of this paper is organized as follows. Section 2 first introduces exchanged folded hypercube, optical bus and linear array. Then WCC and DWC on optical bus for the RWA problem is described. Section 3 proposes the embedding scheme *ES* and the wavelength assignment algorithm WA. Finally, Section 4 concludes this paper.

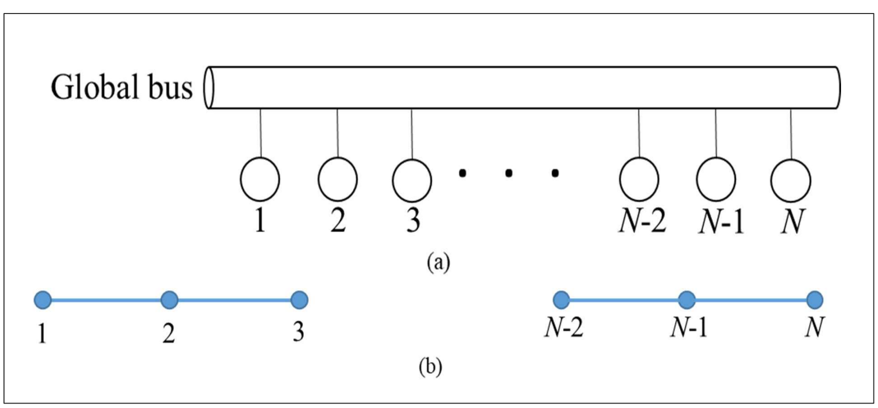


Figure 1. The bus topology modeled by a linear array graph

2. Preliminaries

2.1. The Exchanged Folded Hypercube

Let x denote a positive integer, and $u = u_{x-1} \cdots u_0 \in \{0,1\}^x$ be a binary string of length x. We set $u[i] = u_i$ and $u[i:j] = u_i u_{i+1} \ldots u_j$, where $0 \leq j \leq i \leq x-1$. We also set $[j,i] = \{j, j+1, \ldots, i\}$, where $0 \leq j \leq i$. By the definition of section 2.1 in [17], an equivalent definition for *EFH(s,t)* is given as follows.

Definition 1 ([17]): The s + t + 1-dimensional exchanged folded hypercube $EFH(s,t) = (V, E)$, where s and t are positive integers,

$V = \{a_{s-1} \ldots a_0 b_{t-1} \ldots b_0 c | a_i, b_j, c \in \{0,1\}, i \in [0, s-1]$, and $j \in [0, t-1]\}$ and $E = E_1 \cup E_2 \cup E_3 \cup E_4$. For any two vertices $u, v \in V$, we define E_1, E_2, E_3 and E_4 as follows:

(a) $(u; v) \in E_1$, if and only if $u[0] = v[0]$ and $u \oplus v = 1$, where $\oplus$ stands for the exclusive-OR operator.

(b) $(u, v) \in E_2$, if and only if $u[s+t : t+1] = v[s+t : t+1]$, and $u[0] = v[0] = 1$.

(c) $(u, v) \in E_3$, if and only if $u[t:1] = v[t:1]$, $u[0] = v[0] = 0$, and $(u[s+t:t+1]; v[s+t:t+1]) \in E(Q(s))$.

(d) $(u, v) \in E_4$, if and only if $h(u,v) = s + t + 1$, where $h(u,v)$ stands for the hamming distance between vertices u and v.

By Definition 1, there exists 2^{s+t+1} vertices and $(s + t + 4) \times 2^{s+t-1}$ edges in $EFH(s,t)$. Figure 2 demonstrates $EFH(1,2)$, where the black dashed lines, bold lines, solid lines and red dashed lines correspond to edges in E_1, E_2, E_3 and E_4, respectively. We can see that each vertex u in $EFH(1, 2)$ with $u[0] = 1$ is of degree 4, and with $u[0] = 0$ is of degree 3.

To consider directed edges, we use $E^d(EFH(s,t))$ to denote the directed edges set of $EFH(s,t)$. That is, $E^d(EFH(s,t)) = \{\langle u,v \rangle, \langle v,u \rangle | (u,v) \in E(\mathrm{EFH}(s,t))\}$.

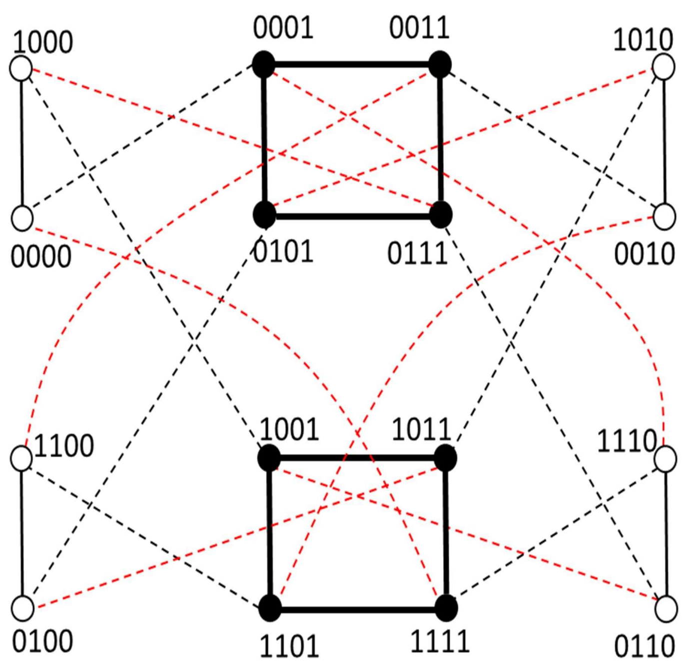


Figure 2. An exchanged folded hypercube EFH(1,2)

Lemma 1 ([17]): *EFH*(*s, t*) is isomorphic to *EFH*(*t, s*).

For simplicity, without loss of generality, we may assume $s \leq t$ in the following by Lemma 1.

2.2. The Optical Bus and Linear Array

Let N and n be positive integers. In this paper, an optical bus with *N* terminals is modelded by a Linear Array of *N* nodes, denoted by LA_n, where $N = 2^n$. The nodes (resp. the links) in LA_n are labeled continuously from 1 to *N* (resp. from ℓ_1 to ℓ_{N-1}). Figure 3 shows LA_2, where $\{\ell_i = (i, i+\ 1)|i \in [1,3]\}$. To consider directed links, we use $E^d(LA_n)$ to denote the directed edges set on LA_n. That is, $E^d(LA_n) = \{\langle i, i+1\rangle, \langle i+1, i\rangle | i \in [1, 2^n - 1]\}$.

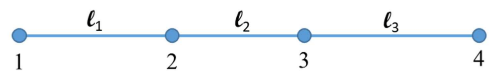


Figure 3. A linear array LA_2

2.3. WCC and DWC on Optical Buses

In the RWA problem, the wavelengths assigned to links on LAn must comply with both the WCC and the DWC. The WCC commands that all the links on a lightpath from the source node to the destiniation node have to use the same wavelength. Fig. 4 shows a lightpath from node 1 to node 3,which does't comply with the WCC. Because this lightpath uses different wavelengths on links <1, 2> and <2, 3>.

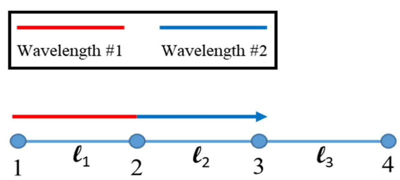


Figure 4. An example that does't commands the WCC

The DWC commands that all lightpaths using the same link must be assigned distinct wavelengths on it. Figure 5 depicts an example of two lightpaths that does't comply with the DWC. The first lightpath is from node 1 to node 3, and the second lightpath is from node 2 to node 4. Clearly, the two lightpaths use he same wavelength (i.e., wavelength #1) on link <2; 3>. Therefore, this example fail to comply with the DWC.

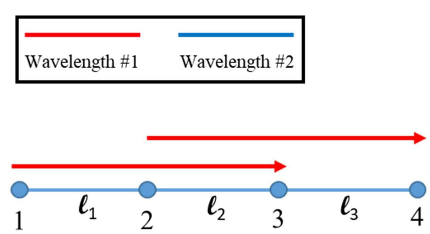


Figure 5. An example that does't commands the DWC

3. Wavelength Assignment for *EFH*(*s,T*) on LA_n

Let $u = u_{s+t} \cdots u_0 \in \{0,1\}^{s+t+1}$ be a vertex belongs to *V* (*EFH*(*s, t*)). And let u' be a substring of *u*, or a string formed by concatenating some substrings of *u*. Hereafter, this paper uses val(u0) to denote the decimal value of u'. By setting eight different values to $u[t+1]$, $u[1]$ and $u[0]$, $V(EFH(s,t))$ is divided into nonoverlapping subsets, denoted by P^k. Let $u = u_{s+t} \cdots u_0 \in \{0,1\}^{s+t+1}$ denote a vertex in P^k, where $k \in [0,7]$. Then the vertex subset P^k can be described as:

$$P^k = \{u^k | val(u[t+1]u[1]u[0]) = k\}$$

If $k \in \{1,3,5,7\}$, it is easy to verify that the subgraph induced by all the vertices in P^k contains 2^{s-1} nonoverlapping copies of $Q(t-1)$. If $s > 1$, let $Q_i^k(t-1)$ denote the $Q(t-1)$ with $val(u[s+t : t+2]) = i$, where $i \in [0, 2^{s-1}-1]$. And the vertex in $Q_i^k(t-1)$ with $val(u[t:2]) = j$ is denoted by $q_{i,j}^k(t-1)$ where $i \in [0, 2^{s-1}-1]$ and $j \in [0, 2^{t-1}-1]$. Otherwise if $s <= 1$, the unique $Q(t-1)$ is denoted by $Q_0^k(t-1)$, and the vertex in $Q_0^k(t-1)$ is denoted by $q_{0,j}^k(t-1)$, $j \in [0, 2^{t-1}-1]$ where $j \in [0, 2^{t-1}-1]$. By a similar reason, if $k \in \{0,2,4,6\}$, we can also define $Q_i^k(s-1)$ and $q_{i,j}^k(s-1)$, where $i \in [0, 2^{t-1}-1]$ and $j \in [0, 2^{s-1}-1]$.

Embedding Scheme ES

```
Input: An exchanged folded hypercube EFH(s,t).
Output: The assigned number AN(u) for all u ∈ EFH(s,t).
begin
Step 1.   for each vertex u ∈ EFH(s,t) do AN(u) ← null;
Step 2.   order[8] ← {1,3,2,6,7,5,4,0};
Step 3.   number ← 0;
Step 4.   for x ← 0 to 7 do
              k ← order[x];
              if k is an odd number then
                  for i ← 0 to 2^(s−1) − 1 do
                      for j ← 0 to 2^(t−1) − 1 do
                          number ← number + 1;
                          AN(q^k_{i,j}(t − 1)) ← number;
              else
                  for i ← 0 to 2^(t−1) − 1 do
                      for j ← 0 to 2^(s−1) − 1 do
                          number ← number + 1;
                          AN(q^k_{i,j}(s − 1)) ← number;
```

Figure 6. The embedding scheme ES

3.1. The Embedding Scheme ES

Figure 6 describes an embedding scheme, called ES, which assigns a number to each vertex u in $EFH(s,t)$. ES gives an assigned number, denoted by $AN(u)$, to each vertex u. In Step 1, the assinged number for vertex u, denoted by $AN(u)$ is initialzed to null value. In Step 2, the 8 elements in the array order are initialized, i.e., order[0] = 1; order [1] = 3 : : : ; order[7] = 0. Hence, in Step 4, vertices in P^1 are processed before vertices in P^3, and vertices in P^3 are processed before vertices in P^2, . . . , etc. In Step 3, the variable number is initialized to zero. In Step 4, each vertex is considered, and assinged a distinct number from $[1, 2^{s+t+1}]$.

Figure 7 shows the numbers assigned to the vertices in $EFH(1,2)$ by the embedding scheme ES. Figure 8 shows the conceptual diagram of embedding $EFH(s,t)$ to LA_n by ES, in which, vertices belonging to P^k are depicted.

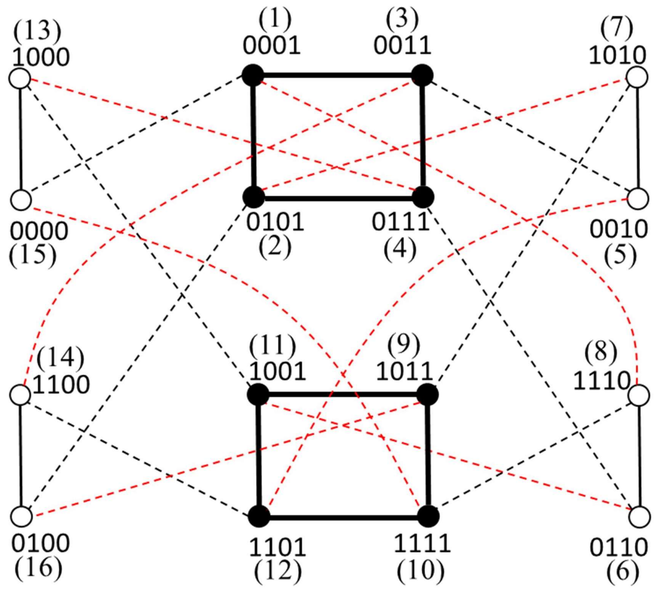


Figure 7. The numbers assigned to vertices in EFH(1,2) by ES

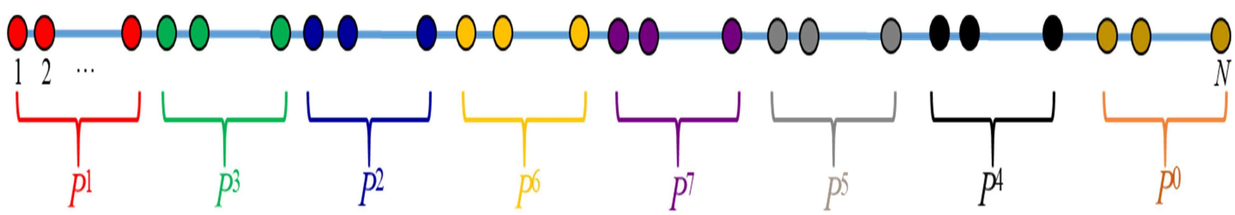


Figure 8. The conceptual diagram of the embedding scheme ES

Let $e^d = \langle u, v\rangle$ be a directed edge in $E^d(EFH(s,t))$ such that $AN(u) = x$ and $AN(u) = y$. We use $P^{ES}(e^d)$ to denote the corresponding routing path of e^d on LA_n. If $x < y$ (resp. $x < y$), then the routing path $P^{ES}(e^d) = \langle x, x+1, x+2, \ldots, y-1, y\rangle$ (resp. $P^{ES}(e^d) = \langle x, x-1, x-2, \ldots, y+1, y\rangle$). We also use $E^d(P^{ES}(d^d))$ to denotes the directed links on $PES(e^d)$.

Property 2: In the embedding scheme *ES*, if $x \neq y$ or $k_1 \neq k_2$ or $i_1 \neq i_2$, then vertices in $Q_{i_1}^{k_1}(x)$ and $Q_{i_2}^{k_2}(y)$ are embedded into two nonoverlapping linear subarrays of LA_n.

Proof : Let $Q_i^k(x)$ be an $Q(x)$ in the subgraph induced by vertices in P^k, where $x \in \{s, t\}$ and $k \in [0, 7]$. It is clear that the vertices in $Q_i^k(x)$ are assgined consecutive distinct numbers in the embedding scheme ES. Hence, the vertices in $Q_i^k(x)$ are embedded into a linear subarray of LA_n. If $x \neq y$ or $k_1 \neq k_2$ or $i_1 \neq i_2$, it is easy to understand that vertices in $Q_{i_1}^{k_1}(x)$ and $Q_{i_2}^{k_2}(y)$ are embedded into two nonoverlapping linear subarrays.

Let $u^k = u_{s+t} \cdots u_0 \in \{0,1\}^{s+t+1}$ denotes a vertex in vertex subset P^k, where $k \in [0, 7]$. Recall that $val(u[t+1]u[1]u[0]) = k$. We also use u_x^k = to denote a vertex in P^k with $val(u[s+t : t+2]u[t : 2]) = x$, where $x \in [0, 2^{s+t-2} - 1]$. If $x \in [0, 2^{s+t-2} - 1]$, we define 8 edges as follows:

$e_x^a = (u_x^1, u_x^3)$, $e_x^b = (u_x^3, u_x^2)$, $e_x^c = (u_x^2, u_x^6)$, $e_x^d = (u_x^6, u_x^7)$, $e_x^e = (u_x^7, u_x^5)$, $e_x^f = (u_x^5, u_x^4)$, $e_x^g = (u_x^4, u_x^0)$ and $e_x^h = (u_x^0, u_x^1)$.

we also define If $x \in [0, 2^{s+t-2} - 1]$, the edge set, $Cyc(x) = \{e_x^a, e_x^b, e_x^c, e_x^d, e_x^e, e_x^f, e_x^g, e_x^h\}$.

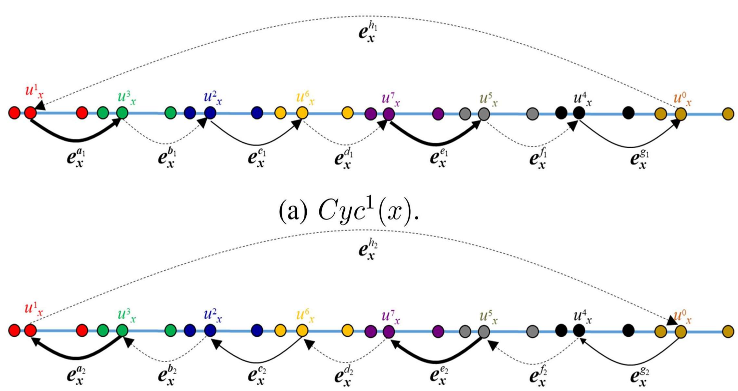
Proposition 3: The eight edges in *Cyc*(*x*) forms a cycle in *EFH*(*s,t*), where $x \in [0, 2^{s+t-2} - 1]$.

Proof : To validate this proposition, we only need to check existence of the 8 edges in *EFH*(*s, t*). By definition, it is celar that $val(u[s+t : t+2]u[t : 2]) = x$. In the following, we list binary string patterns of these edges:

$$
\begin{aligned}
e_x^a &= (u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 01, u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 11),\\
e_x^b &= (u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 11, u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 10),\\
e_x^c &= (u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 10, u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 10),\\
e_x^d &= (u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 10, u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 11),\\
e_x^e &= (u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 11, u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 01),\\
e_x^f &= (u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 01, u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 00),\\
e_x^g &= (u_{s+t}\ldots u_{t+2}1u_t\ldots u_2 00, u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 00),\\
e_x^h &= (u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 00, u_{s+t}\ldots u_{t+2}0u_t\ldots u_2 01),
\end{aligned}
$$

By checking the above binary string patterns, we can see that e_x^b, e_x^d, e_x^f and e_x^h are in E_1, $e^a{}_x$ and $e^e{}_x$ are in E_2; and e_c and e_g are in E_3 (see Definition 1). Thus, the proposition is true.

To consider directional version of Cyc(x), we denote $\mathrm{Cyc}^1(\mathrm{x})$ and $\mathrm{Cyc}^1(\mathrm{x})$ as two directed cycles with reversed directions. Figure 9 (a) and (b) show $\mathrm{Cyc}^1(\mathrm{x})$ and $\mathrm{Cyc}^2(\mathrm{x})$.



(a) $Cyc^1(x)$.

(b) $Cyc^2(x)$.

Figure 9. Two directional cycles with reversed directions

Let $x \in [0, 2^y - 1]$, where *y* is a positive integer. Hereafter, we define $\bar{x} = 2^y - 1 - x$.

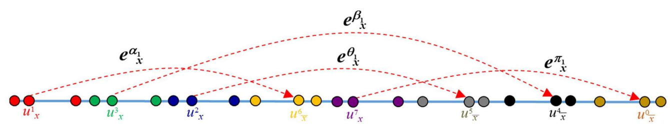
Property 4: If $k \in [0, 7]$ and $x \in [0, 2^{s+t-2} - 1]$, the vertex u_x^k connects to the vertex $u_{\bar{x}}^{\bar{k}}$ through an edge in E_4.

Proof : Let $u_x^k = u_{s+t} \cdots u_0 \in \{0,1\}^{s+t+1}$. By definition, $val(u[t+1]u[1]u[0]) = k$ and $val(u[s+t : t+2]u[t : 2]) = x$. Let $u' = u'_{s+t} \cdots u'_0 \in \{0,1\}^{s+t+1}$ be the vertex connecting to u_x^k by an edge in E4. By Definition 1, we can obtain that $val(u'[t+1]u'[1]u'[0]) = 7 - k = 2^3 - 1 - k = \bar{k}$ and $val(u'[s+t : t+2]u'[t : 2]) = 2^{s+t-2} - 1 - x = \bar{x}$. Clearly, $u' = u_{\bar{x}}^{k}$. Therefore, the property is true.

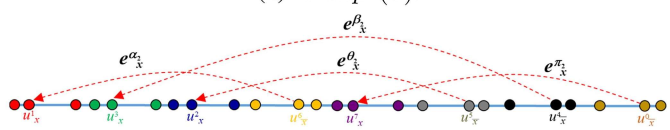
By Property 4, let $e_x^{\alpha} = (u_x^1, u_{\bar{x}}^6)$, $e_x^{\beta} = (u_x^3, u_{\bar{x}}^4)$ $e_x^{\theta} = (u_x^2, u_{\bar{x}}^5)$, and $e_x^{\pi} = (u_x^7, u_{\bar{x}}^0)$ be the four edges in E_4, where $x \in [0, 2^{s+t-2} - 1]$. If $x \in [0, 2^{s+t-2} - 1]$, we also define the edge set, $Comp(x) = \{e_x^{\alpha}, e_x^{\beta}, e_x^{\theta}, e_x^{\pi}\}$. To consider directional version of *Comp(x)*, we denote $Comp^1(x)$ and $Comp^2(x)$ as two directed edge sets. Figure 10 (a) and (b) show $Comp^1(x)$ *and* $Comp^3(x)$ in E_4^d .

3.2. The Wavelength Assignment Algorithm WA

Chen and Shen proposed wavelength assignment algorithms for bidirectional and unidirectional *Q(x) on* LA_x [4], where *x* is a positive integer. In this Subsection, an algorithm in [4], called Algorithm 1, is used to tackle the bidirectional case.



(a) $Comp^1(x)$.



(b) $Comp^2(x)$.

Figure 10. The two directed edge sets in E_4^d

A wavelength algorithm, called WA, is shown in Figure 11. Note that the wavelengths assigned in Step 1 (resp. Step 2) are numbered $1, 2, \ldots, 2^{s+t-1}$ (resp. $2^{s+t-1} + 1, 2^{s+t-1} + 2, \ldots, 2^{s+t} + 2^{s+t-2}$). In Step 3 (resp., Step 4), Algorithm 1 in [4] is called using two input parameters $Q(t\text{-}1)$ and $LA_{t\text{-}1}$ (resp., $Q(s\text{-}1)$ and $LA_{s\text{-}1}$).

Theorem 5: The wavelength assignment algorithm WA needs $2^{s+t} + 2^{s+t-2} + \lfloor 2^t/3 \rfloor$ wavelengths.

Proof : Obviously, all edges in *EFH(s,t)* have been considered by the wavelength assignment algorithm WA. In Step 1, two directed link set, called DS_1 and DS_2, are first initialized to be empty sets. Then, there are 2^{s+t-2} iterations to be executed, and wavelength assignment algorithm WA assigns 2 unused wavelengths for each iteration. Hence, the total wavelengths assigned in Step 1 is 2^{s+t-1}. Similarly, in Step 2, three directed link set, called DS_1, DS_1 and DS_1, are first initialized to be empty sets. Then, there are 2^{s+t-1} iterations to be executed, and wavelength assignment algorithm *WA* assigns 3 unused wavelengths for each iteration. Therefore, the total wavelengths assigned in Step 2 is $2^{s+t-1} + 2^{s+t-2}$. In Step 3 (resp., Step 4), edges in $Q_i^k(t-1)$ (resp., $Q_i^k(t-1)$, are processed, and Algorithm 1 in [4] is called to assign wavelengths. On the basis of the results in [4], each $Q_i^k(t-1)$ (resp., $Q_i^k(s-1)$) in Step 3 (res., Step 4) needs $\lfloor 2^t/3 \rfloor$ (resp., $\lfloor 2^s/3 \rfloor$) wavelengths. From the Property 2, we know that each $Q_i^k(t-1)$ (resp., $Q_i^k(s-1)$) is embedded into a nonoverlapping linear subarray of LA_n. Therefore, the wavelengths assigned in Step 3 and 4 can be reused.

Because $s < t$, only $max([2^s=3], [2^t=3] = [2^t=3]$ wavelengths are required for these two steps. Therefore, the wavelength algorithm WA needs $2^{s+t-1} + (2^{s+t-1} + 2^{s+t-2}) + [2^t/3] = 2^{s+t} 2^{s+t-2} + [2^t/3]$ wavelengths.

Figure 12 shows the wavelength allocation for *EFH*(1, 2), and 11 wavelengths are required by the wavelength assignment algorithm WA. Figure 13 shows the wavelengths assignment to the directed edges in E^d(*EFH*(1, 2)) by the wavelength Figure 11. The wavelength assignment algorithm WA. assignment algorithm WA.

Wavelength Assignment Algorithm WA

Input: An exchange folded hypercube EFH(s,t), and the assigned number AN(u) for all vertex u in EFH(s,t).

Output: The wavelengths assigned to directed link ℓ^d for all ℓ^d in $E^d(LA_n)$.

begin

Step 1. **for** $x \leftarrow 0$ **to** $2^{s+t-2}-1$ **do**
- $DS_1 \leftarrow \emptyset$ and $DS_2 \leftarrow \emptyset$;
- **for** each directed edge $e^d \in \mathrm{Cyc}^1(x)$ **do**
  - $DS_1 \leftarrow DS_1 \cup E^d(P^{ES}(e^d))$;
- **for** each directed edge $e^d \in \mathrm{Cyc}^2(x)$ **do**
  - $DS_2 \leftarrow DS_2 \cup E^d(P^{ES}(e^d))$;
- Assign an unused wavelength to all ℓ^d in DS_1;
- Assign an unused wavelength to all ℓ^d in DS_2;

Step 2. **for** $x \leftarrow 0$ **to** $2^{s+t-2}-1$ **do**
- Set $DS_1 = \emptyset$, $DS_2 = \emptyset$ and $DS_3 = \emptyset$;
- **for** each directed edge $e^d \in (\mathrm{Comp}^1(x) \cup \mathrm{Comp}^2(x))$ **do**
  - **If** ($e^d \in \{e_x^{\alpha_1}, e_x^{\alpha_2}, e_x^{\pi_1}, e_x^{\pi_2}\}$) **then**
    - $DS_1 \leftarrow DS_1 \cup E^d(P^{ES}(e^d))$;
  - **If** ($e^d \in \{e_x^{\beta_1}, e_x^{\beta_2}\}$) **then**
    - $DS_2 \leftarrow DS_2 \cup E^d(P^{ES}(e^d))$;
  - **If** ($e^d \in \{e_x^{\theta_1}, e_x^{\theta_2}\}$) **then**
    - $DS_3 \leftarrow DS_3 \cup E^d(P^{ES}(e^d))$;
- Assign an unused wavelength to all ℓ^d in DS_1;
- Assign an unused wavelength to all ℓ^d in DS_2;
- Assign an unused wavelength to all ℓ^d in DS_3;

Step 3. **for** $k \in \{1,3,5,7\}$ **do**
- **for** $i \leftarrow 0$ **to** $2^{s-1}-1$ **do**
  - call *Algorithm 1* in [4] to assign wavelengths to all ℓ^d in $E^d(P^{ES}(e^d))$ for all $e^d \in E^d(Q_i^k(t-1))$;

Step 4. **for** $k \in \{0,2,4,6\}$ **do**
- **for** $i \leftarrow 0$ **to** $2^{t-1}-1$ **do**
  - call *Algorithm 1* in [4] to assign wavelengths to all ℓ^d in $E^d(P^{ES}(e^d))$ for all $e^d \in E^d(Q_i^k(s-1))$;

Figure 11. The wavelength assignment algorithm WA.

assignment algorithm **WA**.

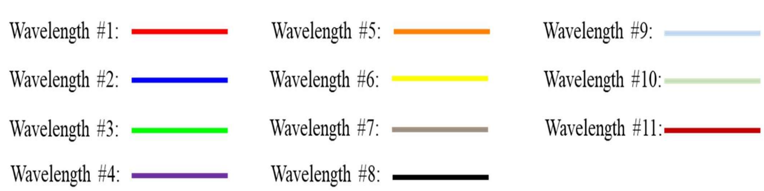


Figure 12. The wavelength allocation for EFH(1,2).

4. Conclusion

In this paper, the RWA problem for conducting exchanged folded hypercube communication pattern on optical bus is studied. To address this problem, the embedding scheme ES and the wavelength assignment algorithm WA are proposed. The author have also shown that the number of wavelengths used by the wavelength assignment algorithm WA is $2^{s+t} + 2^{s+t-2} + \lfloor 2^t/3 \rfloor$. For future works, the author points out two promising research directions:

1. To evaluate the performance of the wavelength assignment algorithm WA. The author would like to derive a tight lower bound of the minium number of required wavelengths by using the congestion estimation technique.

2. Recently, Bhavani and Jena have proposed exchanged folded crossed cube [3]. Qi et, al. have also developed exchanged folded hypercube [17]. Inspired by these works, the author want to propose a more generalized interconnection network architecture which includes both of exchanged folded crossed cubes and exchanged folded hypercubes as special cases.

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