

One-dimensional Signal for Adaptive Linear Prediction

Rumen P. Mironov
The Faculty of Telecommunications
Technical University of Sofia
Boul. Kl. Ohridsky 8
Sofia 1000, Bulgaria
rmironov@tu-sofia.bg



ABSTRACT: A software model of adaptive linear prediction code for the one-dimensional signal is created. The pre-programmed codec is tested on Simulink for MATLAB 6.5. The results of the decoded signal are compared with the results of the real sound signal by calculating the MSE and the SNR.

Keywords: Digital Signal Processing, Linear Prediction, Adaptive Data Compression, Matlab Simulation

Received: 25 April 2023, Revised 3 August 2023, Accepted 15 August 2023

DOI: 10.6025/jes/2023/13/4/103-111

Copyright: with authors

I. Introduction

The basic methods for digital signal compression are lossless and lossy compression. Due to the low coefficient of information reduction, the first group of methods is used most frequently in archiving systems and didn't find widespread use in the information transmission systems. The methods for lossy compression are divided into three groups: statistical, psychoacoustic and transforms methods ([1], [2], [3] and [4]). The psychoacoustic compression methods are based on the shortcomings of the human hearing system and are suitable only when the recipient of the record signals is a man. The transformation methods provide a high degree of information reduction and work mainly on the basis of unitary transformations ([3], [4]), for which after decompression the specific block distortions are received. Statistical compression methods are based on the reduction of the information redundancy of the transmitted signals and proceeds in two stages: decorrelation of communication signals and reduction of binary digits, necessary for transmission of signals ([1], [2]). One of the simplest and most convenient for practical implementation methods for decorrelation is the method for linear prediction ([2], [5]). An adaptive method for linear prediction ([2], [5]).

An adaptive method for coding of one dimensional digital signals, based on the linear prediction and Least Mean Square

(LMS) weight coefficients adaptation [6] of the prediction filter is presented. From the developed mathematical equations a general block scheme of Line Prediction Codec (LPC) is synthesized and experimental results from the simulation by Simulink for Matlab 6.5 environment for test signals are given.

2. Mathematical Description

We will assume that the correlation covers n neighborhood elements of the input digital signal, represented by the stationary series $\{x(i)\}$ with $N > n$ values, which have zero average component and correlation function $R_x(r)$ for: $r = \overline{0, n-1}$. Then the basic equation of the linear prediction will be presented by the following way:

$$\hat{x}(i) = a_1x(i-1) + \dots + a_{n-1}x(i-n+1) = \sum_{k=1}^{n-1} a_kx(i-k), \quad (1)$$

where $\hat{x}(i)$ is the value of the predicted element from the input signal $x(i)$. The prediction error is described by the equation:

$$e(i) = x(i) - \hat{x}(i), \quad (2)$$

and the quantization error - by the equation:

$$e_q(i) = Q[e(i)]. \quad (3)$$

According to the LMS algorithm of Widrow [6], the weight coefficients a_k of the prediction filter can be determined recursively:

$$a_k(i) = a_k(i-1) - \mu \nabla_k = a_k(i-1) - \mu \frac{\partial e^2(i)}{\partial a_k}, \quad (4)$$

where: ∇_k is the gradient of the squared error by the prediction and μ is the adaptation step for the i^{th} step of adaptation.

The derivative of the prediction error is determined by the Equations (1), (2), (3), and (4). The sequence for the components of weights is given by:

$$a_k(i) = a_k(i-1) + 2\mu e(i)x(i-k). \quad (5)$$

The beginning values of the weighted coefficients can be calculated after the minimization of mean square error of the prediction. From Equations (1) and (2) follows:

$$\overline{e^2(i)} = \frac{1}{N} \sum_{i=0}^{N-1} [x(i) - \hat{x}(i)]^2 = E \left\{ \left[x(i) - \sum_{k=1}^{n-1} a_kx(i-k) \right]^2 \right\}, \quad (6)$$

where E is the averaging operator.

The partial derivative of $\overline{e^2(i)}$ with respect to any weight coefficient a_1 can be expressed as:

$$\begin{aligned} \frac{\partial \overline{e^2(i)}}{\partial a_k} &= \frac{\partial E \left\{ \left[x(i) - (a_1x(i-1) + \dots + a_{n-1}x(i-n+1)) \right]^2 \right\}}{\partial a_k} \\ &= -2E \left\{ [x(i) - (a_1x(i-1) + \dots + a_{n-1}x(i-n+1))]x(i-k) \right\} \end{aligned}$$

Once equated to zero and transformed, from the upper expression is obtained:

$$E\{x(i)x(i-l)\} = \sum_{k=1}^{n-1} a_k E\{x(i-k)x(i-l)\}. \quad (7)$$

The autocorrelation function of the digital signal, presented with the series $\{x(i)\}$ is:

$$R_x(r) = \frac{1}{N} \sum_{i=0}^{N-1} x(i)x(i-r) = E\{x(i)x(i-r)\}. \quad (8)$$

From the Equations (7) and (8) the following expression is obtained:

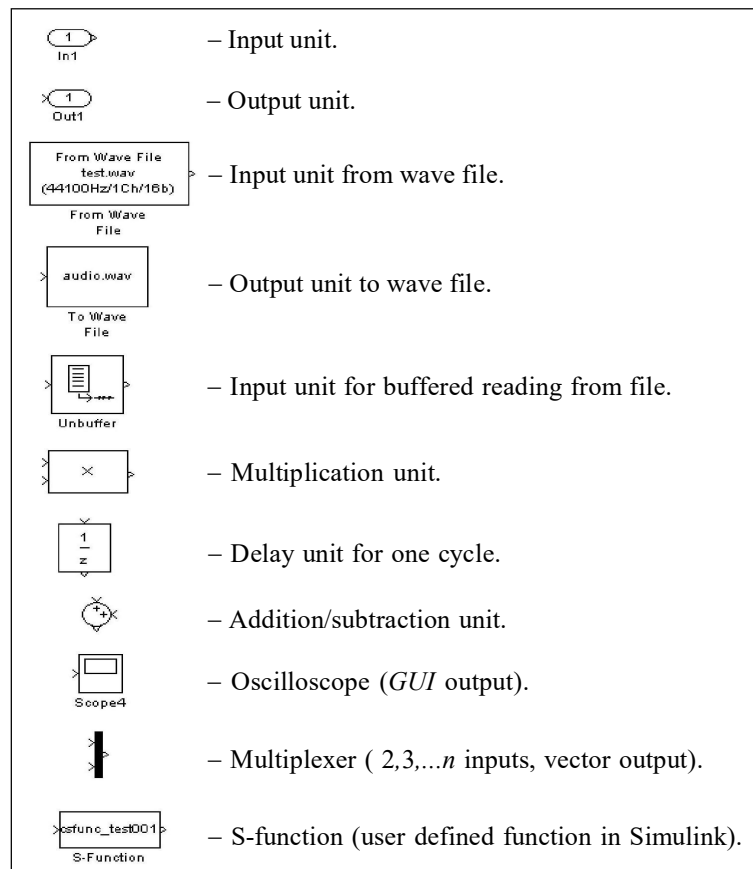
$$R_x(l) = \sum_{k=1}^{n-1} a_k R_x(k-l). \quad (9)$$

For $l = \overline{1, n-1}$ equation (9) represent a linear system with $n-1$ unknowns.

The following equations are used for decoding of signals:

$$\mathbf{x}'(i) = \mathbf{e}'(i) + \hat{\mathbf{x}}'(i), \quad (10)$$

$$\mathbf{e}'(i) = \mathcal{Q}^{-1}[\mathbf{e}_q(i)]. \quad (11)$$



The synthesized by the Equations from (1) to (11) adaptive line prediction codec (*ADPCM*) from 2nd order ($k=2$ and provided that there is no quantization unit) is shown on Figure 1. On Figure 2 and Figure 3 the synthesized coder and decoder blocks for *ADPCM* are shown respectively. The presented schemes are developed through the Simulink package for *Matlab* 6.5 environment and included the units.

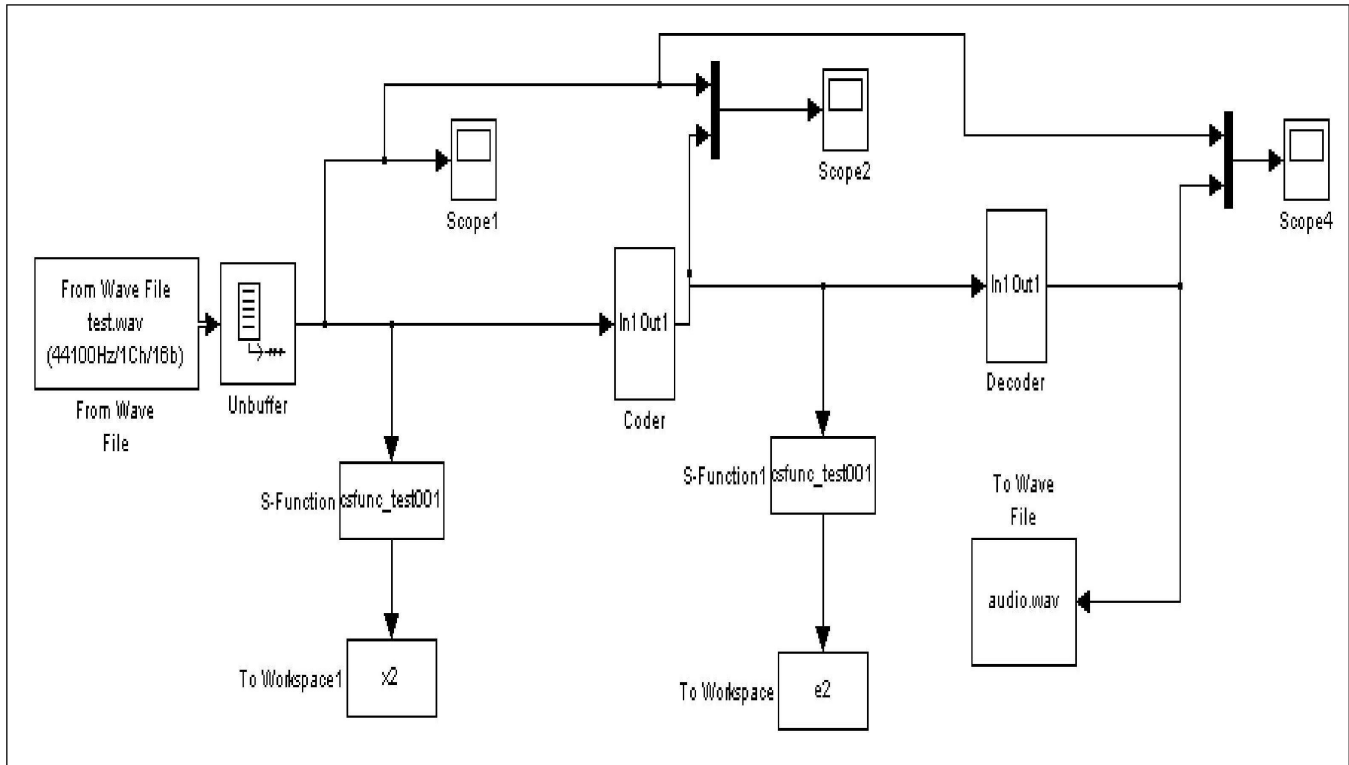


Figure 1. ADPCM Codec

3. Experimental Results

The developed *ADPCM* codec is used for simulation on Matlab 6.5 environment of real audio signals (*WAV* file format, 1 channel (mono), 16 bits, 44.1 KHz sampling rate). The starting values of the weights are obtained from the correlation matrix:

$$[R_x] = \begin{bmatrix} R_x(0) & R_x(1) \\ R_x(1) & R_x(0) \end{bmatrix}$$

for autocorrelation functions $R_x(0), R_x(1), R_x(2)$, calculated from the Equation (8). The starting weight vector $\vec{A}(0) = [a_1(0), a_2(0)]^t$ is calculated from the equation:

$$[R_x]A(0) = \begin{bmatrix} R_x(1) \\ R_x(2) \end{bmatrix},$$

by the Kramer's formulas: $a_1(0) = \frac{\Delta_1}{\Delta}$, $a_2(0) = \frac{\Delta_2}{\Delta}$.

Using Equation (5) the values of the weight vector for the next iterations are calculated by the following:

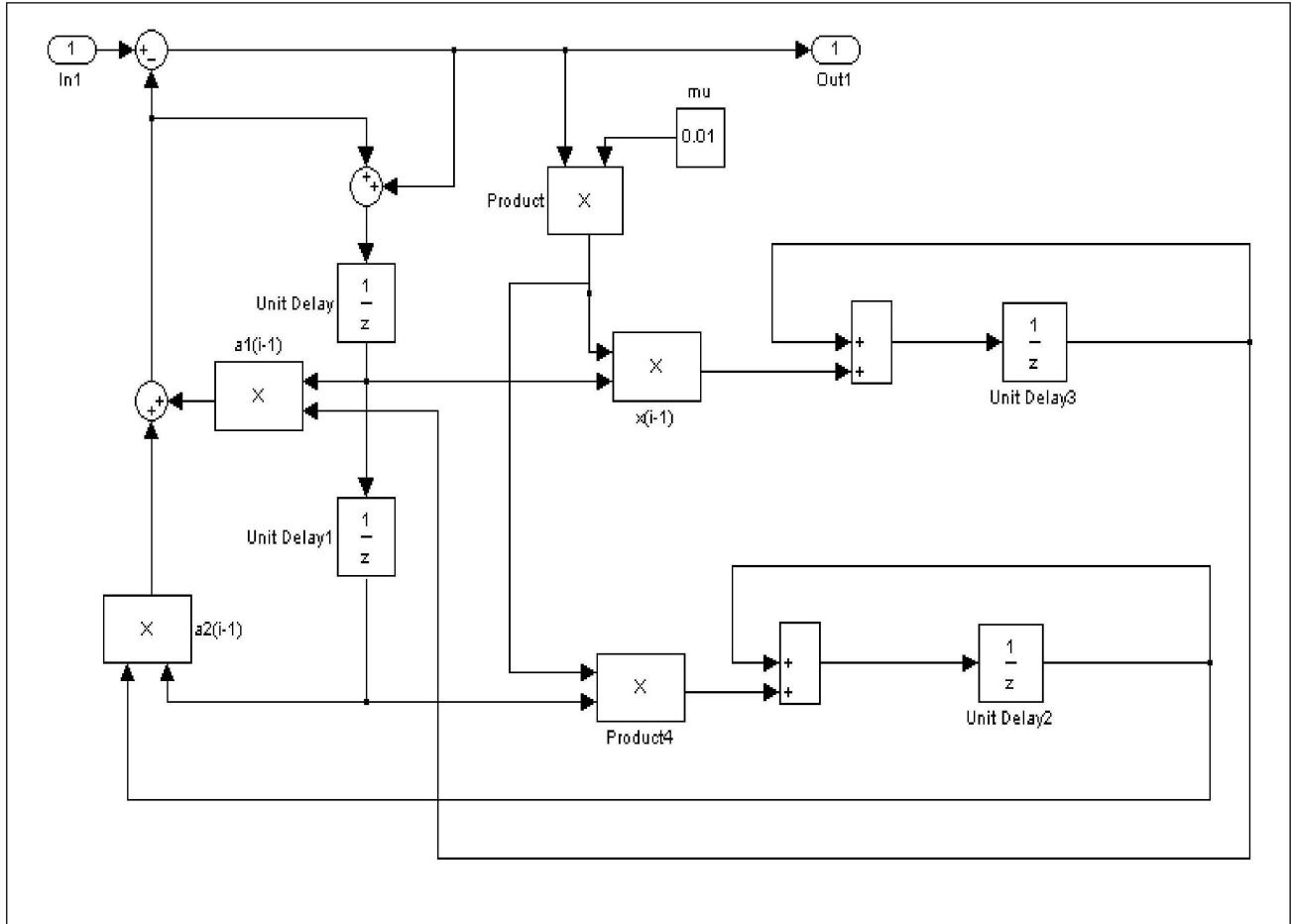


Figure 2. ADPCM Coder

$$\begin{bmatrix} a_1(i) \\ a_2(i) \end{bmatrix} = \begin{bmatrix} a_1(i-1) \\ a_2(i-1) \end{bmatrix} + \mu \cdot e(i) \cdot \begin{bmatrix} x(i-1) \\ x(i-2) \end{bmatrix},$$

where the errors are calculated from the Equation (2) and the predicted values are calculated from:

$$\hat{x}(i) = a_1(i-1) \cdot x(i-1) + a_2(i-1) \cdot x(i-2).$$

The adaptation step depends on the inequality: $0 < \mu < \frac{1}{R_x(0)}$ and the value $\mu = 0.01$ is chosen.

The mean squared error is calculated by the equation:

$$\overline{\varepsilon^2} = \frac{1}{N} \cdot \sum_{i=1}^N e^2(i).$$

The input test signal and output predicted signal are shown on Figure 4 and Figure 5 respectively. On Figure 6 the input and output signals are shown together (visualized from the oscilloscope Scope 4). On Figure 7 the same signals are visualized by the zooming in horizontal direction (from 0.031s to 0.0324s).

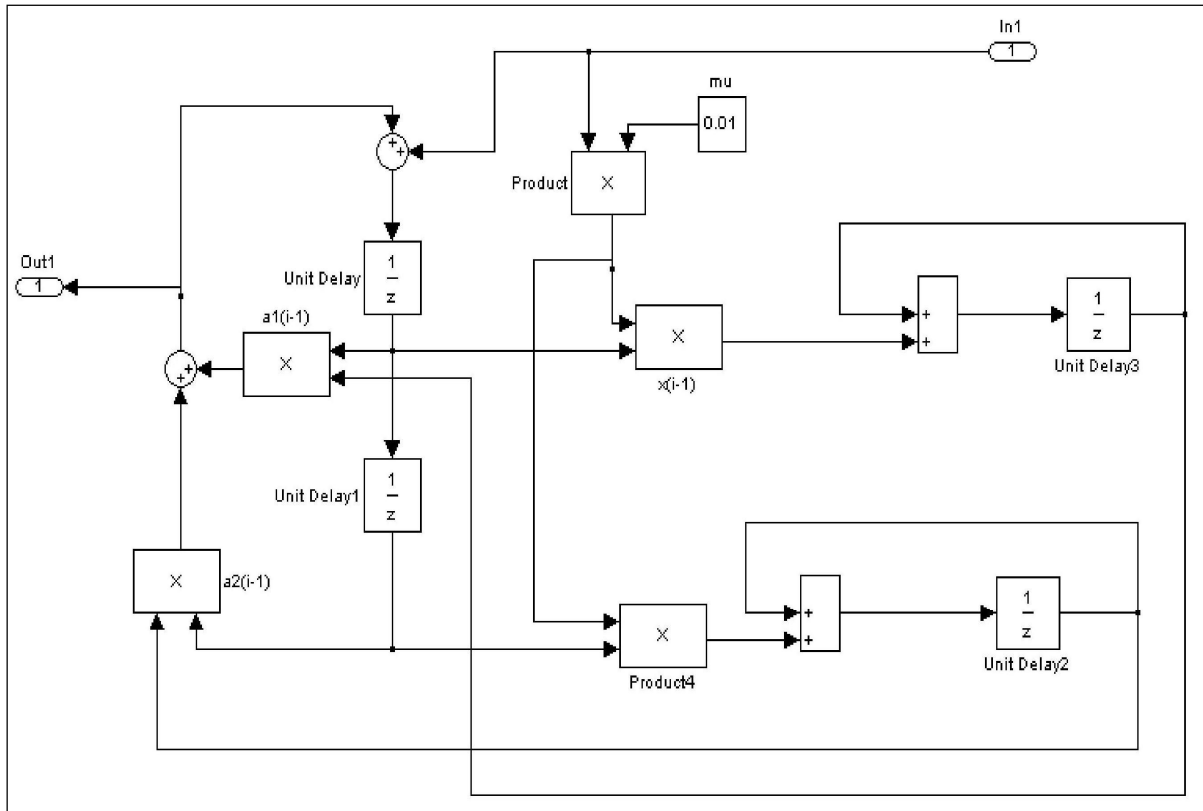


Figure 3. ADPCM Decoder

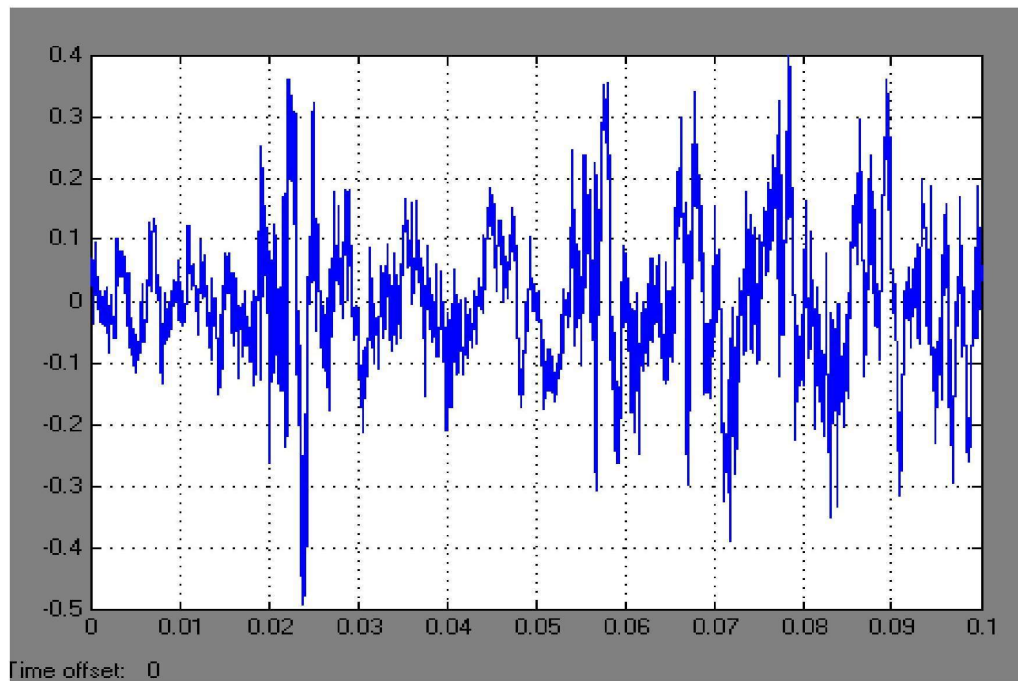


Figure 4. Input signal $x(i)$

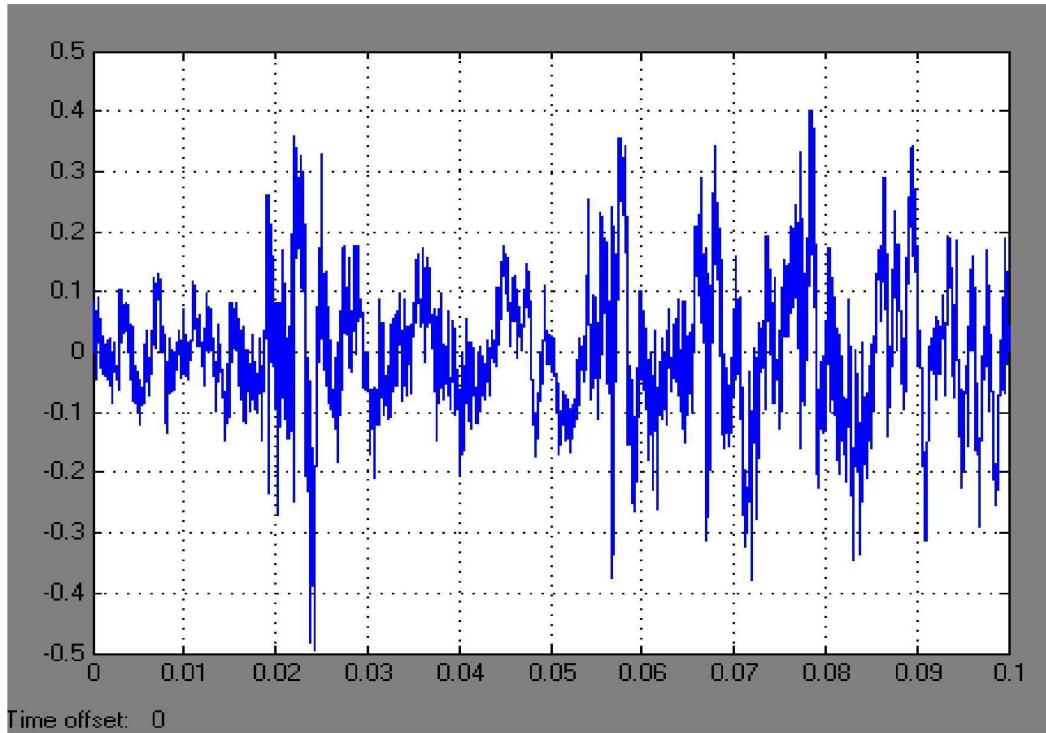


Figure 5. Output predicted signal $\hat{x}(i)$

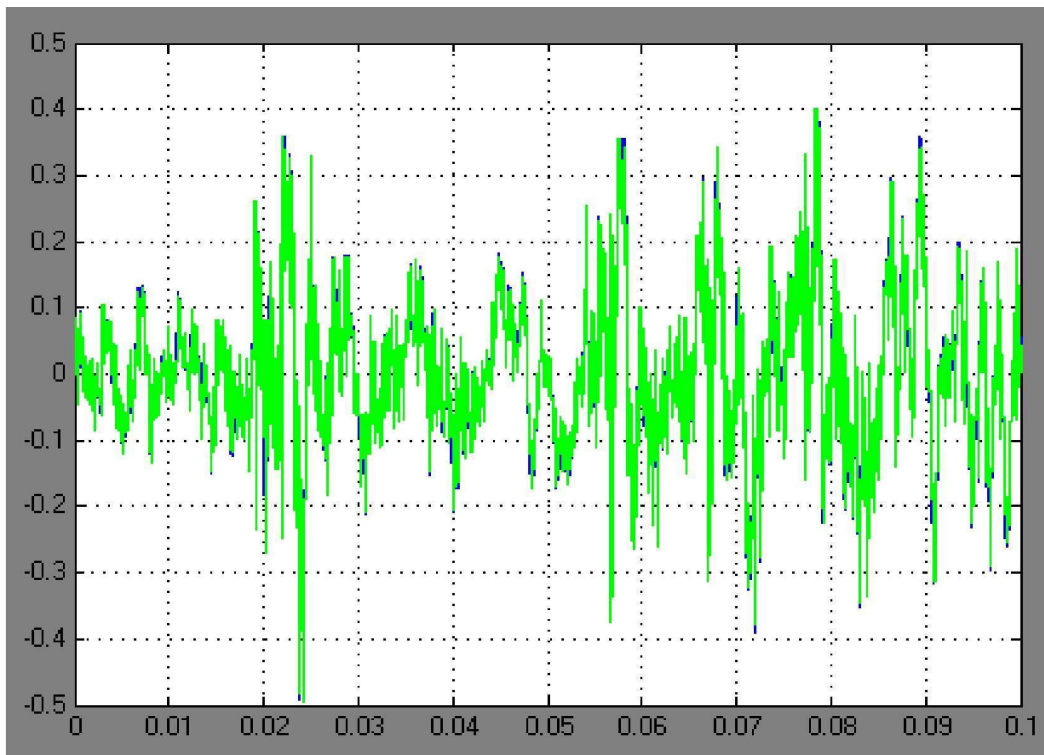


Figure 6. Input and output predicted signal $\text{MUX}\{x(i), \hat{x}(i)\}$

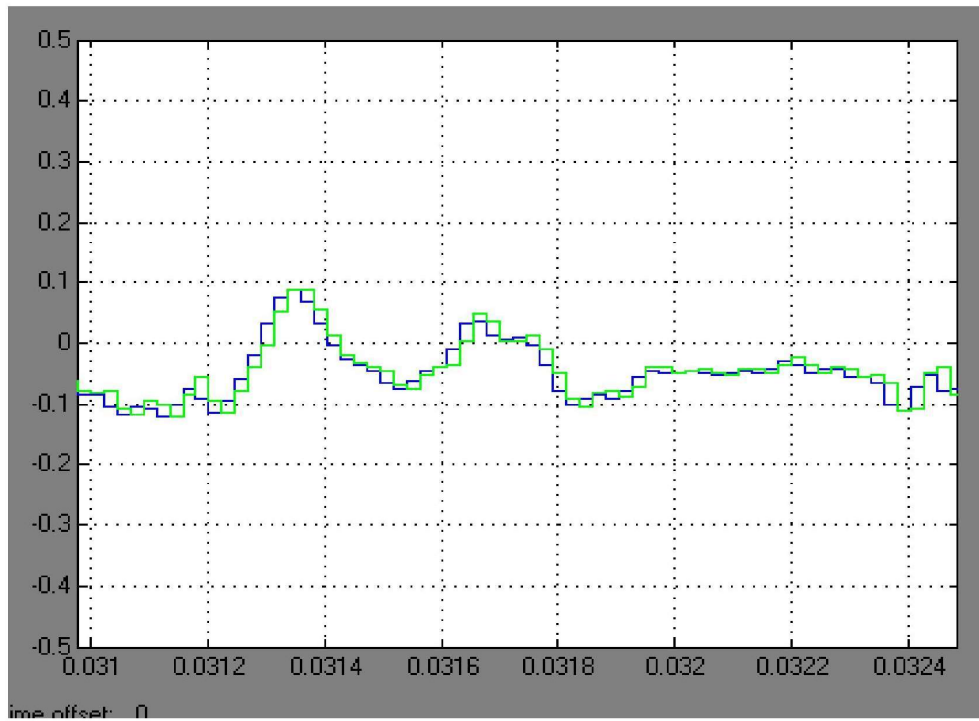


Figure 7. Input and output signal $MUX\{x(i), \hat{x}(i)\}$ (zoomed)

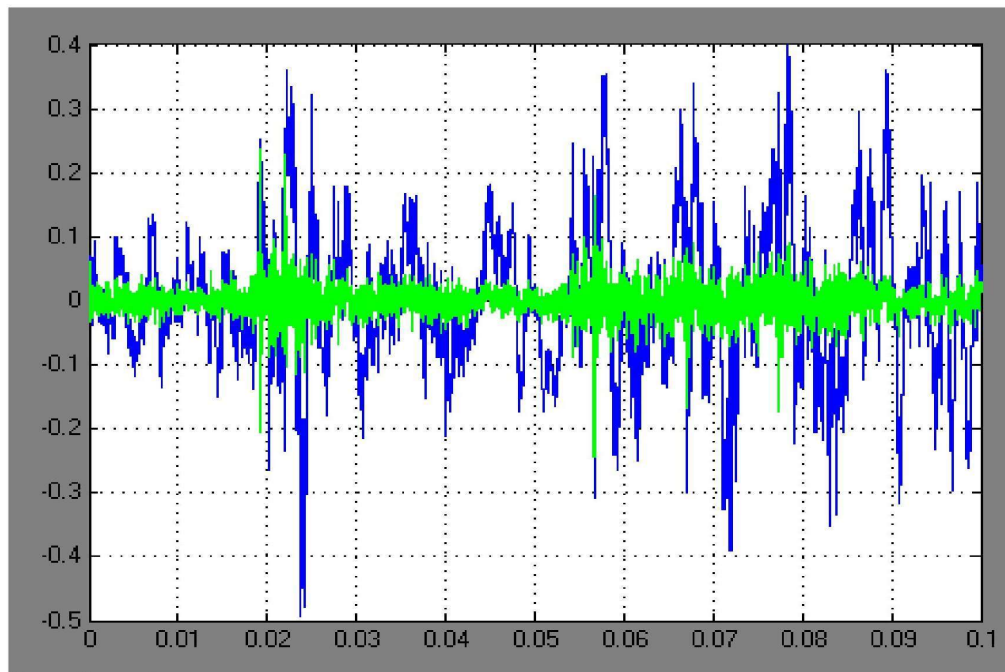


Figure 8. Input and error signal $MUX\{x(i), e(i)\}$

On Figure 8 the input signal and the error signal are visualized together.

4. Conclusion

An adaptive method for coding of one-dimensional digital signals, based on the linear prediction and Least Mean Square (*LMS*) weight coefficients adaptation of the prediction filter is presented. From the developed mathematical equations an algorithm and a general block scheme of adaptive linear prediction codec is synthesized and experimental results from the simulation by Simulink for Matlab 6.5 environment for test signals in WAV format are given

The developed *ADPCM* codec provides minimum processing error and led to increase of *PSNR* with about 0.3dB in comparison with 2 coefficients non-adaptive prediction codec. The presented simulation model can be used in digital signal processing for spectral analysis, coding and transmission of one-dimensional signals and in distance learning by the using a Matlab Web Server.

The developed image codec is used in laboratory work on the disciplines: “Image and Signal Processing” and “Audio and Video Communication on Internet” and in the experimental work in laboratory “Electronic System for Visual Information” in Technical University of Sofia.

Acknowledgement

The authors thank the National Fund for Scientific Research of the Bulgarian Ministry of Education and Science for the financial support by the contract VU-I-305/2007.

References

- [1] Stranneby, D., Walker, W. (2004). *Digital signal processing and applications*. Elsevier.
- [2] Mitra, S. (2006). *Digital signal processing: A computer-based approach*. Mc Graw Hill.
- [3] Madisetti, V., Williams, D. (Eds.). (1998). *The Digital Signal Processing Handbook*. CRC Press.
- [4] Poularikas, A. D. (2000). *The Transforms and Applications Handbook (2nd ed.)*. CRC Press.
- [5] Blanchet, G., Charbit, M. (2006). *Digital Signal and Image Processing Using MATLAB*. ISTE.