The Transmission of Differentially Encoded Offset Quadrature Phase Shift Keying on Gamma-Shadowed Nakagami-m Fading Channel

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ABSTRACT: *The transmission of differentially encoded offset quadrature phase shift keying on gamma-shadowed Nakagami-m fading channel is discussed in this paper. This leads to the formation of coherent detection of differentially encoded quadrature phase. The carrier signal recovery imperfection has been considered in the phase error in the phase-locked loop. The Tikhon distribution represents the phase error. The bit error probability expressions (BEPs) are discussed and the numerical results are provided.*

Keywords: DE-QPSK, DE-OQPSK, BEP, Phase Noise, Phase Locked Loop

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1. Introduction

In wireless communication one of the main problem is fading, which is described by several statistical models as Rayleigh, Rician or Nakagami model. Nakagami model is more general than Rayleigh and Rician. Because of that, it is very often used in observations. The basis in all these fading models is the assumption that the average signal power measured is constant. However, the existence of multiple scattering may lead to the case where the received average power becomes random. This phenomenon is called shadowing. At first the shadowing was modeled using lognormal distribution. However, it was often inconvenient for further analyses because the obtained composite probability density function (PDF) is in integral form. Newly, gamma distribution was proposed because it is mathematically more corresponding model. Since fading and shadowing occur simultaneously in wireless systems, it is necessary to have models that can describe the faded-shadowed channel [1]-[7]. In this paper, we considered the composite signal modeled by gamma-shadowing Nakagami-*m* fading distribution.

Quadrature phase-shift keying (QPSK) and offset quadrature phase-shift keying (OQPSK) are widely used modulation techniques. Because of the symmetry in two dimensional signal constellations, ambiguities about the exact phase orientation of the received signal set exist at the receiver. In PSK systems, this ambiguity is resolved using differential encoding (DE). Differentially encoded QPSK (DE-QPSK) and differentially encoded OQPSK (DE-OQPSK) were introduced by researches as a mean of resolving the carrier phase ambiguity of the data before transmission. One of the methods for detecting DE signals is coherent detection [8]. The phase-locked loop (PLL) is used for carrier signal recovery. The phase error is a difference between the incoming signal phase and the recovered carrier signal phase in PLL. It is a statistical process described by Tikhonov distribution. When the receiver is not ideal, a certain phaseerror appears.

The expressions for bit-error probability (BEP) of DEQPSK and DE-OQPSK for the ideal carrier-synchronization over additive white Gaussian noise (AWGN) channel was shown and compared in [9]. After the ideal case, the conditional BEPs of both modulations are given and combined with the different statistics of the phase error in the maximum *a posteriori* (MAP) estimation carriersynchronization loops to obtain the expressions of average BEPs for both modulations [9].

In this paper, we consider DE-QPSK and DE-OQPSK signals transmission over the gamma-shadowed Nakagami-*m* fading channel. The expressions for BEP of DE-QPSK and DE-OQPSK, while the imperfect carrier signal recovery has been taken into account through the phase error, are analyzed and numerical results are presented.

2. Channel Model

As mentioned, we consider transmission of the signal over the channel affected by gamma-shadowed Nakagami-*m* fading.

Let the received signal envelope *r* has Nakagami distribution given by [1]

$$p_{r/\Omega}(r/\Omega) = \frac{2m^m r^{2m-1} e^{-\frac{m}{\Omega}r^2}}{\Gamma(m)\Omega^m}, \quad r > 0, \tag{1}$$

where *m* is the Nakagami parameter, Ω is the average power $\Omega = E[r^2]$ with *E* denoting mathematical expectation and $\Gamma(.)$ is the gamma function. The *m* parameter refers to the fading severity. With lower values of *m*, the fading is stronger. In the case m=1, we have Rayleigh fading, and $m = \infty$ is the no-fadingcase.

In the case when the shadowing is present, Ω is random variable and has gamma distribution given by [1]

$$p_{\Omega}(\Omega) = \frac{m_s^{m_s} \Omega^{m_s-1} e^{-\frac{m_s}{\Omega_s}\Omega}}{\Gamma(m_s)\Omega_s^{m_s}}, \quad \Omega > 0, \tag{2}$$

where $\Omega_s = E[x]$ is the gamma shadow area mean power. The parameter m_s refers to the shadowing severity. With lower values of m_s, the shadowing influence is stronger. In the case $m_s=\infty$, shadowing is not exist.

The composite envelope *r* of the gamma-shadowed Nakagami-*m* faded signal is:

$$p(r) = \int_0^{\infty} p_{r/\Omega}(r/\Omega) p_{\Omega}(\Omega) d\Omega. \tag{3}$$

Substituting (1) and (2) in (3), we have:

$$p(r) = \frac{4}{\Gamma(m)\Gamma(m_s)} \left(\frac{mm_s}{\Omega_s}\right)^{\frac{m+m_s}{2}} \times r^{m+m_s-1} K_{m_s-m}\left(2r\sqrt{\frac{mm_s}{\Omega_s}}\right) \tag{4}$$

where $K_v(.)$ is the modified Bessel function of the second kind and order v and $\Omega_s = E[r^2] = \overline{r^2}$ is the average power.

The instantaneous SNR per symbol, ρ, and the average SNR per symbol, ρ_0, are related by:

$$\frac{\mu^2}{\overline{\mu^2}} = \frac{\rho}{\rho_0}, \quad \mu > 0, \ \rho > 0. \tag{5}$$

The distribution of the SNR in gamma-shadowed Nakagami-m fading channel can be obtained using (4) and (5) by applying standard technique of transforming random variables:

$$p(\rho) = \frac{2}{\Gamma(m)\Gamma(m_s)} \left(\frac{mm_s}{\rho_0}\right)^{\frac{m+m_s}{2}} \times \rho^{\frac{m+m_s-2}{2}} K_{m_s-m}\left(2\sqrt{\frac{mm_s}{\rho_0}\rho}\right). \tag{6}$$

Remark that ρ is the instantaneous SNR per symbol, and ρ_0 is the average SNR per symbol. The average SNR per bit is $\rho_{0b}=\rho_0/\log_2 M = \rho_0/2$ in the case of quadrature modulation formats.

3. Average BEP Performance in the Presence of Carrier Phase Error Over the Gamma- Shadowed Nakagami Fading Channel

3.1. DE-QPSK

For DE-QPSK, the expression for conditional BEP in the presence of a phase error ϕ_c, due to imperfect carrier synchronization, in the channel under the influence of fading is (similarly as in [9]):

$$P_b(\phi_c;\rho)|_{DE-QPSK} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c - \sin\phi_c)\right) \times \left[1 - \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c - \sin\phi_c)\right)\right]$$

$$+\frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c + \sin\phi_c)\right) \times \left[1 - \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c + \sin\phi_c)\right)\right] \tag{7}$$

where ρ represents SNR per symbol.

In order to evaluate the average BEP for DE-QPSK, the PDF of phase error should be known. The conditional BEP must be averaged over that PDF.

The appropriate PDF of loop's phase error in the form of the Tikhonov distribution for DE-QPSK is given by [9]:

$$p_{\phi_c}(\phi_c;\rho)\Big|_{QPSK} = \frac{4\exp(\rho_{eq}(\rho)\cos 4\phi_c)}{2\pi I_0(\rho_{eq}(\rho))},$$

$$\rho_{eq}(\rho) = \frac{\rho_{PLL}(\rho) S_L\big|_{QPSK}(\rho)}{16}, \ -\frac{\pi}{4} \le \phi_c \le \frac{\pi}{4}, \tag{8}$$

where ρ_{eq} represents an equivalent loop's SNR, ρ_{PLL} is loop's SNR that can be expressed as $\rho_{PLL}(\rho)= P/N_0B_L = (E_b/N_0)/(B_LT_b) = (\rho/2)/(B_LT_b)$ (B_L denotes the one-sided loop bandwidth). Degradation term referred to as "squaring loss", S_L is given by [9]:

$$S_L|_{QPSK}(\rho) = \frac{\left[\operatorname{erf}\left(\sqrt{\frac{\rho}{2}}\right) - 2\sqrt{\frac{\rho}{2\pi}}\exp\left(-\frac{\rho}{2}\right)\right]^2}{1+\rho-2\left[\sqrt{\frac{\rho}{2}}\operatorname{erf}\left(\sqrt{\frac{\rho}{2}}\right)+\frac{1}{\sqrt{\pi}}\exp\left(-\frac{\rho}{2}\right)\right]^2} \tag{9}$$

To obtain average BEP, it is necessary to average conditional BEP (7) over PDF of phase error (8). Under the influence of fading, the instantaneous SNR is random variable. It is also required to do averaging of (7) over PDF of instantaneous SNR per symbol (6). The BEP of DE-QPSK is:

$$P_b|_{DE-QPSK} = \int_{\rho=0}^{\infty}\int_{\phi_c=-\pi/4}^{\pi/4} P_b(\phi_c;\rho)\Big|_{DE-QPSK} \times p_{\phi_c}(\phi_c;\rho)\Big|_{QPSK}\, p(\rho)\,d\phi_c\,d\rho\,. \tag{10}$$

3.2. DE-OQPSK

For DE-QPSK, the expression for conditional BEP in the presence of a phase error ϕ_c, due to imperfect carrier synchronization, in the channel under the influence of fading is (similarly as in [9]):

$$\begin{aligned} P_b(\phi_c;\rho)|_{DE-OQPSK} == &\left[\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\rho}{2}}\cos\phi_c\right)\right. \\ &+\frac{1}{4}\operatorname{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c-\sin\phi_c)\right) \\ &\left.+\frac{1}{4}\operatorname{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c+\sin\phi_c)\right)\right] \\ &\times\left[1-\frac{1}{4}\operatorname{erfc}\left(\sqrt{\frac{\rho}{2}}\cos\phi_c\right)\right. \\ &-\frac{1}{8}\operatorname{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c-\sin\phi_c)\right) \\ &\left.-\frac{1}{8}\operatorname{erfc}\left(\sqrt{\frac{\rho}{2}}(\cos\phi_c+\sin\phi_c)\right)\right]. \end{aligned} \tag{11}$$

The appropriate PDF of loop's phase error in the form of the Tikhonov distribution for DE-OQPSK is given by [9]:

$$p_{\phi_c}(\phi_c;\rho)|_{OQPSK} = \frac{2\exp(\rho_{eq}(\rho)\cos 2\phi_c)}{2\pi I_0(\rho_{eq}(\rho))},\ \rho_{eq}(\rho) = \frac{\rho_{PLL}(\rho)S_L\big|_{OQPSK}(\rho)}{4},\ -\frac{\pi}{2}\le\phi_c\le\frac{\pi}{2}\,. \tag{12}$$

"Squaring loss", S_L, for DE-OQPSK is given by [9]:

$$S_L|_{OQPSK}(\rho) = \frac{\left[\mathrm{erf}\left(\sqrt{\frac{\rho}{2}}\right) - \sqrt{\frac{\rho}{2\pi}}\exp\left(-\frac{\rho}{2}\right)\right]^2}{1 + \frac{\rho}{2} - \left[\sqrt{\frac{\rho}{2}}\mathrm{erf}\left(\sqrt{\frac{\rho}{2}}\right) + \frac{1}{\sqrt{\pi}}\exp\left(-\frac{\rho}{2}\right)\right]^2} \quad (13)$$

Similar to DE-QPSK, it is necessary to average conditional BEP (11) over PDF of phase error (12) to obtain average BEP for DE-OQPSK. Under the influence of fading, the instantaneous SNR is random variable, and it is also required to do averaging of (11) over PDF of instantaneous SNR per symbol (6). The BEP of DE-QPSK is:

$$P_b|_{DE-OQPSK} = \int_{\rho=0}^{\infty} \int_{\phi_c=-\pi/2}^{\pi/2} P_b(\phi_c;\rho)\Big|_{DE-OQPSK} \quad (14)$$

$$\times\, p_{\phi_c}(\phi_c;\rho)\Big|_{OQPSK}\, p(\rho)\, d\phi_c\, d\rho\,.$$

4. Numerical Results

Figure 1shows DE-QPSK BEP dependence on SNR per bit for different values of $B_L T_b$. The system has better performance for lower value of $B_L T_b$. In Figure 1 the best performance is for the case when $B_L T_b$ has the lowest value ($B_L T_b$= 0.01) and the arameters m and m_s are higher. It is noticed that influence of $B_L T_b$ is more expressed for higher values of m and m_s, i.e. with the lower influence of fading and shadowing.

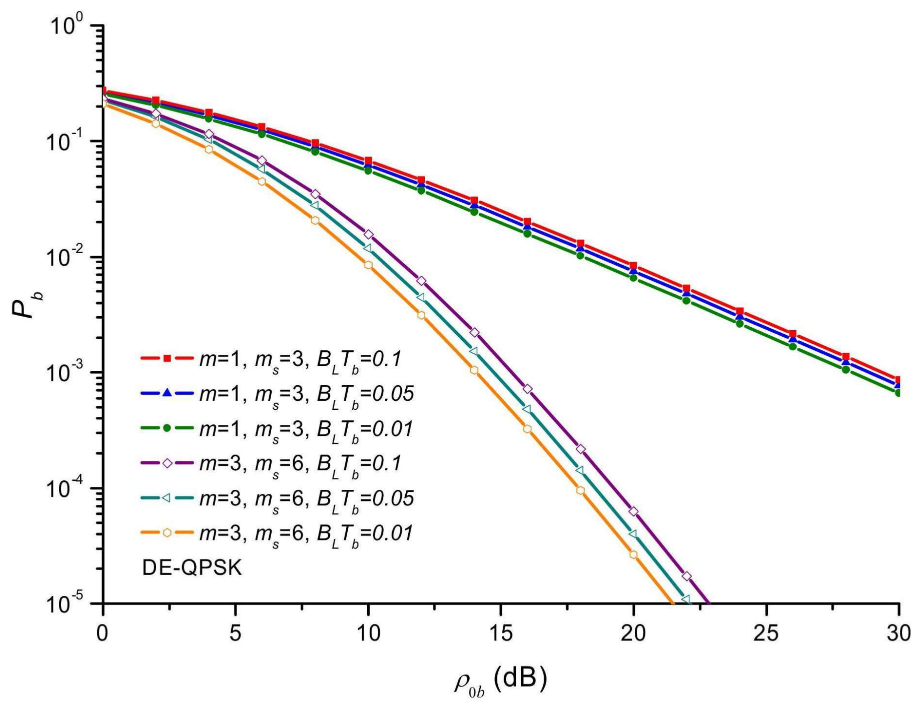

Figure 1. DE-QPSK BEP dependence on SNR per bit for different values of $B_L T_b$

The same BEP dependence for DE-OQPSK is shown in Figure 2. The conclusion is the same: System has better performance for lower value of $B_L T_b$.

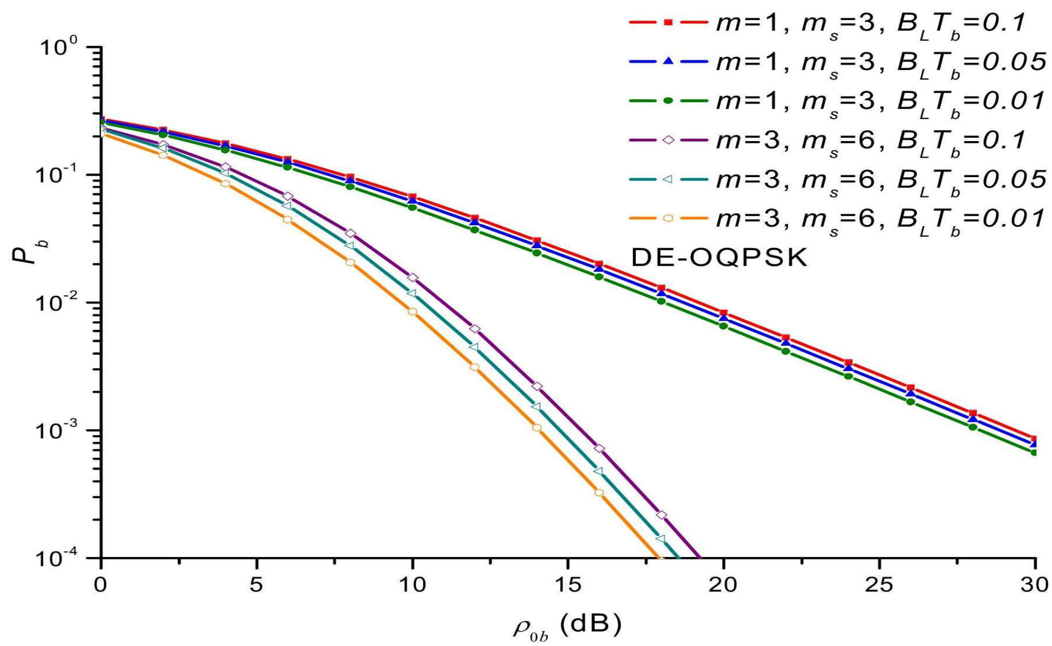

Figure 2. DE-OQPSK BEP dependence on SNR per bit for different values of $B_L T_b$

Figure 3 shows DE-QPSK BEP dependence on SNR per bit, for different values of the shadowing parameter m_s. When the values of the parameter m_s is lower, the influence of shadowing is bigger and we have worse system performance.

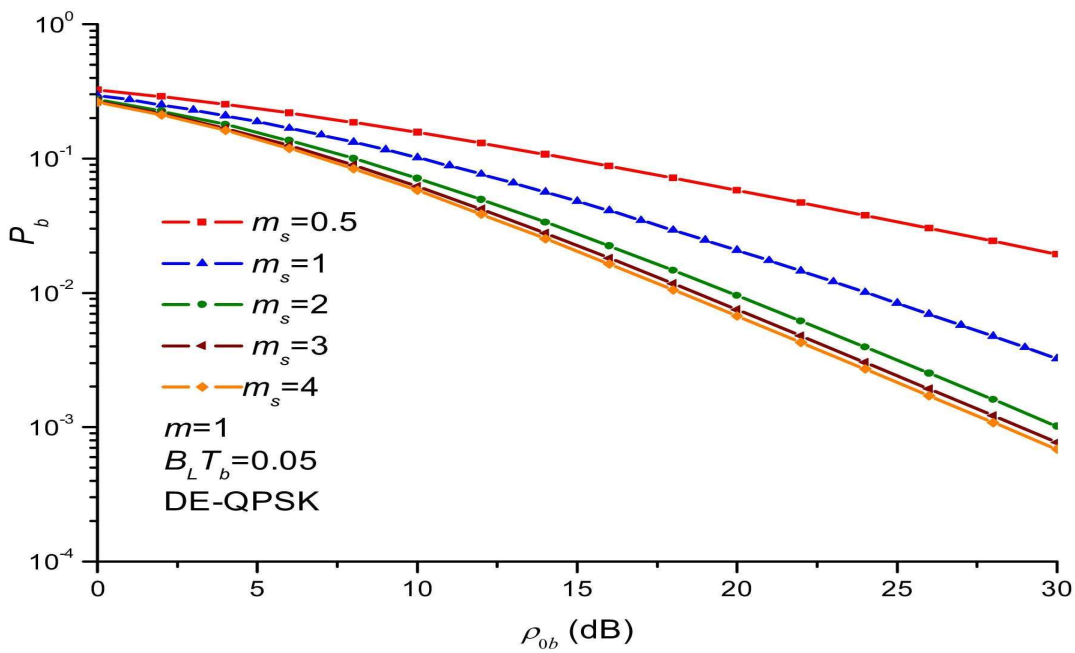

Figure 3. DE-QPSK BEP dependence on SNR per bit for different values of m_s

Figure 4 shows DE-OQPSK BEP dependence on SNR per bit, for different values of the parameter *m*. The performance of the system is the worst when the $m = 0.5$. With decreasing value of fading parameter *m*, we have severe fading.

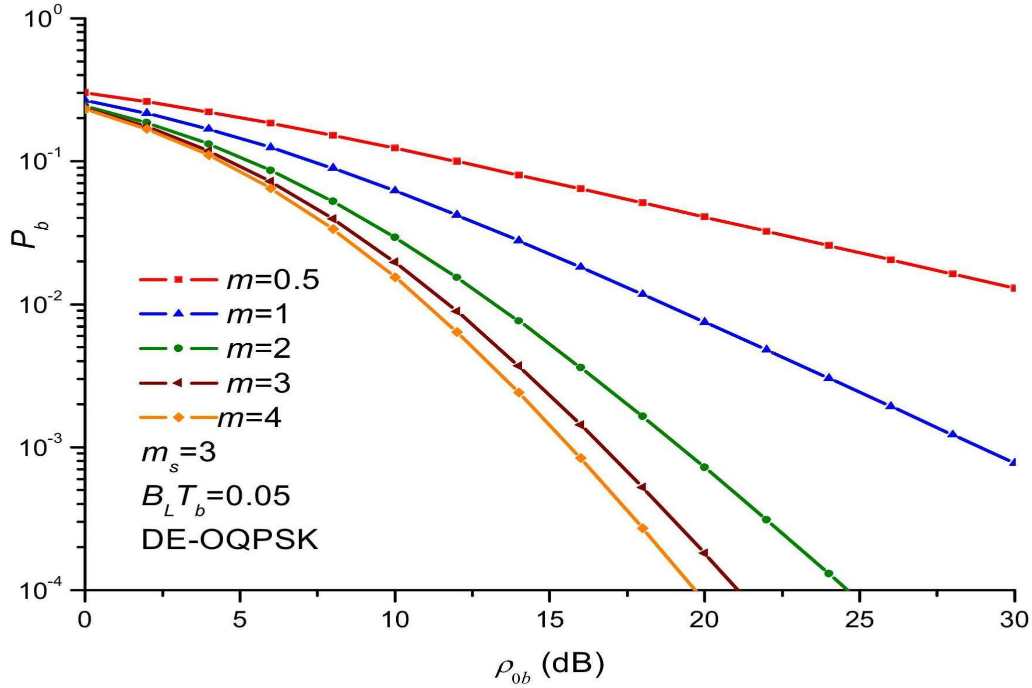

Figure 4. DE-OQPSK BEP dependence on SNR per bit for different values of *m*

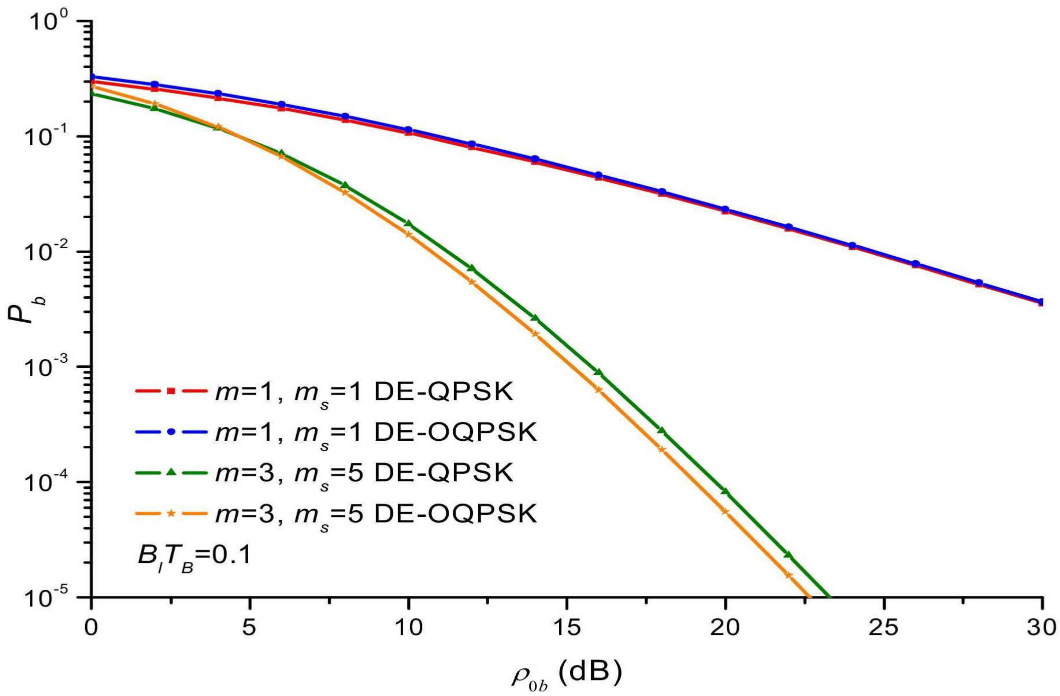

Figure 5. DE-OQPSK and DE-OQPSK BEP dependence on SNR per bit

The DE-QPSK and DE-OQPSK BEP dependences on SNR per bit are shown in Figure 5. We have greater difference between DE-QPSK and DE-OQPSK when the parameters are higher, i.e. when the impacts of fading and shadowing are lower. When the impact of fading and shadowing ($m = 3$ and $m_s = 5$) is lower, the DE-QPSK and DE-OQPSK BEPs have same value when SNR per bit is 4.5dB. For lower values of SNR, system has better performance for DE-QPSK. When the SNR per bit is higher than 4.5dB, the system has better performance for DE-OQPSK.

5. Conclusion

In this paper, we have derived the expressions for BEP of DE-QPSK and DE-OQPSK when the phase error introduced by loop's finite SNR is taken into account and the signal is transmitted over the gamma-shadowed Nakagami-*m* fading channel. The effects of the parameter *BLTb* and the fading and shadowing parameters on the BEP have been noted.

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