Real-time Data Dependant Fuzzy Linear Model for Enterprise Security

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ABSTRACT: *In enterprises, material flow, production time and storage for delivery are required to confirm security and safety. We have used the real-time data with the fuzzy linear model for creating an aggregate production planning and tested the model for enterprise safety issues. We found that adequate security is achieved in the proposed model.*

Keywords: Aggregate Production Planning, Fuzzy Optimization, Uncertain Production, Uncertain Customer Demand

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1. Introduction

Aggregate production planning (APP) is one of the most important part of operations management in competitive supply chains. It concerns matching supply with forecasted customer demand over a planning period, which is usually one year in practice. Generally, the aim is to determine required resources, which include production rate, warehouse levels, work force level, overtime, etc., in such a way as to meet customer demand.

In the literature, it has been assumed most often, that all the parameters which are associated with the APP process, such as customer demand, processing times, production capacities etc., are deterministic in nature (for example, [1]). In order to handle uncertainties which characterise real world APP environments, and a randomness in customer demand, in particular, various stochastic optimisation models have been proposed [2]. Furthermore, one can find in the literature that different types of uncertainties encountered in APP problems, such as imprecise demand, production capacities with tolerance, fuzzy processing times can be specified by production managers using imprecise linguistic terms. They have led to the development of a number of fuzzy APP models and applications of fuzzy optimisation techniques [3]

In this paper, we propose a new fuzzy model for optimal APP in the presence of uncertainty. The novelty of the model is that the objective is to minimise the fuzzy total time required for production, storing manufactured products and their preparation for delivery to the customer. We introduce uncertain factors to take into consideration uncertainty in customer demand which is forecasted and can fluctuate around these values and uncertainty in manufactured quantities. As all the time parameters listed above, customer demand deviations and the parameters which describe the output of manufacturing process are fuzzy, both the associated objective function and constraints become fuzzy, too. We adapt and apply one of the methods for transforming the fuzzy linear programming optimisation model, with the fuzzy objective function and fuzzy constraints, into a crisp optimisation model with both the crisp objective function and crisp constraints [4].

The paper is organized as follows. Literature review on APP models and methodologies used and methods of modelling uncertain APP parameters is presented in Section 2. Problem statement is given in Section 3. The fuzzy aggregated production and inventory planning model are described in Section 4, while Section 5 contains case study and analyses of results of different experiments carried out using the proposed model. The conclusion is given in Section 6.

2. Literature Review

It is well recognised in the literature that treating uncertainty in APP models in an appropriate way brings an advantage to handling real world APP problems and brings them nearer to the practice [3]. Majority of the APP models handle uncertainty using a classic probability theory approach, and consider only one type of uncertainty which is based on randomness and frequency of a random event occurrence.

Linear mixed integer programs (MIPs) were developed to solve two production planning problems with demand uncertainty [5], when the manufacturer had a flexibility to accept or reject an order. The MIP method in production planning problems for multi-period and multi-items in maketo- order manufacturing system was used in [6]. Avraamidou and Pistikopoulos [7] developed a bi-level mixed integer linear programming model for a supply chain under demand uncertainty.

A very important issue in modern production planning is energy consumption. Today, the most manufacturers invest significant money assets to optimise and reduce energy consumption. In [8], a multi-objective linear programming problem with three objective functions including operational expense, energy expense and carbon emission, was analysed.

Zadeh proposed a new approach to handle different types of uncertainty, by introducing the concept of fuzzy sets [9]. It has been demonstrated in the literature that fuzzy sets can be successfully applied to modelling uncertainty where available information is vague or cannot be defined precisely due to the limited knowledge. One can find some good examples in the literature on how fuzzy sets are applied in supply chain management problems, for example in supply chain partners' collaboration [10], in MRP (material requirement problems) [3], in serial supply chains [11], etc. Tang et al. considered both uncertainty in customer demand and production capacity and modelled them as fuzzy values in a multi-product APP model [12] A fuzzy multi-objective mixedinteger non-linear programming model for a supply chain was proposed in [13]. Fuzzy customer demand was considered in three objective functions that minimised the total supply chain cost, total maximum product shortages, and the rate of changes in human resources.

3. Problem Statement

A problem is to generate the optimum aggregate production and inventory plan for a supplier for a given planning time horizon. The supplier operates in a "make-to-order" manner and has to prepare a production and inventory plan in such a way as to satisfy customer demand and optimise an associated performance measure in the considered time horizon.

The planning time horizon is discretised into a series of subsequent discrete time periods. The APP determines 3 quantities to be generated for each time period in the planning time horizon: (1) optimal production quantity to be manufactured, (2) the safety stock quantity that should be kept in the warehouse and (3) the quantity that should be delivered to the customer.

If the same production line is used for manufacturing of different products for more than one customer, an efficient use of the production line is of paramount importance for the production process.

All these uncertainties have to be taken into account when generating the optimal production and inventory plan.

4. Fuzzy Aggregated Production and Inventory Planning

4.1. Notation

The following notation is used:

- i – index of a time period in a planning horizon, $i = 1,\dots,n,$
- D_i – customer demand in period i, $i = 1,\dots,n,$
- $\tilde{t}_p$ – fuzzy production time per unit of product (in minutes), with trapezoidal membership function $\tilde{t}_p = (t_{p1}, t_{p2}, t_{p3}, \dots)$,
- $\tilde{t}_s$ – fuzzy preparation time for shipping to customer per unit of product (in minutes), with trapezoidal membership function $\tilde{t}_s = (t_{s1}, t_{s2}, t_{s3}, t_{s4})$,
- $\tilde{t}_t$ – fuzzy preparation time for shipping to customer per unit of product (in minutes), with trapezoidal membership function $\tilde{t}_t = (t_{t1}, t_{t2}, t_{t3}, t_{t4})$,
- $\tilde{w}_i^d$ fuzzy factor for uncertain customer demand deviation from forecasted value in period i, $i = 1,\dots,n,$ with triangular membership function $\tilde{w}_i^d = (w_{il}^d, w_{im}^d, w_{iu}^d)$,
- $\tilde{w}_i^p$ fuzzy factor for uncertain production quantity output in period i, $i = 1,\dots,n,$ with triangular membership function $\tilde{w}_i^p = (\tilde{w}_{il}^p, w_{im}^p, w_{iu}^p)$,
- T^l – minimum "days of inventory" in the warehouse,
- T^u – maximum "days of inventory" in the warehouse,
- C – machine capacity.

Decision variables:

- P_i – quantity manufactured in period i,
- Ss_i – safety stock in period i,
- Q_i – quantity delivered to customer in period i.

4.2. Fuzzy App LP Model

The objective is to minimize the total material lead time $\tilde{Z}$ including the production time $\tilde{t}_p p_i$, warehouse time $\tilde{t}_s Ss_i$ required for storing safety stock of manufactured products and time for preparation of delivery to customers $\tilde{t}_t Q_i$, as follows:

$$min\ \tilde{Z} = \sum_{i=1}^{n} \tilde{t}_p p_i + \tilde{t}_s Ss_i + \tilde{t}_t Q_i \tag{1}$$

The following constraints are considered:

Uncertain customer demand $\tilde{w}_i^d\ D_i$ in each time period i is satisfied using the uncertain production $\tilde{w}_i^p P_i$ or safety stock Ss_i :

$$Ss_i + \tilde{w}_i^d D_i \geq \tilde{w}_i^p P_i\ ,\ i=1\ ,\ \dots\ ,\ n \tag{2}$$

The safety stock Ss_{i+1} in each time period $i + 1$ is equal to the stock in the previous period Ss_i increased by uncertain production in the previous period, $\tilde{w}_i^p P_i$, and reduced by uncertain customer demand, i.e., quantity delivered to the customer in the previous period, $\tilde{w}_i^d D_i$:

$$Ss_{i+1} = Ss_i + \tilde{w}_i^p P_i - \tilde{w}_i^d D_{i,}\ i = 1, \dots, n \tag{3}$$

Installed machine capacity C produces uncertain $w_i^p P_i$ units per period i :

$$\tilde{w}_i^p P_i \geq 0\ ,\ i = 1,\dots,n \tag{4}$$

$$C \geq \tilde{w}_i^p P_i, \; i=1,\ldots,n \tag{5}$$

The safety stock Ss_i in period i is defined by a supplier's target to cover between T^l and T^u days of uncertain customer demand $\tilde{w}_i^d\, D_i$ in that period:

$$Ss_i \geq T^l \tilde{w}_i^d D_i, \; i=1,\ldots,n \tag{6}$$

$$T^l \tilde{w}_i^d D_i \geq Ss_i, \; i=1,\ldots,n \tag{7}$$

The delivery Q_i in each period i must be equal to uncertain customer demand $\tilde{w}_i^d D_i$ in order to operate with the maximum service level - 100%.

$$Q_i = \tilde{w}_i^d D_i, \; i=1,\ldots,n \tag{8}$$

Decision variables P_i , Ss_i and Q_i in each time period i are non-negative:

$$P_i \,,\, Ss_i \,,\, Q_i \geq 0, \; i=1,\ldots, n \tag{9}$$

4.3. From the Fuzzy App Optimization Model to a Crisp App Optimization Model

We applied a method developed by Jimenez et al [4] to transform the fuzzy APP model into a crisp APP model. We adapted it in such a way as to handle fuzzy parameters in the objective function with trapezoidal membership functions.

The transformation includes 3 steps as follows.

Step 1. The decision maker specifies the feasibility degree β of constraint satisfaction he/she is ready to accept. Let us assume that the lowest feasibility degree that the decision maker is ready to consider is Neither acceptable nor unacceptable solution - β of course, it can be changed to any other feasibility degree β from interval [0, 1].

The crisp optimisation model is solved iteratively for each feasibility degree β = 0.5., 0.6,...,0.9,0.95,0.99 and 1 where each solution is β-feasible, i.e., the minimum of feasibility achieved for all constraints is β. The b-feasible solution P_i, Ss_i and Q_i are found as follows.

First, fuzzy parameters $\tilde{t}_p$, $\tilde{t}_s$ and $\tilde{t}_t$ in the objective function are mapped into their crisp expected values. They are calculated as the middle points of the Expected intervals.

Step 2. The decision maker specifies tolerance thresholds to obtained fuzzy objective function values achieved for different β-satisfaction of constraints. The shortest time $\underline{Z}$ will be achieved for the lowest constraints' satisfaction $\tilde{\beta}$ = 0.5 and the longest time Z for the highest constraints' satisfaction β = 1. We assume that the tolerance function G is linear between these two tolerance thresholds, the shortest time $\underline{Z}$ and the longest time Z . The membership function is:

$$\mu_{\tilde{G}}(Z) = \left\{ \begin{array}{ll} 1, & z < \underline{Z} \\ \dfrac{\overline{Z}-z}{Z-\underline{Z}}, & \underline{Z} \leq z \leq \overline{Z} \\ 0\,, & z > \overline{Z} \end{array} \right\} \tag{10}$$

We propose the following formula to calculate tolerance $K_{\tilde{G}}(Z\ (\tilde{\beta}))$ to obtained objective function value $Z\,(\tilde{\beta})$ when the feasibility of constrains is β.

$$K_{\tilde{G}}(\tilde{Z}(\beta)) = \frac{\overline{Z} - EV(\overline{Z}(\beta))}{\overline{Z}-\underline{Z}} \tag{11}$$

Step 3. Balance between the feasibility degree of constraints β and the satisfaction degree of solution, is $K_{\tilde{G}}(Z(\tilde{\beta}))$ calculated as:

$$\beta \, . \, K_{\tilde{G}}(\tilde{Z}(\beta)) \tag{12}$$

The solution P_i , Ss_i , Q_i , $i=1,...n$ which achieves the highest balance $\max\limits_{\beta=0.5,0.6,..,0.9,0.95,0.99,1} \beta . K_{\tilde{G}}(\tilde{Z}(\beta))$, is recommended.

5. Case Study

We considered a first tier supplier in the automotive industry located in Serbia, which has become an increasingly important industrial sector in the recent years. The factory supplies window regulators to a number of European car manufacturers.

We analysed one production line which manufactures multi products for two different customers. All products belong to the same product family. They are packed in two types of plastic containers specified by the customers. The developed fuzzy APP model is applied to determine the minimal time required for production and logistics processes. The planning horizon is selected to be a period of 12 weeks. Customer demand forecast for 12 weeks is a typical mid-term forecast used in the automotive industry for production planning. A longer period of customer demand has huge uncertainty and is not reliable for sustainable production planning.

The result of fuzzy APP model is presented in Table 1. The calculation is performed using formulas (1-12) and simplex method of classical LP solver. An algorithm is developed in software Visual Studio 2015 in C++ programming language. The performance of computer: Intel processor i3-2120 (3M Cache, 3.30 GHz), 8G RAM memory (2133 MHz).

Feas. degr. β	Decision variables			Fuzzy objective function value				Tolerance $\mu_{\tilde{G}}(Z)$	Balance $K_{\tilde{G}}(Z(\beta))$	Object.func value Z
	$\sum p_i$	$\sum Ss_i$	$\sum Q_i$	Z_1	Z_2	Z_3	Z_4			
0.5	188776	123372	194883	54839	58009	62338	68889	0.733	0.3665	61382
0.6	194107	126022	198581	56235	59489	63927	70646	0.666	0.3995	62948
0.7	199546	128672	202279	57654	60992	65541	72429	0.598	0.4183	64538
0.8	205098	131323	205977	59094	62519	67180	74241	0.528	0.4225	66153
0.9	210766	134141	209674	60562	64074	68851	76089	0.458	0.4118	67799
0.95	213644	135620	211523	61305	64863	69698	77027	0.422	0.4006	68634
0.99	215968	137007	213003	61909	65503	70386	77791	0.393	0.3886	69313
1	216552	137365	213372	62061	65664	70559	77983	0.385	0.3852	69484

Table 1. Results of the Fuzzy APP Model

Fuzzy factor $\tilde{w}_i^p$ is symmetrical triangular fuzzy number and modeled by logistics expert in enterprise as 10% of production output deviation (0,9, 1, 1,1). Fuzzy factor $\tilde{w}_i^p$ is symmetrical triangular fuzzy number and obtained as previous calculation based on customer demand deviation in enterprise in period of 12 weeks before testing time window of 12 weeks:

$$f_i = \frac{D_i}{\sigma} \, , \, i = 1 \, , \, ... \, , \, 12 \tag{13}$$

Where f_i is demand fluctuation, D_i customer demand prediction different for every week i, and σ is standard deviation of D_i, i=1,....,12. Production time $\tilde{t}_p$, time for safety stock storing in warehouse t_s, and time for preparation of shipment to customer $\tilde{t}_t$ are measured in enterprise and presented as nonsymmetrical trapezoidal fuzzy numbers: $\tilde{t}_p$ = (0,20,0,21,0,23,0,25), $\tilde{t}_s$ = (0,020,0,023,0,028,0,04), $\tilde{t}_t$ = (0,075,0,077,0,082,0,086) minutes per unit product. The target of safety stock keeping

Week, i	Prod.planning with using maximal prod. capacity (C=19000 pcs/week)			Prod. planning with using safety stock target (3 days of coverage)			Results of experiment from fuzzy APP model			Realised production plan in enterprice		
	Ss_i	P_i	Q_i	Ss_i	P_i	Q_i	Ss_i	P_i	Q_i	Ss_i	P_i	Q_i
1	10000	19000	16516	10000	15962	16516	17031	10070	16516	10000	17450	16516
2	12484	19000	15744	9446	16424	15744	10585	16585	15744	10934	17800	15744
3	15740	19000	16877	10126	16424	16877	11426	19588	16877	12990	17750	16877
4	17863	19000	16122	9673	19667	16122	14137	19089	16122	13863	18350	16122
5	20741	19000	22030	13218	15651	22030	17104	12564	22030	16091	17740	22030
6	17711	19000	11398	6839	13637	11398	7638	17272	11398	11801	17850	11398
7	25313	19000	15130	9078	16185	15130	13513	19443	15130	18253	15950	15130
8	29183	19000	16888	10133	19943	16888	17826	18106	16888	19073	16840	16888
9	31295	19000	21980	13188	18087	21980	19044	18245	21980	19025	17530	21980
10	28315	19000	15492	9295	15952	15492	15309	18264	15492	14575	17440	15492
11	31823	19000	16258	9755	16328	16258	18081	18262	16258	16523	17530	16258
12	34565	19000	16374	9824	11550	16374	20085	17610	16374	17795	18360	16374
Σ	275033	228000	200809	120576	195809	200809	181778	205098	200809	180923	210590	200809

Table 2. Comparing with Two Strategies of Production and Inventory Planning

days used in calculation is betwwen $T^l = 3$ and $T^u = 5$ days. the machine capacity is $C = 19000$ pcs/week for 5 working days in a week.

The optimal value of objective function in fuzzy APP model is 66153 minutes for whole time window of 12 weeks (5512 minutes/ weekly; 1102 minutes/daily; 18,4 h/daily) for feasibility degree b = 0,8 (Table 1). Testing is performed in enterprise for both common used strategies of production/inventory planning and compared with realized production plan in enterprise and result of fuzzy APP model. For comparing purpose an initial safety stock value in week 1 for both strategies and realized production plan is supposed $Ss_1 = 10000$ pcs . The result of fuzzy APP model for week 1 is $Ss_1 = 17031$ pcs.

In Table 2 .are presented data for both strategies of production and inventory planning in enterprise. For every strategy and real-world data the delivered quantity is the same $\sum Q_i = 200809$ pcs of goods for period of 12 weeks. The quantity of safety stock $\sum Ss_i$ and produced parts $\sum P_i$ are different. They are highest in strategy of using maximal capacity 275033 and 228000 pcs, respectively. The safety stock quantity is lowest in strategy with 3 days of inventory, $\sum Ss_i = 120576$ pcs.

An objective function value (Table 3) which represents the total time required for all production and logistics operation in delivery goods to customers is the highest and in same time the most unfavorable in strategy with maximal capacity usage; it is 74476 minutes for 12 weeks (6206 minutes weekly, 1241 minutes daily, 20,7 hours daily). The lowest total time is in strategy with using safety stock target 62760 minutes for 12 weeks (5230 minutes weekly, 1046 minutes daily, 17,4 hours daily). The outcome of fuzzy APP model shows better result than strategy prod. planning using maximal prod. capacity and realized prod. plan in enterprise. Comparing with strategy with using safety stock target of 3 days the outcome is worsebecause the safety stock target used in fuzzy APP model is between 3 and 5 days. Normally the objective function is higher.

Prod.planning with using maximal prod. capacity (C=19000 pcs/week)		Prod. planning with using safety stock target (3 days of coverage)		Results of experiment from fuzzy APP model		Realized production plan in enterprise	
object. function.	Inven cover	object. function.	Inven cover	object. function.	Inven cover	object. function.	Inven cover
74476	6,9	62760	3	66640	4,5	67822	4,6

Table 3. objective Function Value and Inventory Coverage

4. Conclusion

The review of the published APP models in literature showed that they did not consider and analysed the material flow time in the APP problems. All of the APP models have been developed to minimize operational cost in manufacturing, dealing with impacts on production, inventory or delivery costs. However, in some industrial sectors, the material flow time is a very important factor and cannot be neglected, because it has a big impact on the total measure of manufacturer performance. We consider a real world APP problem in the automotive industry and develop a fuzzy LP model which considers a material flow as the measure of performance. Results obtained using the proposed APP model are better compared to the practical results; the total material flow time is shorter using the proposed APP model. Practical application of the APP model in the factory would contribute to optimised production and inventory plan with higher customer satisfaction with the service level. Finally, the cash flow in the factory can be much improved.

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