

# Spectral Efficiency of Linear Multiuser Receiver over UWB High Data Rate Channel based on Channel Division Multiple Access



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**ABSTRACT:** *This contribution is focused on the design and analysis of an innovative multiple access scheme that exploits the intrinsic properties of the wireless environment to improve the multiuser environment, entitled Channel Division Multiple Access (ChDMA). Such a design allows simultaneous multiuser accessing to a common destination by considering the channel impulse response (CIR) of each user as a signature. Benefitting from the natural diversity of the wireless environment, the receiver is able to detect and demodulate the received signals by using the channel as a code under the assumption of the channel state information. We analyze the performance of the ChDMA integration in MBOFDM UWB high data rate channel in a random environment. The spectral efficiency criterion is chosen to investigate the performance analysis using MMSE receiver. Additionally, we analyze an asymptotic analysis of MMSE detector behavior taking into account the channel eigenvalues distribution with the associated spectral efficiency. A concept of power delay profile (PDP) has emerged from this work to investigate its impact into the spectral efficiency.*

**Keywords:** ChDMA, MBOFDM, UWB, Spectral Efficiency, Eigenvalue Distribution, MMSE, PDP

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## 1. Introduction

For a very large bandwidth, the capacity derivation for UWB channel cannot be the same as for a classical narrowband channel. The main difference results from two frequency dependent attenuation origins: distance and frequency selectivity of channel. Indeed, the attenuation due to distance is proportional to the squared in verse of frequency, and cannot therefore be considered constant over the full UWB bandwidth. Besides, the various realizations of the channel act as linear filters that attenuate randomly the transmitted signal at different frequencies. This must be taken into account in the capacity computation [1].

Our purpose in this work is to review, firstly, the MBOFDM UWB high data rate system. Then, we present ChDMA Characteristics with the channel model. We shall examine system spectral efficiency with a selection of MMSE detector in a random environment in the receiver. The MMSE linear detector is particularly interesting as it allows maximizing the signal-to-interference ratio (SIR) among all linear receivers. The key performance measure here is the eigenvalue distribution of the channel matrix. In section III,

we evaluate the performance analysis by means of spectral efficiency in large system approach. In section IV, we present the PDP impact in spectral efficiency.

## 2. Context and Assumptions

### 2.1 Multiband OFDM solution

The multiband technique proposed in the WiMedia Alliance MBOFDM scheme divides the UWB spectrum into 14 bands of 528 MHz each, as illustrated in Figure 1. The first 12 bands are then grouped into four band groups consisting of three bands each. The last two bands are grouped into a fifth band group [2]. Initially, most of the studies have been performed on the first three subbands from 3.1 to 4.8GHz. An OFDM signal can be transmitted on each subband using a 128 point inverse fast Fourier transform (IFFT). Out of the 128 subcarriers used, only 100 are assigned to transmit data. The multiuser access is performed with time frequency codes (TFC) which provides frequency hopping from a subband to another at the end of each OFDM symbol. Hence, at a given instant, if we consider a 3 user system, each user occupies one of the first three subbands. The TFC allows every user to benefit from frequency diversity over a bandwidth equal to three subbands [2].

An OFDM signal can be transmitted on each subband of the 128 subcarriers used; only 100 are assigned to transmit data. The multiuser access is performed with time frequency codes (TFC) which provide frequency hopping from a subband to another at the end of each OFDM symbol [2].

In the MBOA solution case, the signal generated at the output of the IFFT expresses [2] is:

$$S_{OFDM}(t) = \sum_{n=-\infty}^{+\infty} \sum_{i=-N_{ST/2}}^{N_{ST/2}} X_n(i) p_c(t - iT_{CP}) e^{j2\pi n \Delta_f(t - iT_{CP})} \quad (1)$$

$\Delta_f$ ,  $N_{ST}$  and  $T_{CP}$  represent respectively the subcarriers spacing, the total number of used subcarriers and the spacing

between two consecutive OFDM symbols.  $X_n(i)$  is a complex symbol belonging to QPSK constellation and is transmitted by subcarrier n during the ith OFDM symbol. It represents a data, a pilot or a reference symbol,  $p_c(t)$  is a rectangular window defined by [2] as:

$$\begin{cases} p_c(t) = 1. \text{ for } 0 \leq t \leq T_{FFT} \\ p_c(t) = 0. \text{ for } T_{FFT} \leq t \leq T_{FFT} + T_{CP} + T_{GI} \end{cases}$$

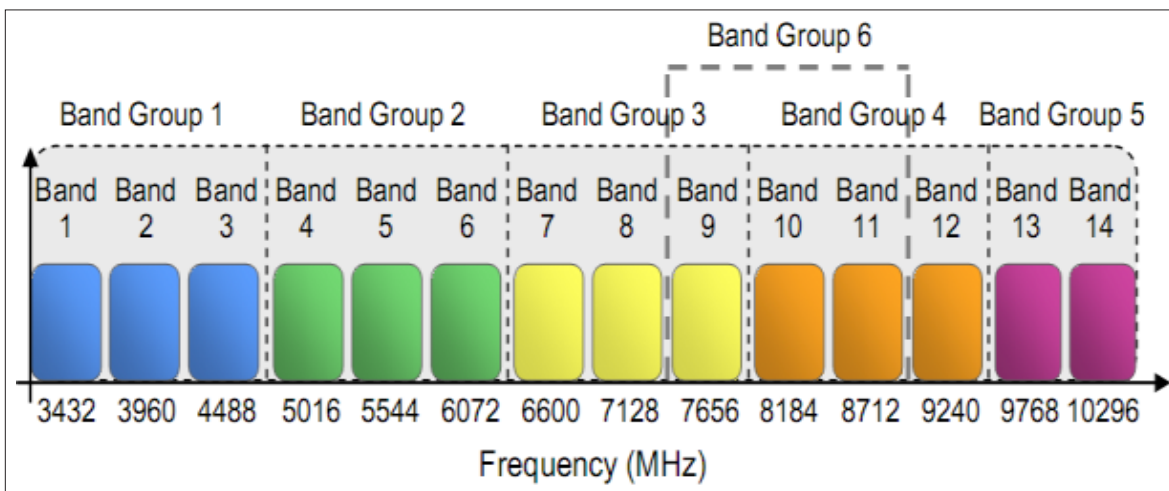


Figure 1. UWB spectrum bands in the MB-OFDM system

Transmitted data rates in each subband vary from 53.3 to 480 Mbit/s, which are listed in Table 1 [2].

Data rate	Modulation	Coding Rate (R)	Conjugate Symmetric	Time Spreading Factor	Coded bits
53.3	QPSK	1/3	Yes	2	100
80	QPSK	1/2	Yes	2	100
110	QPSK	11/32	No	2	200
160	QPSK	1/2	No	2	200
200	QPSK	5/8	No	2	200
320	QPSK	1/2	No	1	200
400	QPSK	5/8	No	1	200
480	QPSK	3/4	No	1	200

Table 1. MBOFDM Data Rates

### 2.2 802.15.3a Channel model

The proposed model for MBOFDM is 802.15.3a high data rate channel which is adopted from Saleh-Valenzuela model [3]. Mathematically, the impulse response of the multipath model is given by [4]:

$$h_k(t) = X_k \sum_{z=0}^{Z_k} \sum_{b=0}^{B_k} \alpha_k(z, b) \delta(t - T_k(z) - \tau_k(z, b)) \quad (2)$$

Where:

•  $X_k$  is the lognormal shadowing of the  $k^{th}$  channel realization,  $T_k(z)$  is the delay of cluster  $z$  and  $\alpha_k(z, b)$ ,  $\tau_k(z, b)$  represent respectively the gain and the delay of multipath  $b$  within cluster  $z$  respectively.

### 2.3 ChDMA-UWB system principle analysis

The idea behind the ChDMA is to use the channel impulse response (CIR) of each user as a signature, i.e., the signature code is given by the channel and the users are separated as a CDMA system. It is important to note that the signatures are given by the environment and by user's position, which means that they are uniquely determined. This signature location dependent property provides decentralized flexible multiple access as the codes are naturally generated by the radio channel [5].

The previous multiple access schema don't benefit of using low duty cycle transmissions. For this reason, in this paper we consider a multiuser communication system which exploits the duty cycle assumption and sends a modulated peaky signal. The new multiple access considered system is based in the integration of ChDMA in the 802.15.3a model. The ChDMA proposition works because UWB channels have a large coherence time (typically about 100  $\mu$ s) relatively to their delay spread (typically around 15-40 ns, depending on the user environment) [6]. Additionally, for the IEEE 802.15.3a standard, the received signal is composed of many delayed replicas of the original one.

Following the clustered ray model, each ray is considered as a single replica of the signal due to the distortion accompanied the environment, yields that each user occupies an appropriate position as presented in Figure 2 [6] where we consider an uplink channel between user  $k$  and the destination. For this reason, the transmitted signal identification is easily obtained where the receiver benefits of the different position occupied by each user independently, as it knows the channel it will be able to detect and to demodulate the received waveform.

## 3. Spectral Efficiency Performance Analysis

The spectral efficiency  $\gamma$  is defined as the number of bits per chip summed over the users that can be reliably transmitted [10]. It is expressed as the bits per second per Hertz (bit/s/Hz) supported by the system.

### 3.1 Linear Minimum Mean Square Error (MMSE) Receiver

The concept of linear MMSE detection originates from turning the problem of detection of transmitted symbols in a CDMA

system into a problem of linear estimation [16].

(3)

In the following, we consider the uplink communication, because the MMSE receiver requires a lot of information about all user channel conditions. The  $SINR_{k,n}$  at the output of the linear MMSE detector is given by [8]:

$$SINR_{k,n} = h_{k,n}^H (H_{k,n} H_{k,n}^H + \sigma^2 I)^{-1} h_{k,n}$$

Where  $n$  is related to  $n$ th symbol from  $N$  and  $k$  related to the  $k$ th user from  $K$ ,  $H_{k,n}$  is the matrix obtained from  $H$  suppressing the column  $h_{k,n}$ . The instantaneous spectral efficiency is given by [5]:

$$\begin{aligned} \gamma &= \frac{1}{N} \sum_{i=1}^N \log_2(1 + SINR_{k,n}) \\ &= \frac{1}{N} \sum_{i=1}^N \log_2(1 + h_{k,n}^H (H_{k,n} H_{k,n}^H + \sigma^2 I)^{-1} h_{k,n}) \end{aligned} \quad (4)$$

We must mention that the signal to noise ratio  $1/\sigma^2$  is related to the spectral efficiency  $\gamma$  following the author in [10] by:

$$\frac{1}{\sigma^2} = \frac{N}{K} \gamma \frac{E_b}{N_o}$$

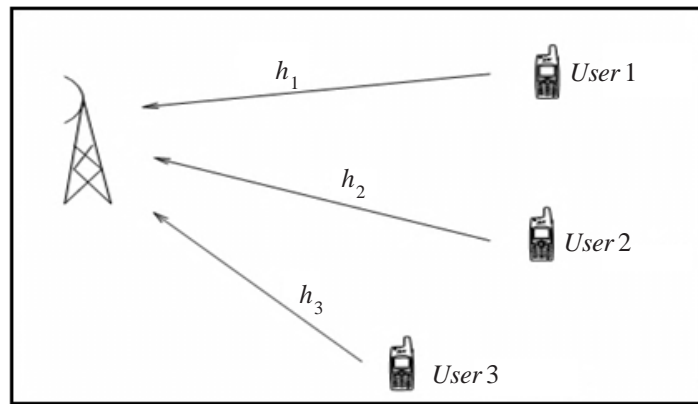


Figure 2. Channel Impulse Division Multiple Access with three users

### 3.2 Asymptotic analysis of eigenvalues

Deeper insight on the linear MMSE spectral efficiency performance behavior is obtained by  $K, N \rightarrow \infty$  with constant ratio  $\beta = K/N$ . The study of the eigenvalue distribution of random matrices has triggered important consideration in many numerous works. For this reason, the performance of MBOFDM ChDMA UWB communication system is determined via the eigenvalues of the covariance matrix  $C_N = H_N H_N^H$ . We study the eigenvalues distribution performance of linear MMSE multiuser receivers in random environments, where signals from the users arrive in “random directions”. The eigenvalue distribution is described in Figure 3 [12]:

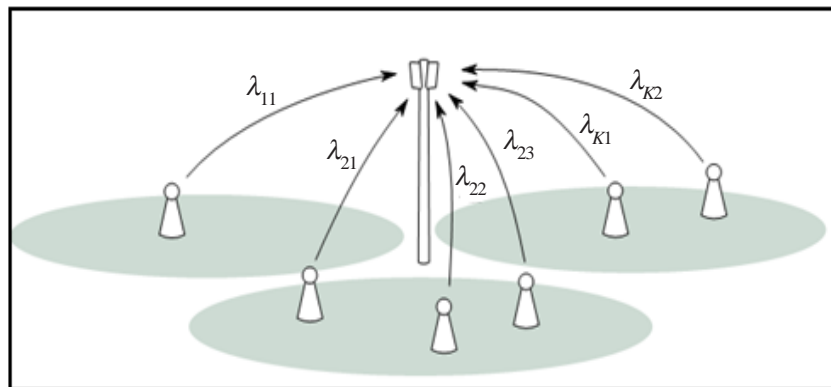


Figure 3. Eigenvalue distribution

The formula for the MMSE demodulator could be described via considering the transformation of  $C_N$  using the singular value decomposition (SVD). We denote the SVD representation of the channel correlation matrix  $C_N$ :

$$C_N = SDS^T$$

Where  $S$  is a unitary matrix containing the eigenvectors of  $HH^H$  and  $D$  is a diagonal matrix, containing the singular values  $\lambda_i$  with  $i=1, \dots, N$  representing the eigenvalues of  $HH^H$ . Second, following results from random matrix theory [9], it can be deduced that the empirical distribution of the eigenvalues of  $C_N$  converges to some limiting distribution  $F$  we obtain:

$$SINR_{k,n} = h_{k,m}^H (\lambda_i + \sigma^2 I)^{-1} h_{k,n} \quad (5)$$

We assume a large system, i.e., we let  $K, N \rightarrow \infty$  while  $K/N$  is finite and converges to a specific value. Yields in the asymptotic analysis, when  $N, K \rightarrow +\infty$ , the spectral efficiency can be expressed in terms of the eigenvalue distribution of  $HH^H$  and it is given by:

$$\Upsilon_{mmse} = 1/N \sum_{i=1}^N \log_2 \left( 1 + \frac{\lambda_i}{\lambda_i + \sigma^2} \right) \quad (6)$$

We must mention that in [21] (Lemma 9) if we consider the key performance criterion as the signal to interference (SIR) of user  $k$  under the MMSE detector we obtain:

$$\sum_{i=1}^K \frac{SIR_k}{1 + SIR_k} = \sum_{i=1}^K \frac{\lambda_k}{1 + \lambda_k}$$

By this way (6) could be described by:

$$\Upsilon_{mmse} = 1/N \sum_{i=1}^K \log_2 \left( 1 + \frac{SIR_i}{SIR_i + \sigma^2} \right) \quad (7)$$

We should point out that (6) converges in probability to:

$$\Upsilon_{mmse} = \int \log_2 \left( 1 + \frac{\lambda_i}{\lambda_i + \sigma^2} \right) dF_H^N(\lambda) \quad (8)$$

Where  $F_H^N$  defined by the author in [13] by:  $R \rightarrow R$  is the empirical eigenvalue distribution (e.e.d) of the random matrix  $H$ , i.e.

$$F_H^N(\lambda) = \frac{1}{N} \cdot |\{\lambda_i : \lambda_i \leq \lambda\}|$$

Let  $G_A(z)$  the Stieltjes Transform of  $F_H^N$  as defined by [22]:

$$G_A(z) = \int \frac{1}{\lambda_i + \sigma^2} dF_H^N(\lambda) \text{ for } z \in \mathbb{C}^+$$

Where  $\mathbb{C}^+ = \{z \in \mathbb{C}^+ | \text{Im}[z] > 0\}$

Yields:

$$\begin{aligned} \Upsilon_{mmse} &= \int_0^\infty \log_2 \left( \frac{\lambda_i + \sigma^2 + \lambda_i}{\lambda - z} \right) dF_H^N(\lambda) \\ &= \int_0^\infty \log_2(2\lambda + \sigma^2) dF_H^N(\lambda) - \int_0^\infty \log_2(\lambda + \sigma^2) dF_H^N(\lambda) \end{aligned} \quad (9)$$

Following Theorem 6 in [14], we have that  $F$  converge if  $N \rightarrow \infty$  to a distribution function  $G$ , we obtain the asymptotic output SNR of the MMSE receiver convergence of each user to:

$$h_{k,m}^H (H_{k,m} H_{k,m}^H + \sigma^2 I)^{-1} h_{k,m} \rightarrow \rho = \int \frac{1}{\lambda + \sigma^2} dG(\lambda) \quad (10)$$

Finally we obtain:

$$\gamma_{\infty} \rightarrow \int_0^{\infty} \log_2(1+\rho) dG(\lambda)$$

Additionally, for asymptotically large random matrices such upper bounds are provided by random matrix theory .If the channel matrix is composed of independent identically distributed random entries. The eigenvalues of HHH are asymptotically upper bounded by [11]:

$$\lambda_k < (1 + \sqrt{\beta})^2$$

By this way the bounds of (8) are given by:

$$\int_0^{\infty} \log_2(2\lambda + \sigma^2) dG(\lambda) < \int \log_2(2(1 + \sqrt{\beta})^2 + \sigma^2) dG(\lambda)$$

$$\int_0^{\infty} \log_2(\lambda + \sigma^2) dG(\lambda) < \int \log_2((1 + \sqrt{\beta})^2 + \sigma^2) dG(\lambda)$$

Yields :

$$\Upsilon_{mmse} < \int \log_2(2(1 + \sqrt{\beta})^2 + \sigma^2) - \log_2((1 + \sqrt{\beta})^2 + \sigma^2) dG(\lambda)$$

We obtain:

$$\Upsilon_{mmse} < \log_2 \frac{(2(1 + \sqrt{\beta})^2 + \sigma^2)}{((1 + \sqrt{\beta})^2 + \sigma^2)}$$

Following Theorem 2 presented in [15] we have that under the condition of large system the minimal value of eigenvalue is given by:

$$\lambda_k > (1 - \sqrt{\beta})^2$$

The minimum bounds of (8) are given by:

$$\int_0^{\infty} \log_2(2\lambda + \sigma^2) dG(\lambda) > \int_0^{\infty} \log_2(2(1 - \sqrt{\beta})^2 + \sigma^2) dG(\lambda)$$

$$\int_0^{\infty} \log_2(\lambda + \sigma^2) dG(\lambda) > \int_0^{\infty} \log_2((1 - \sqrt{\beta})^2 + \sigma^2) dG(\lambda)$$

Let the minimal bound is given by:

$$\Upsilon_{mmse} > \log_2 \frac{(2(1 - \sqrt{\beta})^2 + \sigma^2)}{((1 - \sqrt{\beta})^2 + \sigma^2)}$$

If we consider the case of  $0 < \beta < 1$ , and following the author in [17], the density of F(x) is given by :

$$\begin{cases} f_{\beta}(\lambda) = (\sqrt{\lambda - (1 - \sqrt{\beta})^2} \cdot ((1 + \sqrt{\beta})^2 - \lambda)) / 2\pi\beta\lambda, \\ \quad \text{for } (1 - \sqrt{\beta})^2 < \lambda_k < (1 + \sqrt{\beta})^2 \\ 0, \text{ otherwise.} \end{cases}$$

Then, if we consider the case of  $0 < \beta < \infty$ , we obtain:

$$\begin{cases} F_H^N(\lambda) = 1 - 1/\beta \text{ for } 0 < \lambda_k < (1 + \sqrt{\beta})^2 \\ F_H^N(\lambda) = 1 - 1/\beta + \int_{(1 - \sqrt{\beta})^2}^{\lambda} f_{\beta}(\lambda) dt \text{ for } (1 - \sqrt{\beta})^2 < \lambda_k < (1 + \sqrt{\beta})^2 \\ F_H^N(\lambda) = 1 \text{ when } \lambda_k < (1 + \sqrt{\beta})^2 \end{cases}$$

Additionally, Following the Toeplitz structure of the matrix  $C_N$  where the following eigen-decomposition holds when  $N \rightarrow \infty$  [18]:

$$C_\infty = \lim_{N \rightarrow \infty} F_N^H D F_N$$

Where  $F_N$  being the Fourier matrix and  $D$  could be defined following the decomposition in [14] according to its elements  $D_{n,n}$ :

$$\begin{cases} (PDP * L) / (W_c * T_1) & \text{with } (NW_c T_1) * (1 - 1) / L \leq n - 1 \leq (NW_c T_1) * 1 / L \\ D_{n,n} = 0 & \text{otherwise.} \end{cases} \quad (11)$$

Where:

- PDP refers to power delay profile representing the exponential decay of each cluster; also it reflects the decay of the total cluster power with the delay. The detailed definition is given following the author in [19]:

$$E [|\xi_{1,l} \beta_{k,1}|^2] = \Omega_0 e^{-\frac{T_1}{T}} e^{-\frac{\tau_{k,l}}{\gamma}}$$

With  $\Omega_0$  represents the average energy of the first path of the first cluster,  $T$  and  $\gamma$  are, respectively, defined as constants that characterize the exponential decay of each cluster and each ray.

- $W_c$  represents the frequency resolution,
- $T_1$  is the arrival time of the first path of the  $l$ th cluster and  $L$  is the number of multipath.

#### 4. Impact of Power Delay Profile

In this section we evaluate the impact of a power delay profile (PDP) on the spectral efficiency. In fact, if we refer to (11) and replace PDP with his previous expression in (12), we can represent the results of spectral efficiency with a random generation of the channel by employing the matrix  $D$ . We assume, respectively, the number of frequency samples  $N$ , the frequency resolution  $W_c$ , the number of multipath  $L$ , 128, 40 MHz and 100. Fig 4 presents the impact of the PDP on the spectral efficiency. As the SNR increase, the spectral efficiency of the simulated channel increase and this depend intimately of the system parameters. The result shows that the energy is equally spread over all the bandwidth. In this respect, PDP has been shown to be a useful measure, and we have employed it to identify the optimum number of parameters to represent the performance. This increase of spectral efficiency is also due to the attractive choice of OFDM for UWB communication because it can capture the multipath energy efficiently.

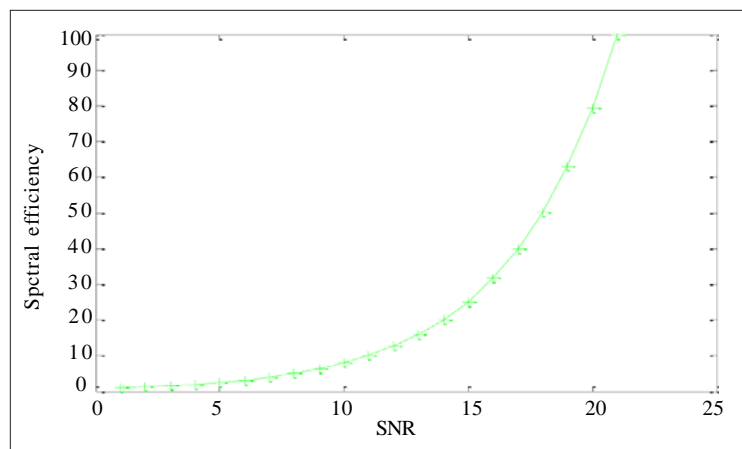


Figure 4. Impact of the power delay profile

The obtained result shows that the clusters are disjoint because we have an increase of spectral efficiency. However, we can find generally some overlap between the  $l$ th and  $(l+1)$ <sup>th</sup> clusters.

## 5. Results

In this section, we present the simulation results obtained by averaging over 500 Monte Carlo iteration, we consider the mode I of the MBOFDM channel employing the first three subbands of 528 MHz (from 3.1 GHz to 4.684 GHz). Additionally, each realization of the channel model related to each user is generated independently and assumed to be time invariant during the transmission of a frame. We simulated the MMSE spectral efficiency using the CM1 UWB channel model where the parameters are presented in Table 2 [20]. We consider at the receiver a per subcarrier MMSE frequency domain equalizer. Figure 5 depicts the spectral efficiency performance results obtained over the CM1 channel.

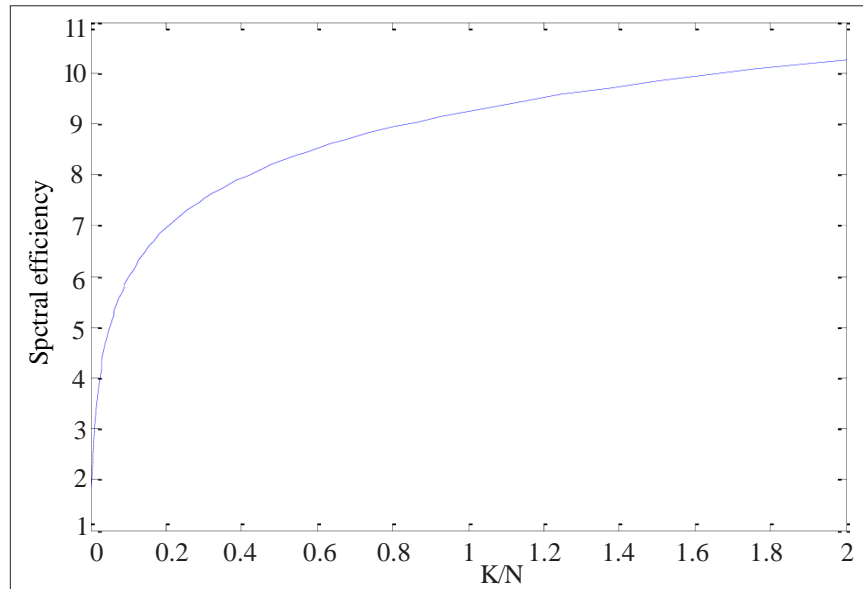


Figure 5. Validation of ChDMA-UWB channel model with MMSE detector ( $E_b/N_0 = 5\text{dB}$ )

The obtained results show that the performance of the MBOFDM ChDMA-UWB high data rate system is critically dependent of the receiver structure as the number of users increase, the spectral efficiency increases as well. The increase in spectral efficiency comes from the multiuser diversity effect provided by ChDMA system; if we consider many users transmitting independently, we could find a user with strong channel yields that the channel is used in the most efficient manner where the total throughput is maximized. By this way, we obtain a maximized multiuser diversity gain. We must mention also that QPSK is the best practical choice in the MBOFDM ChDMA-UWB channel coherent regime when the receiver knows the channel.

## 6. Conclusion

In this paper, we provided an analysis of the eigenvalues distribution performance of the MMSE receiver in MBOFDM ChDMA-UWB random environment in terms of spectral efficiency as the system size grows large. ChDMA system presents a very good technical solution to be used as UWB physical layer for short range high data rate wireless applications. The large system limit bounds of the spectral efficiency of the MMSE receiver are determined by way of evaluating the eigenvalue distribution. Additionally, we investigated the impact of power delay profile (PDP) on the spectral efficiency of MBOFDM ChDMA-UWB high data rate system.

As future work, a detailed performance evaluation of the impact of the modulation and the environment should be done. Furthermore, it is also of interest to focus in other types of receivers, which are more adapted for this multiple access scheme. A more ambitious and interesting open problem is the studying mutual information criteria as well channel estimation mismatches.

## References

[1] Mohamed Kamoun, Sylvain Chaillouz, Jeremy Gosteau. (2005). Laurent Mazet Marc de Courville Pierre Duhamel, Data rate upper bounds for UWB link with IEEE802.15.3a channel model. ICU 2005. *IEEE International Conference*.



- [2] Antoine STEPHAN. (2008). Resource Allocation Strategies and Linear Precoded OFDM Optimization for Ultra-Wideband Communications, PhD . dissertation, *National Institute of Applied Sciences (INSA)*.
- [3] Saleh, A., Valenzuela, R. (1987). A statistical model for indoor multipath propagation, *IEEE J. Select. Areas Commun.* 5 (2) 128-137, Feb.
- [4] Foerster, J. (2003). Channel modeling sub-committee report final, IEEE document P802.15-02/490r1, Intel R&D, Feb.
- [5] Raul L. de Lacerda Neto, Aawatif Menouni Hayar, Merouane Debbah. (2007). Channel Division Multiple Access: New multiple access approach for UWB Networks, European Ultra Wide Band Radio Technology Workshop, Grenoble, France, p. 10-11, May.
- [6] Raul Liberato De Lacerda Neto. (2008). Exploiting the wireless channel for communication, PhD Nice-Sophia Antipolis University.
- [7] Seyed Mohammad Sajad SADOUGH. (2008). Systemes OFDM Ultra Large Bande: Estimation de canal et detection ameliorée prenant en compte les imprecisions d'estimation , PhD. dissertation, Paris-sud M. Luc , Vandendorpe University.
- [8] Laura Cottatellucci, Ralf R. Muller, Merouane Debbah. (2007). On the Capacity of Asynchronous CDMA Systems. *Conference Record of the Forty-First Asilomar*.
- [9] Marcenko, V. A., Pastur, L. A. (1967). Distribution of eigenvalues for some sets of random matrices, *Math. USSR-Sb*, 1, p. 457–483.
- [10] Verdu, S., Shamai, S. (1999). Spectral Efficiency of CDMA with Random Spreading, *IEEE Trans. on Information Theory*, p. 622–640, Mar.
- [11] Ralf R. Muller. (2003). Applications of Large Random Matrices in Communications Engineering, *In: proceeding of the international conference on advances in Internet, Processing systems and Interdisciplinary Research (IPSI)*, Sveti Stefan, Montenegro.
- [12] Romain Couillet. (2009). Random Matrices in Wireless Communications, Course 2: System performance analysis: capacity and rate regions.
- [13] Peacock, Matthew, J. M., Collings, Iain, B., Honig, Michael, L. (2006). Asymptotic Spectral Efficiency of Multiuser Multisignature CDMA in Frequency-Selective Channels, *IEEE transactions on information theory*, 52 (3), March.
- [14] Jamie Evans, David N. C. Tse. (2000). Large System Performance of Linear Multiuser Receivers in Multipath Fading Channels, *IEEE transactions on information theory*, 46 (6), September.
- [15] Bai, Yin, Y. Q. (1993). Limit of the smallest eigenvalue of a large dimensional sample covariance matrix, *Ann. Probab.*, 21 p. 1275–1294.
- [16] Xie, Z., Short, R. T., Rushforth, C. K. (1990). A family of suboptimum detectors for coherent multiuser communications, *IEEE Journal on Selected Areas in Communications*, 8 (4) 683, May.
- [17] Silverstein, Jack, W. (1984). Some Limit Theorems on the Eigenvectors of Large Dimensional Sample Covariance Matrices, *Journal of Multivariate Analysis*, 15, 295-324.
- [18] Gray, R. M. (1977). Toeplitz and Circulant Matrices: A Review, Technical Rept. No. 6504-1, Inform. Sys. Lab., Stanford Univ., Stanford, CA, April.
- [19] Batra, A., Balakrishnan, J., Aiello, G. R., Foerster, J. R., Dabak, A. (2004). Design of a multiband OFDM system for realistic UWB channel environments, *IEEE Trans. Microwave Theory Tech.*, 52, p. 2123–2138, Sep.
- [20] Emeric Gueguen, Matthieu Crussiere, Jean-François Helard. (2007). An OFDM-CDMA Scheme for High Data Rate UWB Applications, VTC Spring, 2905-2909.
- [21] Hanly, Stephen V., David N. C. Tse, Member. (2001). Resource Pooling and Effective Bandwidths in CDMA Networks with Multiuser Receivers and Spatial Diversity, *IEEE Transactions on Information Theory*, 47 (4) May.
- [22] Wegmann, R. (1976). The asymptotic eigenvalue-distribution for a certain class of random matrices, *Journal of Mathematical Analysis and Applications*, 56, p. 113–132.