

# Digital Filter for Measuring the Transfer Coefficient Amplitude of the Interference Measures

Georgy Mihov and Dimitar Badarov  
The Faculty of Electronic Engineering and Technologies at Technical University of Sofia  
8 Kl. Ohridski Blvd  
Sofia 1000, Bulgaria  
[gsm@tu-sofia.bg](mailto:gsm@tu-sofia.bg) , [dbadarov@tu-sofia.bg](mailto:dbadarov@tu-sofia.bg)



**ABSTRACT:** *The interference frequency measures are presented with test procedures. In the interference signal, we have prepared the mean values of the digital filter for measuring the transfer coefficient. We have implemented this system in the MATLAB environment. The power-interference frequency measure is used for the second and third-order harmonics. It also added the elimination of the DC set. We have finally created the amplitude of the interference measures.*

**Keywords:** Mains Interference, Frequency Measurement, Harmonic Suppression

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## 1. Introduction

A method for measurement of a mains interference (hum) frequency in electrocardiographic (ECG) signal is represented in [1] that is further developed for mains frequency measurement [2, 3]. It determines the deviation  $dF$  of the mains frequency  $F$ , calculating the change of the transfer coefficient  $K(f)$  of an averaging filter – see Figure 1.

The method has advantages in front of other existing methods [4, 5], especially when existing recordings with relatively low sampling frequency are used. The errors induced by harmonic content of the mains interference and by its amplitude changing are analyzed in [6].

In the present work a computer based [7, 8] test methodology allowing detailed investigation of the method is proposed.

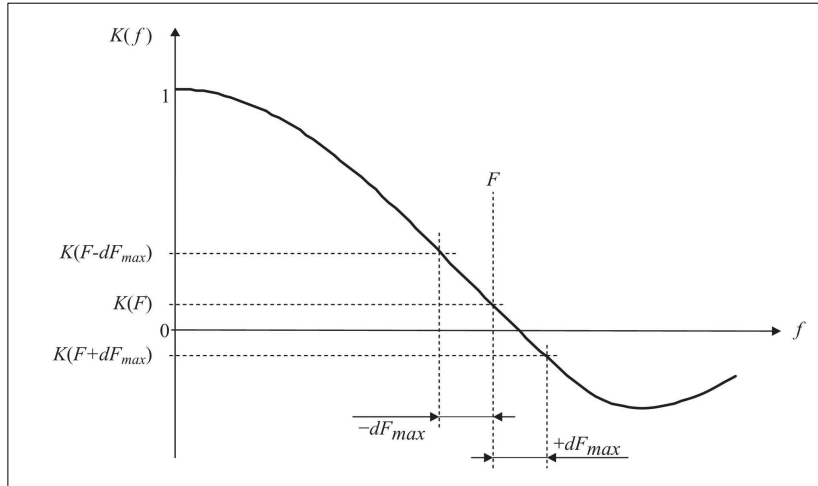


Figure 1. An averaging filter transfer coefficient changing due to the mains frequency deviation

## 2. Harmonic Suppression Filters

### 2.1. Third Harmonic Suppression

For the purpose of third harmonic filtration, a digital filter  $Y$  is synthesized by summation of two filters by the methodology proposed in [9]. Two “three-point” filters are used  $Y1$  and  $Y2$  which have frequencies of their first zeroes on the both sides of the third harmonic of the mains interference  $F_3$ . The two filters are summed by multiplication with complementary to unity coefficients  $(1 - k_y)$  and  $k_y$ .

$$Y_i = (1 - k_y) \cdot Y1_i + k_y \cdot Y2_i ; \quad \left| \begin{array}{l} Y1_i = \frac{B_{i-m_3} + 2B_i + B_{i+m_3}}{4} \\ Y2_i = \frac{B_{i-m_3-1} + 2B_i + B_{i+m_3+1}}{4} \end{array} \right. , \quad (1)$$

where  $m^3$  is the rounded to the lower or equal integer number of samples in the half period of the third harmonic  $F_3$  of the mains frequency.

The transfer coefficient of the resultant filter is also expressed by summation of the transfer coefficients of the two filters multiplied by the complementary to unity coefficients  $(1 - k_y)$  and  $k_y$ .

$$K(f) = (1 - k_y) \cdot K1(f) + k_y \cdot K2(f) , \quad \left| \begin{array}{l} K1(f) = \cos^2 \frac{m_3 \pi f}{Q} \\ K2(f) = \cos^2 \frac{(m_3+1) \pi f}{Q} \end{array} \right. , \quad (2)$$

The coefficient  $k_y$  is defined by applying the condition the first derivative of the transfer function of the summed filter

$K^l(f)|_{f=F_3} = (1 - k_y) \cdot K1^l(f)|_{f=F_3} + k_y \cdot K2^l(f)|_{f=F_3} = 0$  to have a zero value for the third harmonic of the mains interference  $f = F_3$ , from where:

$$k_y = \frac{K1^l(F_3)}{K1^l(F_3) - K2^l(F_3)} ;$$

$$\left| \begin{aligned} K1^l(F_3) &= -\frac{\pi m_3 F_3}{Q} \operatorname{Sin} \frac{2\pi m_3 F_3}{Q} , \\ K2^l(F_3) &= -\frac{\pi(m_3+1)F_3}{Q} \operatorname{Sin} \frac{2\pi(m_3+1)F_3}{Q} ; \end{aligned} \right. \quad (3)$$

and when  $f=F_3$  the parameter  $k_y$  have a value:

$$k_y = \frac{\operatorname{Sin} \frac{2\pi m_3 F_3}{Q}}{\operatorname{Sin} \frac{2\pi m_3 F_3}{Q} - \frac{(m_3+1)}{m_3} \operatorname{Sin} \frac{2\pi(m_3+1)F_3}{Q}} , \quad (4)$$

This way the summed filter  $Y$  by equation (1) has a minimum of the transfer coefficient for the frequency of the third harmonic of the mains interference  $F_3$  but it is not zero. For that purpose, it is recurrently modified by the methodology proposed in [1]:

$$Y^*_i = Y_i - (X_i - Y_i) \frac{K(F_3)}{1 - K(F_3)} , \quad (5)$$

where  $K(F_3)$  is the transfer coefficient of the summed filter calculated by (2)

$$K(F_3) = (1 - k_y) \cdot K1(F_3) + k_y \cdot K2(F_3) , \quad \left| \begin{aligned} K1(F_3) &= \cos^2 \frac{m_3 \pi F_3}{Q} \\ K2(F_3) &= \cos^2 \frac{(m_3+1) \pi F_3}{Q} \end{aligned} \right. \quad (6)$$

The modified filter  $Y^*$  acquires transfer coefficient of zero for the third harmonic of the mains frequency  $F_3$  keeping tangential character to the frequency axis. The synthesis of digital filter for suppression of the third harmonic of the mains frequency  $F_3 = 150 \text{ Hz}$  is represented on Figure 2. The plot contains the first  $Y1$  (blue curve - b) and the second  $Y2$  (red curve - r) from the initial “three-point” filters, the summed filter  $Y$  (green curve - g) and the modified filter  $Y^*$  (black curve - k).  $H$  is the impulse vector of the summed filter  $K(F_3)$  containing the weight coefficients of the impulse response.

The filter for the third harmonic reduces the amplitude of the mains frequency with about 0.7. The conducted experiments showed that the amplitude of the steady value  $F$  of the mains frequency does not affect the accuracy of the measurement.

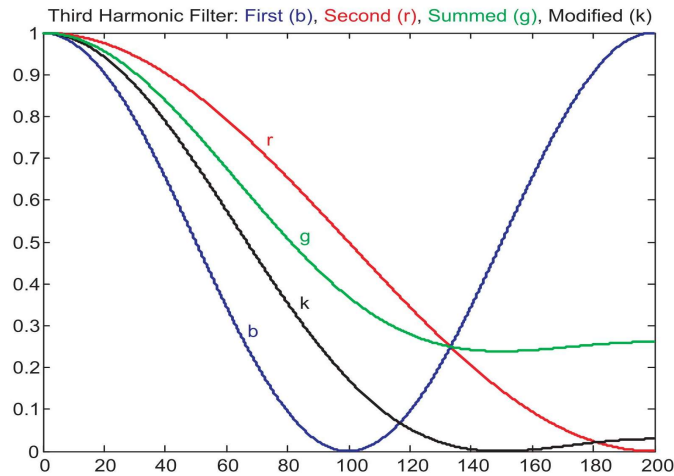


Figure 2. Synthesis of filter for third harmonic of the mains frequency  $F_3 = 150 \text{ Hz}$  for sampling frequency  $Q = 400 \text{ Hz}$ ,  $H=[0.0858 \ 0.2426 \ 0.3431 \ 0.2426 \ 0.0858]$

## 2.2. Second Harmonic Suppression

In similar way we can build digital filter for suppressing the other harmonics of the extracted mains signal. The synthesis of digital filter for second harmonic of the mains frequency  $F_2 = 100 \text{ Hz}$  is represented on Figure 3. The plots are the same as the ones shown on Figure 2. The frequency of the second harmonic is substituted in equations (1-6) along with the number of samples in the half period of the second harmonic  $m_2$ .

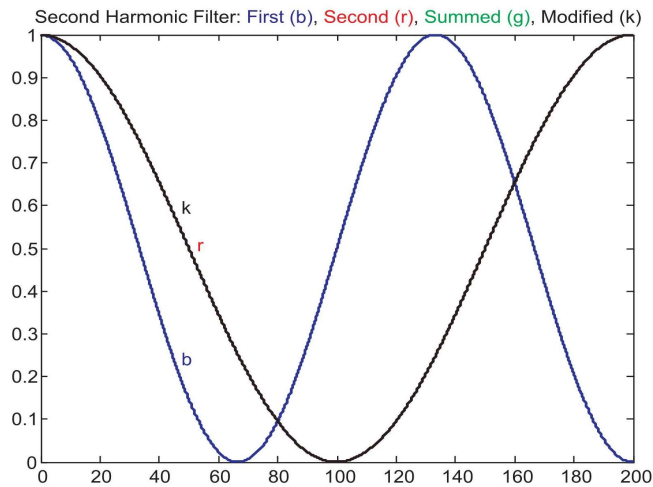


Figure 3. Synthesis of filter for second harmonic of the mains frequency  $F_2 = 100 \text{ Hz}$  for sampling frequency  $Q = 400 \text{ Hz}$ ,  $H=[0.25 \ 0 \ 0.5 \ 0 \ 0.25]$

The filter for the second harmonic suppression reduces the amplitude of the mains frequency with about 0.5 but this does not affect the accuracy of the measurement of the mains interference frequency.

A very important property can be seen in Fig. 3 with sampling frequency  $Q = 400 \text{ Hz}$ . The coefficient  $k_y$  equals one and due to that the complementary to unity coefficient  $(1 - k_y)$  for the other filter equals zero. This means that the summed “three-point” filter  $Y$  is identical with the second  $Y_2$  and the modified filter  $Y^*$  - see equation (1-6). On the figure the three filters are represented one over another. For that reason, only the modified filter is visualized. This can be seen also from the  $H$

vector of the impulse response which contains only one of the summed filters. This property allows easy construction of digital filter for second harmonic suppression when the sampling frequency is 400 Hz.

### 2.3. Elimination of the DC offset of the signal

The measured mains interference can contain a DC offset. It can be generated by the analog part of the measurement unit or from the output of the analog-to-digital conversion process when we are not using a four-quadrant analog-to-digital converter (ADC). For the purpose of mains frequency extraction from the input signal a simple “three-point” filter is used.

$$Y_i = \frac{-B_{i-m_2} + 2B_i - B_{i+m_2}}{4} \quad (7)$$

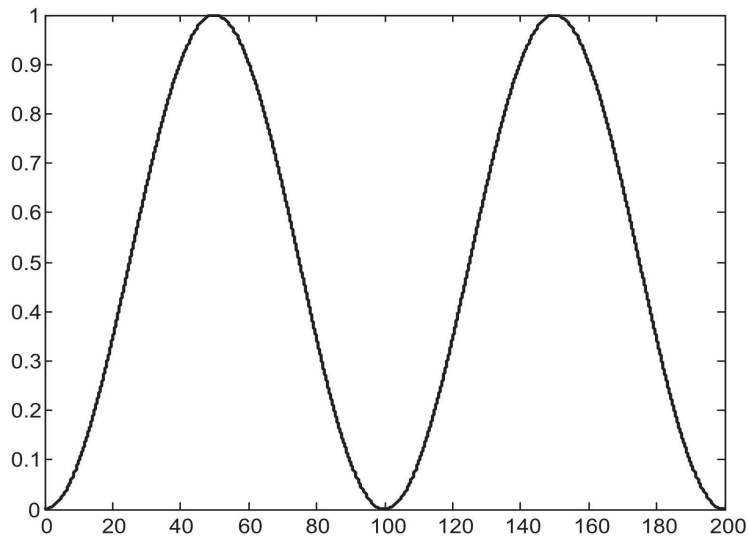


Figure 4. Filter for DC offset elimination for sampling frequency  $Q = 400$  Hz

From Figure 4. we can see that with sampling frequency of 400 Hz the DC offset filter is suppressing all even harmonics.

### 3. Algorithm and Program Implementation of the Method

The algorithm of the test methodology is represented on Figure 5. First the input signal is subjected to mains interference extracting using the filter shown on Figure 4.

The next step is to suppress the second harmonic of the mains interference signal. If the sampling frequency is multiple of the frequency of the second harmonic this step can be skipped because the mains interference extraction procedure suppresses all even harmonics of the interference.

Then it is checked if the frequency of the third harmonic of the interference  $F_3$  is higher than  $Q/2$ . If it is not true, the third harmonic suppression procedure is applied.

The calculation of the mains frequency deviation is accomplished by the application of a “two-point” filter using the procedure given in [3].

The results from the application of the different parts of the test program are shown on Figure 6. The experiment is carried out with a synthesized input signal containing mains interference with changing amplitude by a sinusoidal law with frequency of 0.1 Hz. The signal duration is 30 s which is divided on three parts. In the first 10 s the frequency of the mains interference is 50.25 Hz. In the second 10 s the frequency is 50 Hz and in the last 10 s the frequency is 49.75 Hz.

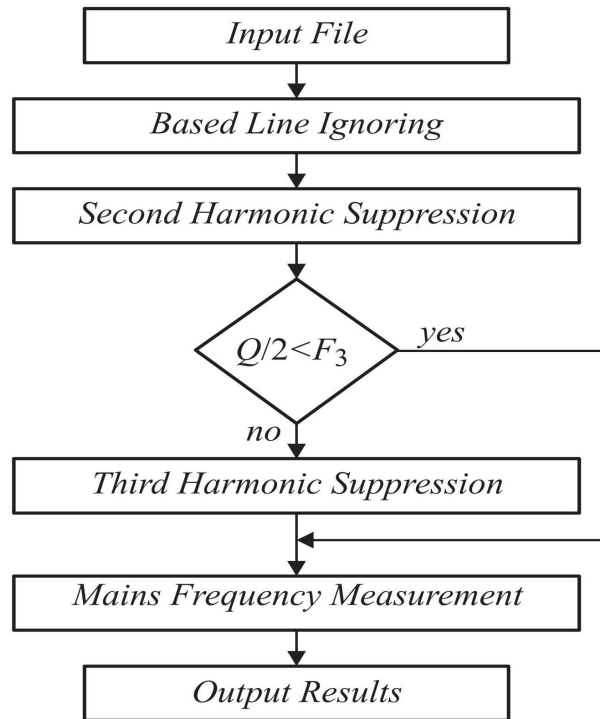
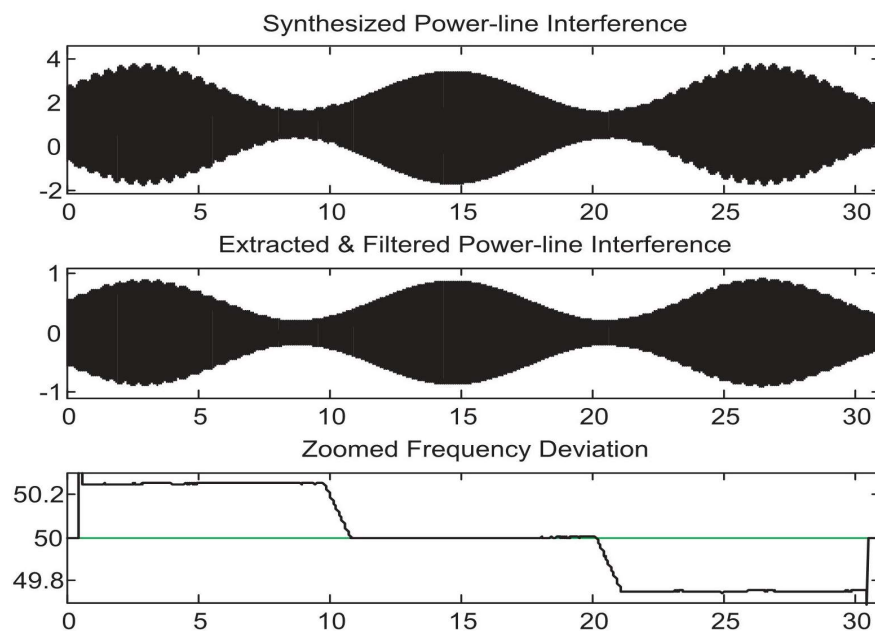
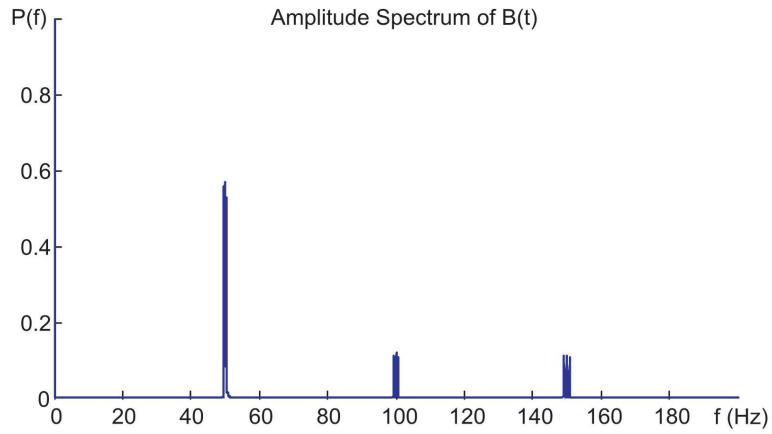


Figure 5. Algorithm of the test methodology for frequency measurement

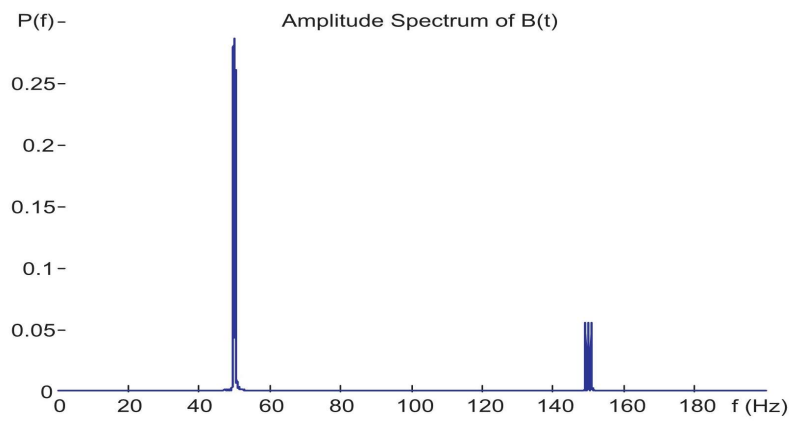
The first subplot of Figure 6. *a* shows the input file, the second subplot shows the file after the procedures for extraction and suppression of the harmonics of the mains interference. The third subplot shows the calculated mains frequency. The figures *b*, *c* and *d* contain the spectrum of the signals calculated by Fourier transform: *b* – of the input signal; *c* – after the extraction and suppression of the second harmonic and *d* – after the suppression of the third harmonic.



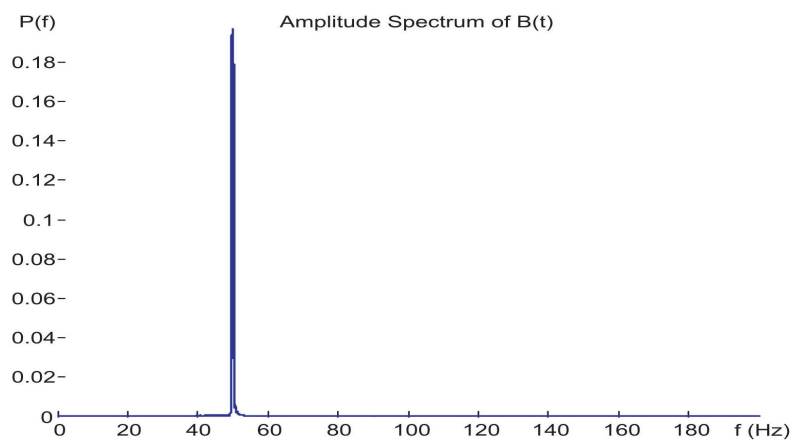
(a)



(b)



(c)



(d)

Figure 6. Test with a synthesized signal and sampling rate  $Q = 400 \text{ Hz}$ : **a**) – visualization of the result; **b**) – spectrum of the input signal; **c**) – spectrum after mains interference extraction and second harmonic suppression; **d**) – spectrum after third harmonic suppression

The next demonstrations are done with real interference signals extracted from old electrocardiographic signals. For the signal BN039\_400.adc the examined epoch is 8s and the calculated frequency values are averaged for period of 1s. For the signal BR024\_200.adc the examined epoch is 160s and the calculated frequency is averaged for a period of 10s.

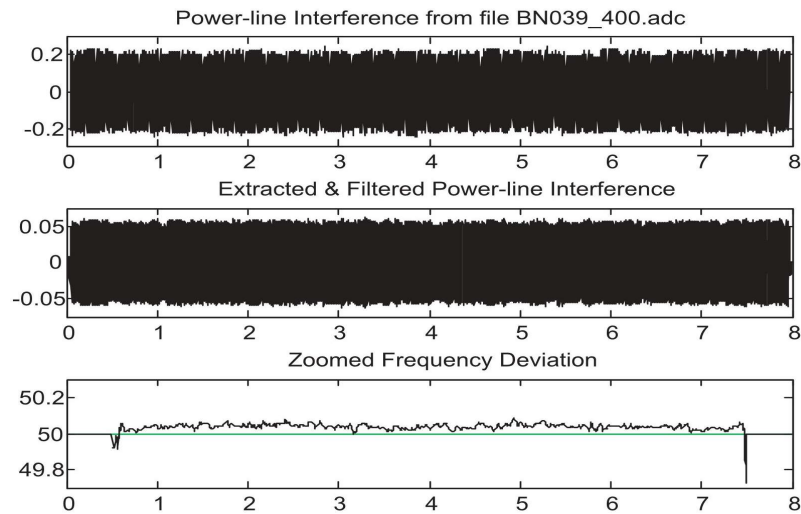


Figure 7. Experiments with real signals with  $Q = 400 \text{ Hz}$

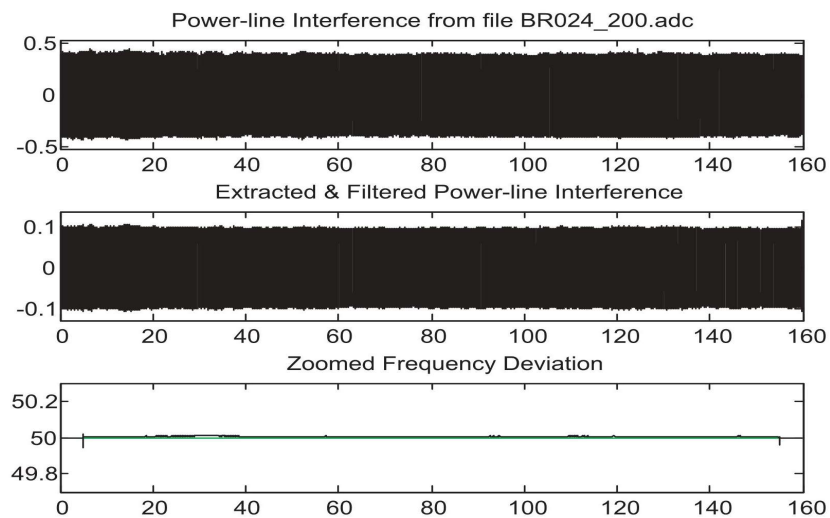


Figure 8. Experiments with real signals with  $Q = 200 \text{ Hz}$

#### 4. Conclusion

A test methodology is developed for examination of the method for mains interference frequency calculation using the transfer coefficient of a digital moving averaging filter.

For the cases in which the sampling frequency is higher than the doubled frequency of the third harmonic a suppression filter is developed. The filter is based on the summation and modification of two digital filters having transfer characteristics tangential to the frequency axis.

By the same methodology a filter for suppressing of the second harmonic is synthesized. The procedure keeps its accuracy without increase of the measurement error.



A filter for elimination of the DC component of the input signal is synthesized based on a “three-point” filter. It is found that when the sampling frequency is two or four times higher than the frequency of the second harmonic, the filter also suppresses the frequencies of all even harmonics of the mains interference.

The methodology is developed and tested in the MatLab environment. Different functions are developed for mains interference extraction and suppression of the second and third harmonic.

## References

- [1] Mihov, G., Ivanov, R. and Levkov, C. (2006) Subtraction method for removing powerline interference from ECG in case of frequency deviation. *In: Proceedings of the Technical University – Sofia*, Vol. 56, pp. 212–217, b. 2.
- [2] Mihov, G. (2012) Power-line interference frequency and amplitude measurement using the subtraction procedure. *Proceedings of the Technical University of Sofia*, 62, 189–198, (in Bulgarian).
- [3] Badarov, D. and Mihov, G. (2015) Mains frequency deviation measurement by using elements of the subtraction procedure based on Xilinx FPGA. *Proceedings of the 24th International Conference Electronics – ET*, 9, 112–115.
- [4] Jovanovic, B. and Damnjanovic, M. (2004) Digital system for power line frequency measurement. *Proceedings of the 48th ETRAN Conference*, 1, 29–32.
- [5] Ovcharov, S., Tyuliev, N. and Yakimov, P. “A system for mains frequency observing”, *ELECTRONICSET’94*, Vol. I. Sozopol: Bulgaria (28–30 September, 1994), pp. 98–103.
- [6] Badarov, D. and Mihov, G. (2017). “Analysis of the Spectrum and Amplitude Error of the Frequency Measurement Using Elements of the Subtraction Procedure”, *IEEE 40-th International Spring Seminar on Electronics Technology ISSE2017*, Sofia, Bulgaria, May 10-14, 2017, 1-6 p.
- [7] Romansky, R. (2012). A Formal Approach for Modelling and Evaluation in the Field of Computing”, *International Transaction on Electrical, Electronics & Communication Engineering*, UK, No 4 (vol. 2) , 2012, pp.1-7.
- [8] Mihov, G.S. and Badarov, D.H. (2017). Testing of digital filters for power-line interference removal from ECG signals”, *ELECTRONICS ET2017*, Sozopol, September 13-15, 2017, pp. 1-6.
- [9] Mihov, G. (2007). Investigation of the FIR Filters Usage in the Subtraction Method for Power-Line Interference Removing from ECG”, *Proceedings of the Technical University – Sofia*, Vol. 57, b. 2, 2007, pp. 84-93.