

# A New Shuffled Frog Leaping Algorithm with Space Zoomed Factor and Gravity Attractor

Deyu Tang<sup>1</sup>, Jin Yang<sup>2</sup>

<sup>1</sup>College of Medical Information and Engineering  
Guang Dong Pharmaceutical University  
Guangzhou, China

<sup>2</sup> College of Computer Science & Engineering  
South China University of Technology  
Guangzhou, China  
scutdy@126.com, goodskyfly@163.com



**ABSTRACT:** Shuffled frog leaping algorithm is a new kind of swarm intelligence optimization algorithm. Due to the local search of the basic shuffled frog leaping algorithm which is the only by the worst frog to search and jump, searching ability of which was limited; therefore it had the low precision, slow convergence speed and easy premature convergence etc. Therefore, in order to enhance the ability of the local searching, this paper presented a gravity attractor, all the frogs in the same memplex could find the best position under its guidance. Considering the transboundary problems in the frog searching for food process, we introduced a space zoomed factor which made the frog out of the searching space could also be put into the searching space, and the social position relation was not changed, thus it improve the search ability of the SFLA. Through the standard function for testing, and compared with the standard SFLA algorithm and szAPSO algorithm, the experimental results show that the improved algorithm not only improves the convergence speed of the algorithm, enhance the searching capability of the algorithm, but also has better stability.

## Categories and Subject Descriptors:

**I.2.11 [Distributed Artificial Intelligence]: G.1.2 [Approximation];** Minimax Approximation and Algorithms

## General Terms:

Swarm Optimization, Optimization Algorithms

**Keywords:** Swarm Intelligence Optimization, Shuffled Frogleaping Algorithm, Search Strategy, Gravity Attractor, Space Zoomed Factor

**Received:** 28 April 2013, Revised 5 June 2013, Accepted 13 June 2013

## 1. Introduction

Shuffled frog leaping algorithm (SFLA) was presented by the scholar Eusuff and Lansey in 2003, according to the simulated frog searching food process, information was shared and exchanged, so a new swarm intelligence optimization algorithm was produced [1]. SFLA is a heuristic cooperative search algorithm based on the population, the implementation of the algorithm is to simulate natural element evolution behavior. As a new optimization method, SFLA algorithm use the shuffled complex evolution algorithms (SCE) as the breadth search implementation framework, and is combined with genetic memetic evolution algorithm (MA) and the characteristics of particle swarm optimization algorithm based on the bird searching food behavior. So it is easy to understand and has fewer parameters (with less than PSO algorithm parameters), fast calculation speed, good global search ability and is easy to implement. This algorithm has been improved and applicated in widely areas, such as the traveling salesman problem [3], 0-1 knapsack problem [4], flowshop scheduling problem [5], image segmentation [6], the grid task scheduling problem [7], economic scheduling problem [8]. However, in the solving problem process by the basic shuffled frog leaping algorithm, for some complex problems, it still has some faults such as the slow convergence speed, low accuracy of optimization, bad local search ability, and as the increase of dimension, eventually affects the efficiency of the algorithm. So the researchers use different methods for the corresponding improvements, such as the paper [9] proposed an improved shuffled frog leaping algorithm based on the PSO algorithm, to a certain extent, improved the effect of shuffled frog leaping algorithm; paper[10] use neighborhood orthogonal crossover operator to

enhance the individual diversity, which improve the speed of convergence. The paper [11] combined with the differential evolution algorithm improve the local search speed of the SFLA, has the very good effect. Frog populations in natural evolution process, the worst frog's thought would be influenced by the best frog's thought. The worst frogs tend to jump to the best frog position looking for food. The frog jumping rule of SFLA algorithm is inspired by the social environment in imitation, but it only achieved the worst frog jumps into the best frog position. According to this rule, the worst frog's position is limited in the middle of a line between its current position and the best frog position. Obviously, the frog jumping rules limit each biological evolution process in local search space. And the other frogs do nothing. This restriction is not only to reduce the speed of convergence, but also easy to cause premature. Therefore, we use a gravity attractor, which guide all the frogs to search the best food.

The local search of shuffled frog leaping algorithm and particle swarm optimization algorithm has similar search method. The transboundary problem of the shuffled frog leaping algorithm has not much research, mainly using the maximum limit method, which is the most simple common method, but is easy to cause the algorithm premature convergence and cannot be guaranteed to frogs be controlled in the search space. In this regard, we studied recent particle swarm optimization algorithm to solve the problem, used for shuffled frog leaping algorithm. In recent years, many scholars put forward different boundary mutation strategy, which is used to solve the problem of particle swarm optimization algorithm. Paper (12) use the physical properties of particle motion to propose three boundary wall: absorption wall, reflecting wall and hide wall; paper (13) integrate the advantage of the absorption wall and hide wall, propose the attenuation wall; paper (14) summarize all these boundary walls and tested the performance of these algorithm and hybrid algorithm. Each frog in the shuffled frog leaping algorithm can be regarded as a particle. Because the frog move behavior directly affects the performance of SFLA algorithm, boundary mutation strategy directly affects the frog move behavior, so using the effective boundary treatment strategy to improve the algorithm's performance is necessary and feasible. In order to solve the defects in SFLA algorithm, this paper use gravity attractor and spatial zoomed factor search strategy, improve the local search ability; and the global shuffled strategy strengthen the individual diversity and evenness in each local cluster. Experiments show that the improved SFLA algorithm greatly improve the accuracy and convergence speed of the algorithm.

## 2. SFLA Algorithm

Shuffled frog leaping algorithm simulate frog population looking for food process. A frog population was classified into some memeplexes, each memeplex exchange the thought of the frog. Combined with the global information exchange and the local depth search, local search makes

thought transfer in local memeplex among individuals, mixed strategy makes the idea of local memeplex exchange. In the shuffled frog leaping algorithm, solution set has a group frog (solution) with the same structure. The population is divided into many memeplexes, the memeplex is considered to be different with different ideological frog collection. According to a certain strategy, frog of memeplex implement local depth search in the solution space. After the defined numbers of the local search iteration, thought in the mixing process was exchanged. Local search and shuffled process continues until it satisfy the convergence condition of definition.

A balance strategy of Global information exchange and the local depth search make the algorithm can jump out of local extreme point, towards the global optimum direction, which became the most important characteristics of shuffled frog leaping algorithm. In SFLA, population has many frogs, each frog represents a solution, the population is divided into many memeplexes have different ideas. Each memeplex is carried out local search respectively, the worst individual  $Q_w$  close to the local best individual  $Q_b$  or the global best individual  $Q_g$  through the memetic evolution. When the local search is executed to a certain stage, each memeplex communicate to implement shuffled process. Repeatedly performs a local search and a shuffling process until they conform to the convergence condition of the definition. The basic SFLA algorithm [1] is as follows:

1. Parameter setting: Size of population, numbers of memeplex, number of local iteration, numbers of global iterations.
2. Generates an initial population.
3. Determine the fitness function  $F(x)$ , used to evaluate the quality of the individuals.
4. In the global iteration process, the frog's. Fitness values was arranged in descending order, and determine the global optimal solution  $Q_g$ , if it meet the convergence condition, then stop the execution; otherwise, proceed to the next step.
5. Frog populations are divided: the frog population was divided into  $m$  memeplexes, each memeplex contains  $n$  frogs, and the number of the population meet  $T = m \times n$ ; then, the first frog is divided into the first memeplex, the second frog were divided into the second memeplex, ..., The  $M$  frog was divided into the  $M$  memeplex, the  $m + 1$  frog is divided into the first clusters, the  $m + 2$  frog was divided into the second memeplex, and so on, until the whole frogs was division. That is:

$$Q_t = \{Q_{t+m(r-1)} \in T \mid 1 \leq r \leq n\}, (1 \leq t \leq m)$$

6. Each memeplex implement local search in the number of the iterations.
  - 1) Determine  $Q_w, Q_b$  according to the following formula to update:

$$D = r \cdot rand \cdot (Q_b - Q_w) \quad (1)$$

$$Q_{w(new)} = Q_w + D, D \in [D_{min}, D_{max}], \quad (2)$$

$rand \in (0, 1], r$  is a constant

2) If produce the solution of better performance, then update  $Q_w$  with  $Q_{w(new)}$ ; Else use  $Q_g$  to replace  $Q_b$ , execute formula (1) (2). if produce the solution of better performance, then update  $Q_w$  with  $Q_{w(new)}$ ; if  $Q_w$  was not improved, then randomly produce a new solution  $Q'_{w(new)}$ , if produce the solution of better performance, then update  $Q_w$  with  $Q_{w(new)}$ .

### 3. Improved SFLA Algorithm

The search rules of shuffled frog leaping algorithm is a simplified search rule of the traditional PSO algorithm, studying search rules (1) (2), we can get the formula:

$$Q_{w(new)} = Q_w + r1 \cdot rand \cdot (Q_b - Q_w)$$

$$\Leftrightarrow Q_{w(new)} = (1 - r1 \cdot rand) \cdot Q_w + r1 \cdot rand \cdot Q_b$$

$$\Leftrightarrow Q_{w(new)} = \alpha \cdot Q_w + \beta \cdot Q_b,$$

$$\alpha + \beta = 1, \alpha, \in (1 - r1 \cdot rand, 1),$$

$$\beta \in (0, r1 \cdot rand]$$

especially, while  $r = 1$ , we obtain:

$$Q_{w(new)} = \alpha \cdot Q_w + \beta \cdot Q_b,$$

$$\alpha + \beta = 1, \alpha, \in (0, 1], \beta \in (0, 1]$$

Therefore, shuffled frog leaping algorithm calculates weighted average position between the current best position of the worst frog and the best frog's position in the present generation, that is to find the gravity of the worst frog's best position, thereby to update the worst frog's position in the last generation. This method is due only to update the worst frog's position, and restricte the local search ability. In the PSO algorithm, paper [15] proved that, when the individual best value  $Q_p$  and the global best value  $Q_b$  remain unchanged, the algorithm is asymptotically stable in the position  $(Q_p + Q_b)/2$ . But because in the process of optimization,  $Q_p$  and  $Q_b$  is dynamic changed, and at the interaction of the particle motion inertia and the social impact, and the influence of random factors, the particles may not to be the stable position by linear movement, but go to the gravity of weighted average position between individual best position and global best position, around the garvity do spiral motion. So, we improve the local search rule of shuffled frog leaping algorithm, using gravity attractor and space zoomed factor to update all frogs's position in each memeplexe, obtain new search rules, which can be mathematically described as follows:

$$Q_c = \alpha \cdot Q_p(t) + \beta \cdot Q_b(t),$$

$$\beta = 1 - \alpha, \alpha \in (1 - r1 \cdot rand, 1), \quad (3)$$

$$\beta \in (0, r1 \cdot rand), r1 \text{ is a constant}$$

$$Q_f(t+1) = Q_e + (Q_f(t) - Q_e) \% (u_l \times v) / v \quad (4)$$

$$(Q_f(t) - Q_e < 0, u_l = D_{min})$$

$$Q_f(t+1) = Q_e + (Q_f(t) - Q_e) \% (u_r \times v) / v \quad (5)$$

$$(Q_f(t) - Q_e > 0, u_r = D_{max} - Q_e)$$

$$v = k \cdot rand, Q^d \in [D_{min}, D_{max}]^d \quad (6)$$

Among formulas above,  $Q_p$  is the current best position of frog,  $Q_b$  is the best position in the memeplexe,  $Q_f$  is the current frog's position.  $C$  is the gravity between the current best position of frog and the best position in the memeplexe,  $K$  is a constant. The frog's position was updated by the gravity attractor, in order to find the optimal position. Frog was constantly updated, so it may deviate from the gravity and emerge from transboundary. In shuffled frog leaping algorithm, this paper uses the space zoomed method to solve the transboundary problem.

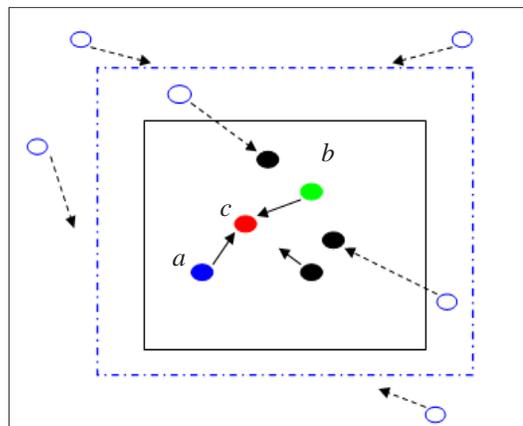


Figure 1. Sketch map of spatial scaling, solid frame is the original search space, the blue line frame is enlarged search space

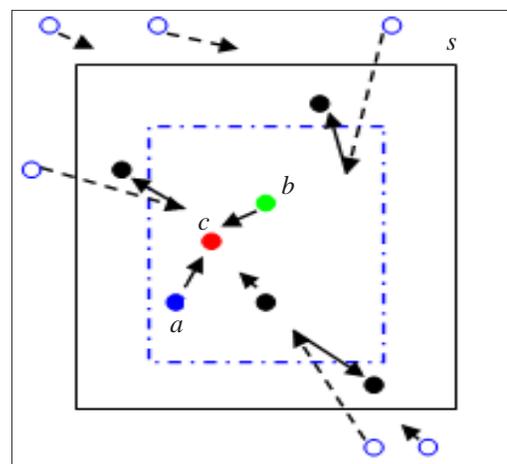


Figure 2. Sketch map of spatial scaling, solid frame is the original search space, the blue line box is the reduced search space

The principle is shown in Figure 1 and Figure 2, in Figure 1, the solid line frame is the original search space, the blue point line frame is the enlarged scale space of original search space, The solid ball is the frog in original search space, hollow ball is the frog flying out the original space. The solid red ball  $c$  is a gravity attractor, blue solid ball  $a$  is the current best position of the frog, green solid ball  $b$  is the best frog's position in memeplexe. As you can see from Figure 1, the search space was enlarged, there are two hollow balls (the frog) was put into the blue point line frame; then the search space is reduced, the two hollow balls was put in the original search space, this two hollow balls become two solid black ball, that is say these two transboundary's frog go into the original search space. From the figure 2, we can see, the search space was reduce, the frog of the old search space and the frog out of the old search space all were absorbed into the new search space. (blue point line frame). Figure 2 shows, there are three hollow balls (the frog) was placed in the blue point line frame; then as the search space amplification, the three balls go into the original search space (solid frame).

### 3.1 Space Zoomed Factor

Space zoomed factor  $v = t \times rand$ ,  $t$  is an integer greater than zero. When  $v > 1$ , the search space was firstly expanded and then reduced; when  $v < 1$ , the search space is firstly reduced and then expanded. In the SFLA, algorithm, each frog is an important one, regardless of its fitness value is bad or best, it will not be eliminated. Direct or indirect social relationship of the frog is complex information transmission way of SFLA algorithm. And the position relationship of the frogs is an important link from the optimization problems to obtain information on the link, so either in the solution of frog's transboundary problem or premature convergence, slow convergence problem, should try to avoid unnecessary interference for population structure in the evolution, which a lot of improvement algorithm does not consider. As seen in Figure 1, the search space is zoomed out, then the frog out of the search space already is also included, becomes effective frog, through the periodic zoomed in or out, all the frogs were controlled in the search space, which not only protect position relationship of frog populations, but also provide a strong guarantee for the algorithm's global search capability.

### 3.2 Gravity attractor

Gravity attractor  $c$  is a new centers of social influence. Space zoomed center of SFLA algorithm is not the original search space geometry center, but the center of gravity attractor  $c$ , it is the weighted average position of the individual best value and global best value. Under gravity attractor's guidance, interest's region around the center of social influence gather more frogs, greatly improve the convergence speed, enhance local exploitation ability of the algorithm.

In the SFLA algorithm, when  $Q_p = Q_b$ , it is no longer subject to other frogs by the formula (3), and begin to stagnate,

does not favor the evolution of the population. Therefore, the improved SFLA algorithm always use the attractor of the global optimal frog as a perturbation, give gravity attractor  $Q_c = r2 \times Q_b + r3$ , in which a random number is  $.r2$  (0.5, 1.5),  $r3 \in (-1, 1)$ . This is benefit to increase activity of global optimal frog, and help to maintain the diversity of the population.

### 3.3 zgSFLA Algorithm's Pseudo code

Pseudo code is as follows:

input: Number of global iterations glo, global best solution  $Qg$ , number of local iterations loc, local best solution  $Qb$ ; the number of memeplexe  $m$ , number of frogs in each memeplexe  $n$ , population  $Q(m * n)$ .

output: global best solution  $Qg$

```
GlobalSFLA ()
{
  For i = 1 to glo
    Sort Shuffle (Q (m * n));
    get (Qg);
    For j = 1 to loc
      For u = 1: m
        For v = 1: n
          Get (Qb);
           $Qc = r1 * Qp (v) + (1 - r1) * Qb$ ; //gravity
           $Qf = Qp$ ;
          If  $Qc = Qb$ 
             $Qc = r2 * Qb + r3$ ;
            // update the best frog in the memeplexe
          end
          LocalSearch (Qc, & Qf);
            if  $f(Qf) < f(Qp)$ 
               $Qp = Qf$ ;
            end
          End//n
        end//m
      end//loc
    End //g
  }
```

Input: gravity attractor  $Qc$ , current frog  $Qf$  in the generation  $t$

Output: new frog  $Qf$  in generation  $t + 1$

```
LocalSearch (Qc, & Qf)
{For w = 1: d//d is dimation
  If  $Qc (w) > Dmax$ 
//transboundary processing for gravity attractor Qc
     $Qc (w) = Dmax$ ;
  If  $Qf (w) > Dmax$ 
// transboundary processing for current frog
     $Qf (w) = Dmax$ 
  end
  Elseif  $Qc (w) < Dmin$ 
     $Qc (w) = Dmin$ ;
  If  $Qf (w) < Dmin$ 
     $Qf (w) = Dmin$ ;
  end
```

```

Else
If  $Qf(w) < Qc(w)$ 
     $U1 = Qc(w) - Dmin;$ 
 $Qf(w) = (Qf(c) - Qc(w)) \% (ul * v) * v + Qc(w);$ 
    //space zoomed processing
Else
     $ur = Dmax - Qc(w);$ 
     $Qf(w) = (Qf(c) - Qc(w)) \% (ur * v) * v + Qc(w);$ 
    // space zoomed processing
End
End
}

```

#### 4. Experiments Analysis

Experiment and simulation is to abstain the minimum value of 5 benchmark functions (the optimal value is 0), Testing software platform is MATLAB7.8 and Windows XP, machine frequency is P4 (210GHz), memory is 3G, hard disk is 120G .The SFLA, szAPSO and zgSFLA algorithms are tested, in order to enhance the comparability of the test, all of the following public parameter settings are the same. The number of global iteration  $T = 500$ , population number is  $P = 200$ , memeplexe's number is  $m = 20$ , individual frog's number in each memeplexe  $n = 10$ , the number of local iteration  $r = 10$ , To szAPSO algorithm for the same comparison, we adds an inner loop, the number of cycle is  $r = 10$ . zgSFLA algorithm parameter  $v = 100$ , random number  $r1 \in (0, 1)$   $r2 \in (0.5, 1.5)$ ,  $r3 \in (-1, 1)$ , variable dimension  $d = 30$ . Three algorithms were run 30 times, We obtain the optimal solution after 30 times continuous operation of the five functions, the global optimal value, the average of the global optimal values, standard deviation is as evaluating indices of the algorithm performance.

(1). Sphere function:  $f1 = \sum_{i=1}^n x_i^2, (x_i \in [-100, 100])$

(2). Rastrigin function:  $f2 = \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10),$   
 $(x_i \in [-5.12, 5.12])$

(3). Ackley function:

$$f3 = -20 \exp\left(-20 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e, (x_i \in [-32, 32])$$

(4). Griewang  $k$  function:

$$f4 = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 (x_i \in [-600, 600])$$

(5). Rosenbroch  $k$  function:

$$f5 = \sum_{i=1}^{n-1} 100 \cdot (x_{i+1} - x_i^2)^2 + (1 - x_i)^2 (x_i \in [-2.048, 2.048])$$

Table 1 lists test results of the above 3 kinds of algorithm.

Table 1 shows that the convergence speed of zgSFLA algorithm is good, improve the quality of solution; improved SFLA algorithm improves the convergence accuracy, and has good robustness, and can effectively avoid the algorithm being trapped in local optimal solution.

Figure 3-7 is the evolution curves of average optimal fitness above 5 functions by using 3 kinds of algorithm to run 30times. The 5 figures show that converges of our zgSFLA algorithm is much faster than SFLA algorithm and szAPSO algorithm, for the Rastrigin function, the SFLA algorithm is superior to szAPSO algorithm; in five different function, zgSFLA algorithm is rarely have flat, which is the key of global shuffled method. To sum up, the proposed algorithm has better convergence speed for either single peak function or multi peak function, it is a kind of reliable and effective global optimization algorithm.

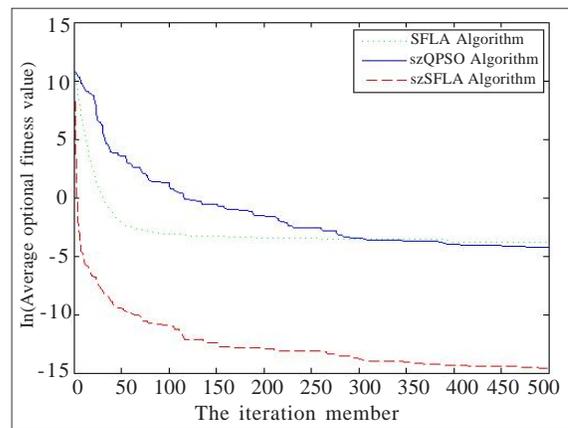


Figure 3. Running result of Sphere function

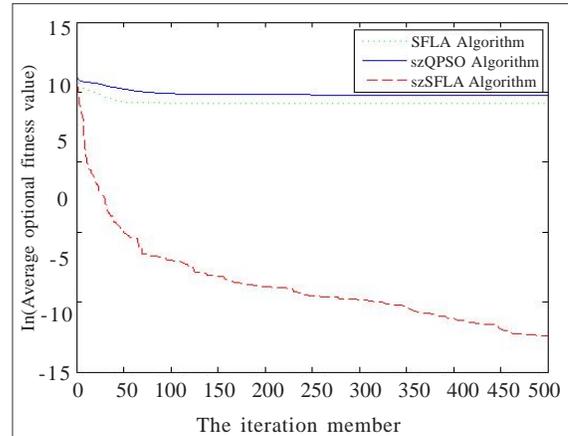


Figure 4. Running result of Rastrigin function

#### 5. Conclusion

zgSFLA algorithm is a new swarm intelligence optimization algorithm. This paper introduces the gravity attractor and spatial zoomed factor, improve local search strategy of SFLA. Gravity attractor guide all the frogs in looking for the right position at the same time, for the frog out of the search space, the use of space zoomed

Function name	Algorithm	Optimal Solution	Average Optimal Solution	Standard Deviation
Sphere	SFLA	0.0039	0.0941	0.1468
	szAPSO	0.00261001526766287	0.0144361854215652	0.0114897963880160
	zgSFLA	4.00414326291755e-08	4.74190641563352e-07	5.79608233656781e-07
Rastrigin	SFLA	45.6290	66.6608285714286	13.3567704395829
	szAPSO	102.264708133047	115.812061102363	6.9321881700543
	zgSFLA	7.40185464920273e-07	4.09512859578242e-06	4.26352440805383e-06
Ackley	SFLA	3.3166	3.61070000000000	0.357775674596993
	szAPSO	0.0233864936012945	0.100580661986391	0.0862204848018264
	zgSFLA	6.46043930929708e-05	9.75981389510495e-05	5.21470439189532e-05
Griewangk	SFLA	1.2738	1.61213333333333	0.199129251157801
	szAPSO	0.000590715949187470	0.00124642471856920	0.000638025521970245
	zgSFLA	7.57585882915635e-10	7.97861499091113e-09	1.04726278240278e-08
Rosenbrock's	SFLA	31.6832	35.2519333333333	4.33031065105804
	szAPSO	0.00576824248496783	0.0429264796326555	0.0568200276632366
	zgSFLA	4.17959480221189e-08	6.67220369290168e-06	1.19510506638086e-05

Table 1. Experimental Results

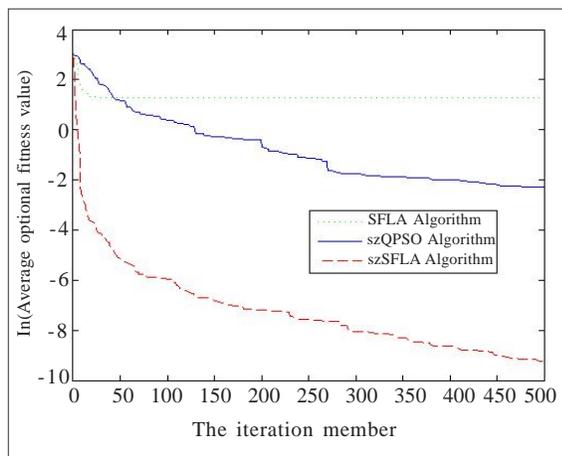


Figure 5. Running result of Ackley function

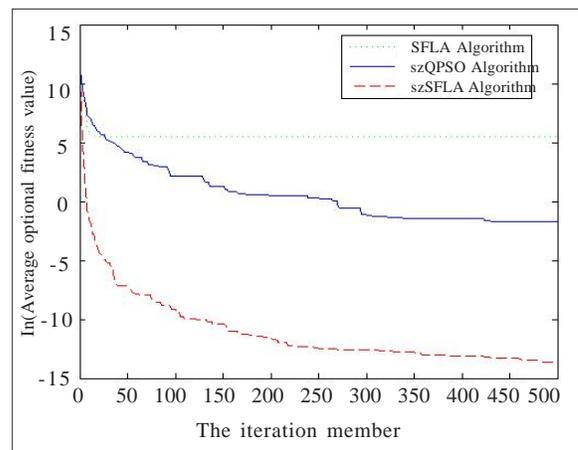


Figure 7. Running result of Griewangk function

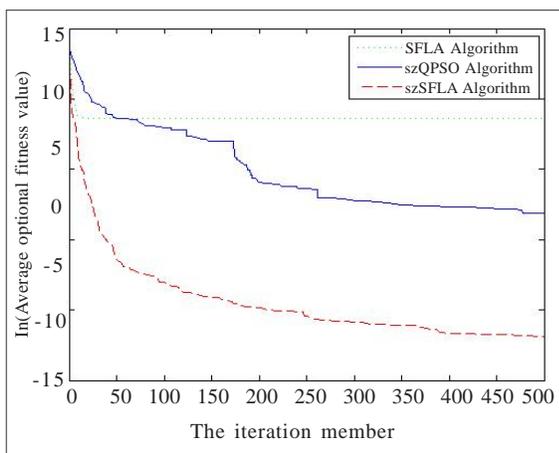


Figure 6. Running result of Rosenbrock's function

method, makes the frog can also be exploited and search, greatly improving the search capability. For gravity attractor parameter  $r1 > 1$ , which can improve the search scope of frog. Selection of  $r1$  is related to application field. Experimental results show that, the improved algorithm has strong search ability, can effectively avoid the premature convergence, has fast convergence speed, good stability. At later work, the algorithm can be applied to many fields.

## References

- [1] Eusuff, M, Lansey, K, Pasha F. (2006). Shuffled frog-leaping algorithm: a memetic meta-heuristic for discrete optimization, 38 ( 2) 129 - 154.

- [2] Hui, Wang., Feng, Qian. (2007). Warm intelligence optimization algorithm, *Chemical Instrument and Automation*, 34 (5) 7- 13.
- [3] Luo, Xue-hui., Yang, Ye., Li, Xia. (2008). Solving TSP with Shuffled Frog-Leaping Algorithm. *Eighth International Conference on Intelligent Systems Design and Applications*. p. 228-232.
- [4] Yang, Zhao., Juan, Shan. Binary Shuffled frog leaping algorithm for solving 0-1knapsack problem. *Computer Engineering and Applications*. 46 (35) 39-41.
- [5] Pan, Quan-Ke., Wang, Ling., Gao, Liang., Li, Junqing. (2011). An effective shuffled frog-leaping algorithm for lot-streaming flow shop scheduling problem. *Int J Adv Manuf Technol*. 52, p. 699–713
- [6] Bhaduri, Antariksha., Bhaduri, Aranya. (2009). Color Image Segmentation using Clonal Selection-based Shuffled Frog Leaping Algorithm, 2009 *International Conference on Advances in Recent Technologies in Communication and Computing*, p. 517-520.
- [7] Yang, Ou., Yuanshu, Sun. (2011). Grid task scheduling strategy based on improved shuffled frog leaping algorithm, *Computer Engineering* 37 (21) 146-151.
- [8] Narimani, Mohammad Rasoul. (2011). A New Modified Shuffle Frog Leaping Algorithm for Non-Smooth Economic Dispatch, *World Applied Sciences Journal*, 12 (6) 803-814.
- [9] Luo, Ping ., Lu, Qiang . (2011). Modified Shuffled Frog Leaping Algorithm Based on New Searching Strategy, 2011 *Seventh International Conference on Natural Computation*, p. 1346-1350
- [10] Qiangying, Meng., Liaguo, Wang. (2011). Shuffled frog leaping algorithm based on orthogonal crossover operator. *Computer engineering and Applications*. 47 (36) 54-58.
- [11] Pengjun, Zhao., Zhe, Shao . (2012). A new improved Shuffled frog leaping algorithm, *Computer engineering and Applications*. 48 (8) 48-52.
- [12] Robinson, J.,Yahya, R. (2004). Particle swarm optimization in electromagnetic. *IEEE Transactions on Antennas Propagation*, 52 (2) 397-407.
- [13] Huang, T., Mohan, A. S. (2005). A hybrid boundary condition for robust particle swarm optimization. *IEEE Antennas and Wireless Propagation Letters*, 4 (1) 112-117.
- [14] Xu, S. H.,Yahya, R. (2007). Boundary conditions in particle swarm optimization revisited. *IEEE Transactions on Antennas and Propagation*, 55 (3) 760-765.
- [15] Yu-hong, Chi., Fu-cun, Sun., Wei-jun,Wang., Chunming, Yu . (2011). An improved Particle Swarm Optimization Algorithm with Search Space Zoomed Factor and Attractor, *Journal of software*, 34 (1) 115-129.