



Digital Data Shape for Efficient Data Storage

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ABSTRACT

This work advocates an innovative coordinate framework that can be easily adapted to various dimensions. The primary goal is to offer a versatile way to represent any shape digitally, ensuring effective and efficient data storage. Potential uses range from identifying objects to analyzing handwritten text. Several illustrative examples and discussions on extending the system to three-dimensional and one-dimensional scenarios are provided. Additionally, the paper points out areas that still require further research.

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1. Introduction

There have been several types of coordinates in the field of mathematics and science for a long time, such as Cartesian coordinates, polar coordinates, cylindrical coordinates, spherical coordinates, etc.) In some cases, one needs abacuses to complete a complex calculation. As an example, the Smith chart is used to determine the variation of the complex impedance with frequency and to carry out impedance adaptations in the transmission lines. This Abacus allows one to calculate complex numbers for this tedious task. Nowadays, with pocket calculators, one does not need such an Abacus anymore, but for the understanding of the phenomena on high-frequency transmission lines, it is very useful for engineers in the field. In this coordinate system, orthogonal circles define the value of a complex impedance with its real part, the resistance, and the imaginary part, the inductance or capacitance.

The idea of sequential coordinates is based on the sequential movement of a pattern or path that evolves as is the case in writing a signature. This sequential movement can be modeled in sequential coordinates for easier recognition later by a computer.

The sequential coordinates contain inclusively the notion of time. This can be used for specific applications as follows:

- **The movement of a robot.** We inclusively follow a robot's path that moves and communicates its sequential coordinates. If one loses contact, it is possible to recover and organize the next moves.

- **Recognition of signatures.** This process is a perfect example of an application of sequential coordinates after the discretization of the path by segments that follow each other.

- **Navigation.** In case a walker lost in the forest, the guide can place him/her on a map with a minimum communication complexity.

In Freeman's early work, a procedure was developed for the description and coding of arbitrary geometric curves, simplifying the manipulation of these curves by computer. Since then, the methodologies and applications have been diversified. The authors of [2] have proposed a performance measure based on the compression ratio for digitizing arbitrary curves. The application of curve scanning for contour extraction has been studied in [3]. [4] studies digitized curves for pattern recognition. An overview of recent work on digitizing geometric curves and more generalized drawings has been presented in [5]. A specific application of coordinate representation is signature verification in [6].

Etienne conducted perimeter estimation of continuous shapes based on their digitization. [7] Le Quentrec focused on the classical problem of controlling information loss during the digitization step. The properties proposed in the literature rely on smoothness hypotheses not satisfied by curves, including angular points. [8]

3D curved electronics are a trend in the microelectronics industry, but conformal printing of circuits on complex curved surfaces remains challenging, especially for electronics comprising concave surfaces with large curvature. [9] (Wang 2023). Quentrec et al proved that there exists a mapping between the boundary of the digitization and one of the continuous shapes such that these boundaries travel together in a cyclic order manner. [10]

Given the wide range of applications and the need for accurate and efficient representations, we believe that the proposed sequential coordinate system can provide significant advantages both in terms of the representation and applications. The rest of this paper is organized as follows. In the following section, we provide the definition of sequential coordinates. Section 3 introduces the extensions to 3 dimensional and the 1 dimensional cases. The conclusions are given in Section 4.

2. Defining Sequential Coordinates

At first, we limit ourselves to two dimensions. An AB starting segment can be traced anywhere on a plane (independence of an origin.) Let us have starting point A and arrival point B. Once the point B is obtained, one can turn left (-) or right at a certain angle. Then, we continue with another segment length towards point C. This will provide coordinates (length, angle) sequentially B ($r_0, 0$); C (r_1, θ_1); D (r_2, θ_2); E (r_3, θ_3) etc. For lengths, we can choose a unit or normalize everything in relation to the length of the AB segment (see Figure 1).

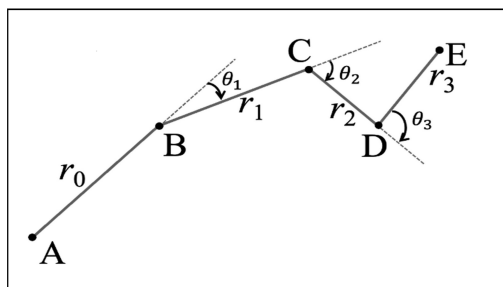


Figure 1. Definition of sequential coordinates which contains implicitly time sequence.
(Departure from one point to the next one)

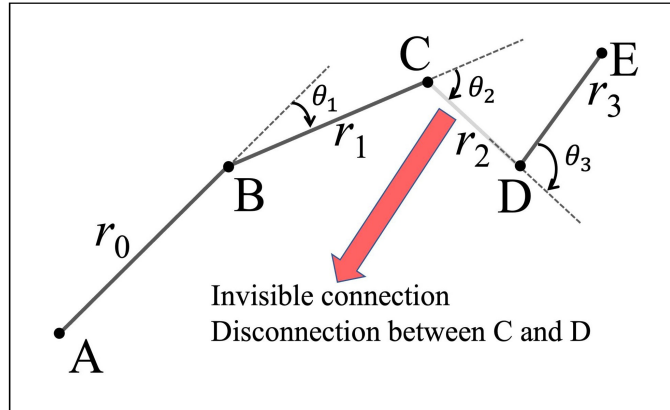


Figure 2. Example of generalization with disconnected contours

We will have $BC/AB = d_1$, $CD/AB = d_2$, $DE/AB = d_3$. The series of coordinates will then be: $\{B(1, 0); C(d_1, \theta_1); D(d_2, \theta_2); E(d_3, \theta_3)\}$.

The same notation can also be used in the case of disconnected contours, representing the corresponding distance by \bar{d} .

For example, the set of coordinates in Figure 2 becomes: $\{B(1, 0); C(d_1, \theta_1); D(\bar{d}_2, \theta_2); E(d_3, \theta_3)\}$

2.1. Recognition of Shapes

In Cartesian coordinates a figure formed by segments can move, rotate in one direction, change scale in a plane, everything will be expressed according to the figures given in Figure 3. It is very difficult to predict that these different figures are similar just by looking at the Cartesian coordinates.

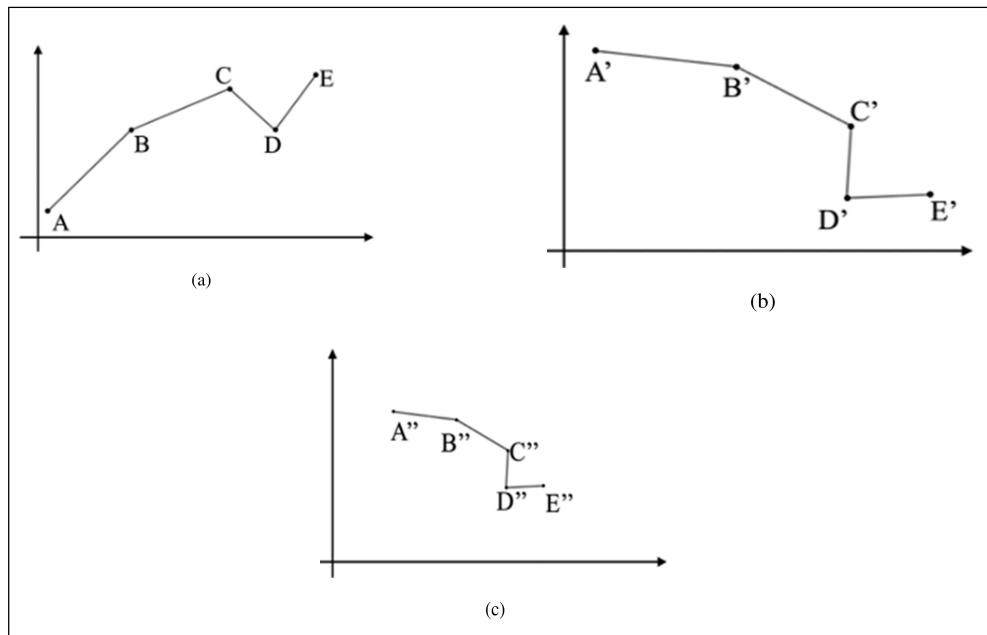


Figure 3. Illustration of various positions and orientations of similar figures without origin problem in sequential coordinates (a) Original shape (b) Rotated shape (c) Shape at another scale and rotated

$A(x^1, y^1); B(x^2, y^2); C(x^3, y^3); D(x^4, y^4); E(x^5, y^5)$

$A'(x'_1, y'_1); B'(x'_2, y'_2); C'(x'_3, y'_3); D'(x'_4, y'_4); E'(x'_5, y'_5)$

$A''(x''_1, y''_1); B''(x''_2, y''_2); C''(x''_3, y''_3); D''(x''_4, y''_4); E''(x''_5, y''_5)$

If we use sequential coordinates we will notice that the lengths change only in the same proportion and the angles remain the same. Since we have no origin we can place these figures anywhere on the plane (see Figure 3).

2.2. Exemplary Cases

In Figure 4 an equilateral triangle (Figure 4(a)) and a square (Figure 4(b)) are shown. The coordinates that define these figures will be:

In the case of the equilateral triangle (by changing the place of length r and angle θ) $\{(0^\circ, r_0); (120^\circ, r_1); (120^\circ, r_2)\}$ avec $r_0 = r_1 = r_2 = r$.

- For the square we simply have $\{(0^\circ, r); (90^\circ, r); (90^\circ, r); (90^\circ, r)\}$.
- For a circle with radius r , we can use the limit $n \rightarrow \infty$

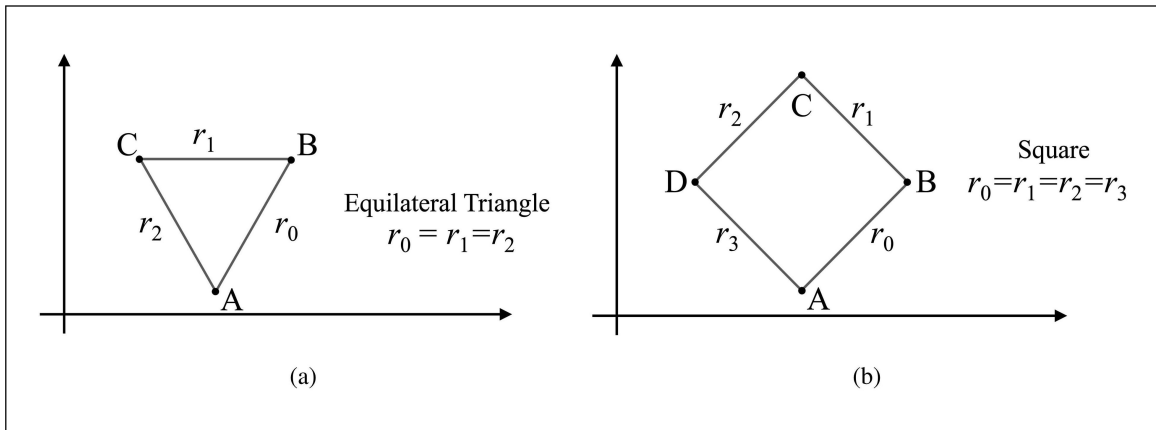


Figure 4. Simple geometric figures, such as equilateral triangles, squares in sequential coordinates, easy to recognize (a) An equilateral triangle (b) A square

$$\lim_{n \rightarrow +\infty} \bigcup_{i=0}^n \left(\frac{360^\circ}{n}, \frac{r}{n} \right). \tag{1}$$

- For a segmented spiral, also using the limit $n \rightarrow \infty$

$$\lim_{n \rightarrow +\infty} \bigcup_{i=0}^n \left(\frac{360^\circ}{M_i}, i \times r \right), \tag{2}$$

for r constant. The values of M_i can be selected at each step to obtain the desired shape.

2.3. Characteristics of the System

The characteristics of such a system are as follows;

- 1) Independence of origin (O)
- 2) Independence of the rotation of the figure one way or the other.
- 3) Easy recognition by similar lengths (r lengths).
- 4) In addition, if one normalizes (e.x. $r_1/r_0 = d_1, r_2/r_0 = d_2, r_2/r_0 = d_3$), the same number will appear in the lengths of the sides. This leads to the recognition of these kinds of figures in an immediate way.

The expressions concerning polygons with rotational symmetry will be of type $\{(r, 0); (r, \theta_1); (r, \theta_1); (r, \theta_1); (r, \theta_1); (r, \theta_1)\}$. Definition of a symbol for this kind of sequence thus becomes:

$$\bigcup_{i=0}^n (r, \theta)_i = \{(r_0, \theta_0); (r_1, \theta_1); (r_2, \theta_2); (r_3, \theta_3); \dots\}, \quad \text{where } \theta_0 = 0. \quad (3)$$

An arbitrary form can be represented by the formulation given above.

3. Extensions to Other Dimensions

In this section, the 3-dimensional and 1-dimensional cases are defined.

3.1. Three Dimensional Case

To move to a third dimension while in a given plane, one can proceed as follows: When we arrive from a point A to a point B in Figure 1, we can draw the normal to the segment AB in this plane and taking this normal as a hinge we can rotate around this normal in space (+) upwards by an angle ϕ and continue with the sequential coordinates in this new plane of space. ϕ will be positive (+) if we have turned upwards or negative (-) if we have turned downwards (See Figure 4).

The expression of the three-dimensional sequential coordinates will be as follows:

$$\bigcup_{i=0}^n (r, \theta, \varphi)_i = \{(r_0, \theta_0, \varphi_0); (r_1, \theta_1, \varphi_1); (r_2, \theta_2, \varphi_2); (r_3, \theta_3, \varphi_3); \dots\}, \quad \text{avec } \theta_0 = 0, \varphi_0 = 0 \quad (4)$$

If we don't change the plan of course $\phi_0 = \phi_1 = \phi_2 = \dots = 0$

3.2. Le cas à 1 dimension

For the one-dimensional sequential coordinates it is sufficient to discretize the curved line and express with the + or - sign the direction of displacement and indicate only the lengths of the

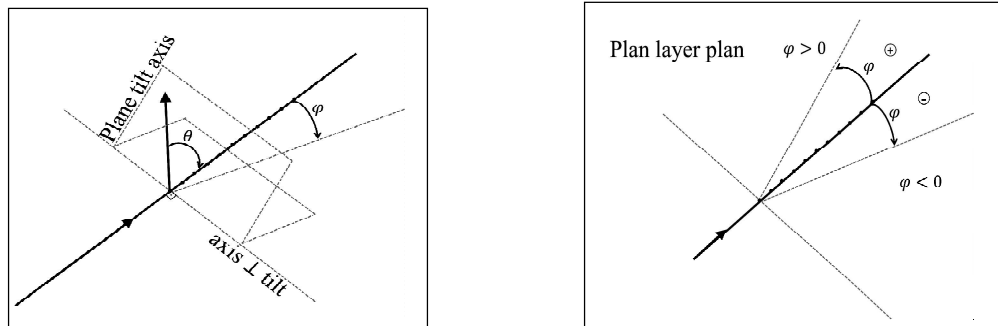


Figure 5. Sequential coordinates in 3 dimensions. Passage from two dimensions to three dimensions in space

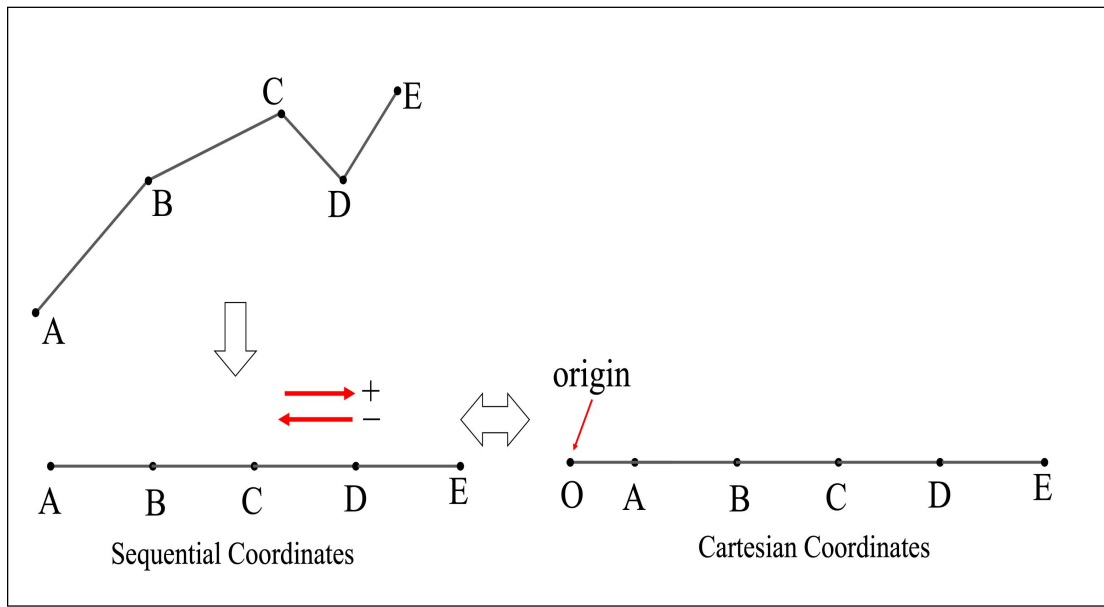


Figure 6. Sequential coordinates in one dimension: independence of the origin

segments with a sign. As there is no origin this sequence will keep its values whatever the starting point (See Figure 5).

As an example of comparing two coordinate systems, if the coordinates of the left-hand representation representing the sequential coordinates in Figure 5 are indicated as (+3); (+7); (-4), the coordinates of the right-hand representation representing the Cartesian coordinates in Figure 5 can be indicated as $|OA| = x_1 = 12$, $|OB| = x_2 = 15$, $|OC| = x_3 = 22$ and $|OD| = x_4 = 18$ assuming the distance between the origin O and the point A in Cartesian coordinates (i.e., $|OA| = x_1 = 12$).

4. Conclusion

In an era of machine learning, it is essential to have efficient representations of arbitrary shapes. This manuscript introduces the concept of sequential coordinates for the first time in the literature. This flexible form allows multidimensional shapes to be represented in an evolutionary manner. Open issues include coordinate sampling and transformation between coordinate systems. The authors believe that non-uniform sampling theory contains the corresponding keys and plan to work on this topic in the future.

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