Performance Analysis of Large-scale Multiuser MIMO Downlink System

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ABSTRACT:*Currently, Multi-user Multiple-In Multiple-Out (MU-MIMO) systems are used in new generation wireless technologies. The numbers of users and applications increase rapidly owing to ongoing improvement in wireless technology. At the same time, wireless communication need the high data rate and link reliability. Therefore, MU-MIMO improvements have to consider 1) providing the high data rate and link reliability, 2) support all users in the same and frequency resource, and 3) using low power consumption. In reality, inter-user interference has a strong impact when more users access the wireless link. Complicated transmission techniques such as interference cancellation are used to maintain a given desired quality of service. Due to these problems, MU-MIMO systems with very large antenna arrays (known as massive MIMO) are proposed. The channel vectors are nearly orthogonal, and the inter-user interference is reduced significantly with massive MU-MIMO systems. Therefore, the users can be served with high data rate simultaneously. In this paper, we focus on the performance analysis of massive MU-MIMO downlink system where the base station uses linear precoding schemes to serve many users over Nakagami-m fading channel.*

Keywords: Massive MU-MIMO, Multi-user MIMO, Linear Fading, Nakagami-m Fading channel, Spectral Efficiency

Received: 12 April 2015, Revised 20 May 2015, Accepted 28 May 2015

1. Introduction

The goal of wireless communication improvement is to provide a high data rate for each user. Theoretically, increasing the number of antennas at the transmitter or receiver can improve the performance of the system in terms of data throughput and link reliability. Multiple-In Multiple-Out (MIMO) technology is introduced in modern wireless broadband standards e.g., Long Term Evolution Advanced (LTE-Advanced). According to 3GPP LTE standard, LTE permits up to 8 antennas at the base station [1]. Besides improving the data throughput and link reliability, MU-MIMO enables to save the transmitter energy, owing to the array gain [2]. On a channel that fluctuates rapidly as a ten of time and frequency, and where the situation allows coding across many channel coherent intervals, the achievable rate scales as $\min(M, K)\log(1+\mathrm{SNR})$ [1] where M is the number of base station antennas, and K is the number of users.

With a multi-user MIMO (MU-MIMO) system, the base station is equipped with multiple antennas and serves several users. Usually, the base station communicates with many users through orthogonal channels. More precisely, the base station communicates with each user in a separate time and frequency resource [2]. However, the higher data rate can be achieved if the base station communicates with the user in the same time-frequency resources. The main challenge of this system is inter-user interference, which significantly reduces the system performance. In the downlink, dirty-paper coding can be used to reduce the effect of the inter-user interference [3], [4]. However, it induces a significant complexity for the implementation.

Recently, massive MU-MIMO technology is attracting substantial attention from both academia and industry [1] – [5]. Most of the studies considered the uplink performance. In this paper, we study massive MU-MIMO downlink system with linear precoding schemes. We consider the system performance when the number of base station antennas and the number of single antennas users are large. We study the system performance in terms of data throughput over a channel model.

Furthermore to study the massive MU-MIMO system performance, we are interested in the performance comparisons amongst linear precoders: Minimum Mean Square Error (MMSE), Zero Forcing (ZF), and Maximum Ratio Transmission (MRT) precoding. Hypothetically, precoding is known as Space Division Multiple Access. Firstly, we derive the optimal linear precoding. Moreover, we simulate the channel model with computer software. Subsequently, we calculate the spectral efficiency to analyze the effect of massive MU-MIMO downlink system over a channel model. Each linear precoding shows the best performance with each signal power regime. For the comparison between MRT and ZF, MRT gives better performance at low signal to noise ratio (SNR) while ZF performs better at high SNR. MMSE gives the best performance across the entire SNR. These properties are used for improving the performance of massive MU-MIMO in different propagation environments

This paper is organized as follows. A brief overview of MU-MIMO Downlink System is presented in Section II. Section III and Section IV demonstrates the performance evaluation and simulation results. Conclusions and Future Work are presented in Section V and Section VI respectively.

2. Brief Overview of MU-MIMO Downlink System

2.1 System Model

We consider un-coded MU-MIMO downlink system with M transmit antennas at the base station (BS) and K single-antenna users in the system. A block diagram of such a system model is illustrates in Figure 1.

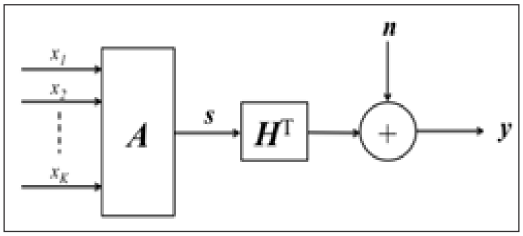

Figure 1. The system model

Let $\boldsymbol{A} \in \mathbb{C}^{M \times K}$ be a linear precoding matrix, and $\boldsymbol{x}$ is a $K \times 1$ information vector, where x_k is a data symbol for user k, where $\mathbb{E}\left[|x_k|^2\right] = 1$. The transmit vector $\boldsymbol{s}$ can be written as $\boldsymbol{s} = \boldsymbol{Ax}$, and its average transmission power is constrained by $\mathbb{E}[\boldsymbol{s}^2] = tr(\boldsymbol{A}^H \boldsymbol{A}) = P_{tr}$. Then, the received vector at the K users is given by

$$\boldsymbol{Y} = \boldsymbol{H}^T\boldsymbol{s} + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{n}$ is a $K \times 1$ additive noise vector. With our model, we assume that the desired signal vector $\boldsymbol{x}$ and the noise vector $\boldsymbol{n}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Further, the transmission power is constrained. We use this model to find the linear precoding matrix in order to study the performance of massive MU-MIMO system.

2.2 Channel Estimation

Typically, channel estimation is done on the downlink. More precisely, each user estimates the channel using pilot sequences transmitted from the base station, and then it feedbacks this CSI to the base station. This channel estimation over-head will be proportional to the number of base station antennas. Therefore, with massive MU-MIMO system, this is very inefficient. Thus, we assume that the channel is estimated at the base station via uplink pilots, assuming channel reciprocity.

2.3 Linear Precoding Schemes

We consider linear precoding schemes, which include MMSE, ZF and MRT.

MMSE Precoding Scheme. MMSE precoding is the optimal linear precoding in MU-MIMO downlink system. This technique is generated by the mean square error (MSE) method. Owing to average power at each transmitted antenna is constrained; Lagrangian optimization method is used for obtaining this precoder. The optimal MMSE precoder is given as [6]

$$\boldsymbol{A}_{MMSE} = \frac{1}{\beta} \boldsymbol{H^*}\left(\boldsymbol{H}^{T}\boldsymbol{H^*} + \frac{K}{P_{tr}} I_{K}\right)^{-1} \tag{2}$$

$$\beta = \sqrt{\frac{\mathrm{tr}\,(BB^{H})}{P_{tr}}} \tag{3}$$

where $\boldsymbol{B} = \boldsymbol{H^*}\left(\boldsymbol{H}^{T}\boldsymbol{H^*} + \frac{K}{P_{tr}} I_{K}\right)^{-1}$

ZF Precoding Scheme. ZF precoding is linear precoding which the inter-user interference can be cancelled out at each other. This precoding is assumed to implement a pseudo-inverse of the channel matrix. The ZF precoder is given as

$$\boldsymbol{A}_{\mathrm{ZF}} = \frac{1}{\beta} \boldsymbol{H^*}\left(\boldsymbol{H}^{T}\boldsymbol{H^*}\right)^{-1} \tag{4}$$

where β is a scalar of Wiener Filter.

$$\beta = \sqrt{\frac{\mathrm{tr}\,(BB^{H})}{P_{tr}}} \tag{5}$$

where $\boldsymbol{B} = \boldsymbol{H^*}\left(\boldsymbol{H}^{T}\boldsymbol{H^*}\right)^{-1}$

MRT Precoding Scheme. One of the common methods is MRT which maximizes the SNR. MRT works well in the MU-MIMO system where the base station radiates low signal power to the users. The MRT precoder can be expresses as

$$\boldsymbol{A}_{MRT} = \frac{1}{\beta} \boldsymbol{H^*} \tag{6}$$

where β is a scalar of Wiener Filter.

$$\beta = \sqrt{\frac{\mathrm{tr}\,(BB^{H})}{P_{tr}}} \tag{7}$$

where $\boldsymbol{B} = \boldsymbol{H^*}$

2.4 Nakagami-m Fading Channel

Nakagami-*m* fading has been widely used to model the fading distribution in various wireless channels. Therefore, Nakagami-*m* fading channel is used for modelling the channel. Typically Nakagami-*m* can be described by the Nakagami-*m* distribution. The PDF of Nakagami-*m* fading channel can be expressed as [7].

$$p_{R}(R) = \frac{R_{P}}{P_{tr}} \, exp\left(-\frac{mR^{2}}{\Omega}\right) \tag{8}$$

where the amplitude of channel $R \geq 0$, $\Omega = E\,(R^{2})$ is an average fading power with $m \geq \frac{1}{2}$ and $\Gamma(m)$ is the gamma function m and Ω can be expressed as

$$m = \frac{E^{2}[R^{2}]}{\mathrm{v\,ar}[R^{2}]} \tag{9}$$

$$\Omega = E\,[R^{2}] \tag{10}$$

The complex Nakagami-*m* coefficient is generated by *m*-Gaussian random variables where each random value has zero mean and varianc $\frac{1}{2m}$ in both the real and imaginary parts $\sim N\left(0, \frac{1}{\sqrt{2\mathrm{m}}}\right)$, $Y \sim N\left(0, \frac{1}{\sqrt{2\mathrm{m}}}\right)$[22]. In the real part, all Gaussian random variables are squared and summed. The result gives a Chi-square distributed random number. This number is given by

$$Z^{2}_{Re} = X_{1}^{2} + X_{2}^{2} + \; ..... + X_{m}^{2} \tag{11}$$

$$h_{Nakagami\text{-}m} = G\, exp\, (j\grave{e}) \tag{12}$$

where G = $\sqrt{Z_{Re}^{2} + Z_{Im}^{2}}$, and $\grave{e}$ *is* (0, 2π).

3. Performance Analysis

This section describes the performances analysis which includes achievable rate, spectral efficiency and energy efficiency. Using this performance analysis, we study the tradeoff between the energy efficiency and spectral efficiency of massive MU-MIMO downlink with linear precoding schemes. We investigate the system performance with different number of base station antennas *M* and number of users *K*.

3.1 Achievable Rate

System performance can be defined by several methods. One of the methods to quantify system performance is achievable rate. The achievable rate is followed by Shannon theorem. This theory tells the maximum rate, which the transmitter can transmit over the channel. The channel is assumed ergodic and all parameters are Gaussian random processes.

From Shannon theorem, the channel capacity over Additive White Gaussian Noise channel is derived by [8]

$$R = log_2\,(1 + \mathrm{SNR}) \qquad (bits/s/Hz) \tag{13}$$

With MU-MIMO downlink system, the transmitter must know the channel state information. CSI is the key of the multi-user communication. Typically, the transmitter transmits multiple data stream to each user simultaneously and selectively with CSI [9]. All the receivers send the channel estimation feedback to the transmitter on the reverse link, so the transmitter obtains CSI. Hence, the transmitter communicates with all the receivers with perfect CSI. With a MU-MIMO system, the interference consists of additive noise and interference between the users. Then, the achievable rate of k^{th} user for MU-MIMO downlink system can be expressed as

$$R_k = \mathbb{E}\,[\log_2\,(1 + \mathrm{SNR_k})] \qquad (bits/s/Hz) \tag{14}$$

We analyse the achievable rate of user 1 because all of the users have the same achievable date properties. Then, we measure the achievable rate for user 1 by measuring the achievable rate of user 1 within 10, 000 channel realization. After we obtain the achievable rates, we average them.

3.2 Spectral Efficiency

With a single cell massive MU-MIMO system with perfect CSI, the spectral efficiency is defined as

$$R_{P\,=}\ \Sigma_{K=1}^{K} R_K \qquad (bits/s/Hz) \tag{15}$$

We calculate the spectral efficiency by multiplication of the achievable rate of user R_k and number of users in the massive MU-MIMO system *K* [1]. The spectral efficiency is given in (16). We use this parameter to study the performance of massive MU-MIMO downlink system.

$$R_p = K \times R_k \qquad (bits/s/Hz) \tag{16}$$

4. Simulation Results

This section describes analyses and compares the performance of single cell massive MU-MIMO downlink system with precoding schemes such as ZF, MMSE and MRT in over Nakagami-*m* fading channel. We conducted the simulation using MATLAB-2013a. Each simulation used 10,000 channel realizations to produce a correct and smooth result. We considered a single cell massive MU-MIMO system, which means no intra-cell interference, in two different scenarios over Nakagami-*m* fading channel with ZF, MRT, and MMSE optimal linear precoding schemes. In scenario 1, all the results are shown- in terms of spectral efficiency versus number of base station antennas *M*. In scenario 2, all the results are shown in term of spectral efficiency versus number of single antenna users *K*. The results show that by increasing the *m* parameter it improves the system performance in term of spectral efficiency.

Scenario 1 - *W*e increase the number of base station antennas *M* with different *m* parameter and set the number of single antennas users. We set the number of users, *K* =2 and increase the number of base station antennas between 2 to 20. We set the SNR to 0 dB. All the results are shown in Figure 2, Figure 3, Figure 4 and Figure 5 in the section below.

Scenario 2 - *We* increase the number of single antenna users *K* with different *m* parameter and set the number base station antennas. We set the number of base station antennas, M = 20 and increase the number of single antenna user between 2 to 20. Furthermore, we set the SNR at 0 dB. All the results are shown in Figure 6, Figure 7, Figure 8 and Figure 9 in the section below.

Figure 2 shows, when *m* = 2 the achievable rate of user 1 with MMSE gives better performance than *m* = 1. When *m* parameter is larger, the spectral efficiency with MMSE increases significantly. For example, at *M* = 10, the spectral efficiency with MMSE increased about 1.3 bits/s/Hz compared to *M* = 5.

Figure 3 and 4 show the spectral efficiency versus the number of base station antennas with different parameters *m* for ZF and MRT. All the results correspond to scenario 1. When we increase the number of base station antennas, the spectral efficiency increased significantly.

In Figure 5 all precoding schemes for scenario 1 are compared. MMSE gives the best spectral efficiency with different parameters of *m* at 0 dB. At smaller number of base station antennas, the spectral efficiency between ZF and MRT are very close with all the parameters for *m*. When we increase *M*, ZF gives better spectral efficiency compared to MRT with all the *m* parameters.

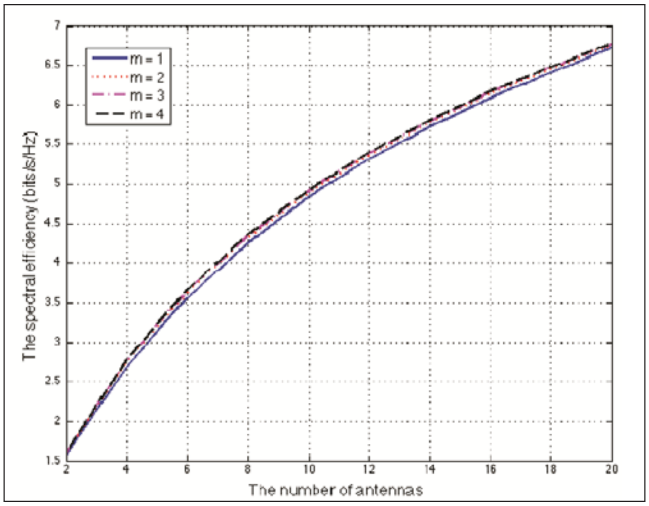

Figure 2. The spectral effiiciency over Nakagami-*m* fading channel with different number of base station antennas for MMSE at K = 2 and SNR = 0 dB.

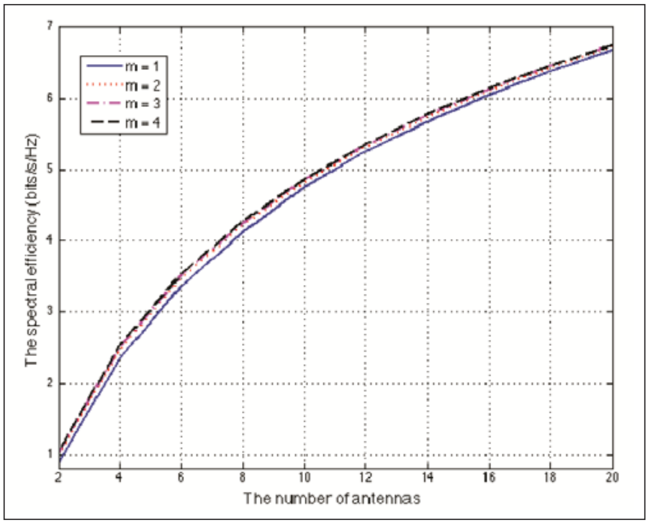

Figure 3. The spectral efficiency over Nakagami-*m* fading channel with different number of base station antennas for ZF at *K* =2 and SNR = 0 dB.

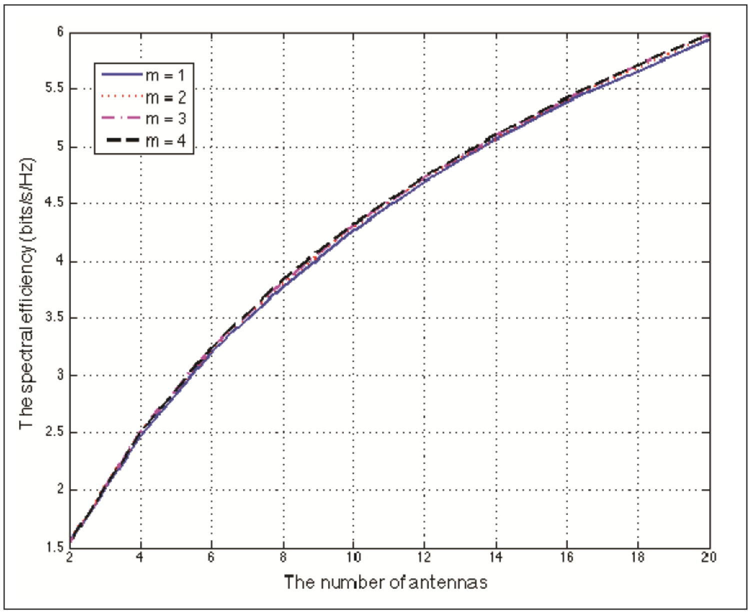

Figure 4.The spectral efficiency over Nakagami-*m* fading channel with different number of base station antenna for MRT at K =2 and SNR = 0 dB.

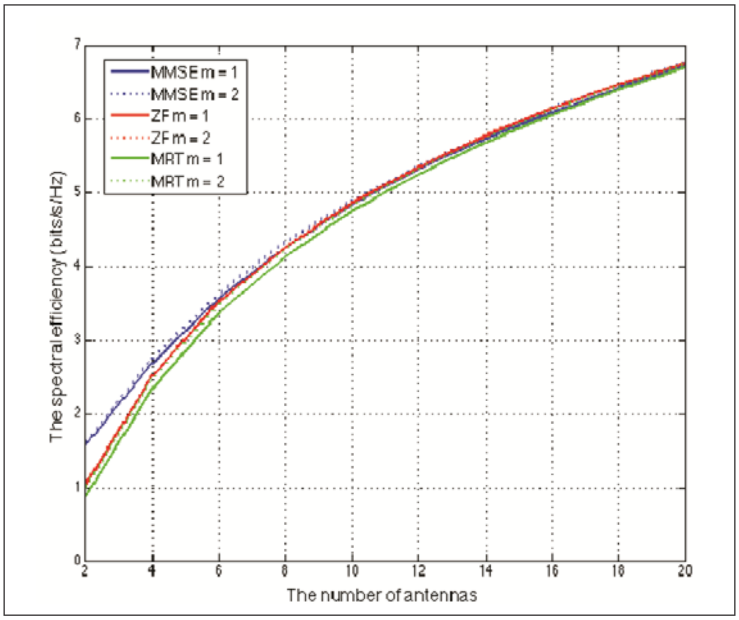

Figure 5. The spectral efficiency versus the number of base station antennas over Nakagami-m fading channel with MMSE, ZF, and MRT at K = 2 and SNR = 0 dB.

Figure 6 shows the spectral efficiency of massive MU-MIMO system with MMSE over Nakagami-*m* fading channel. The spectral efficiency drops when many single antenna users access the system. At the beginning, the spectral efficiency with MMSE increases by increasing the number of single antenna users. The spectral efficiency increases gradually till K =14. At K > 16, the spectral efficiency slightly reduces. The optimal numbers of single antenna users for MMSE are about 16 users.

Figure 7 shows the result of spectral efficiency with ZF. The spectral efficiency increases rapidly when the number of single antenna users increase. At K > 12, spectral efficiency decreased gradually. The optimal numbers of single antenna users in massive MU-MIMO system with ZF are about 12 users.

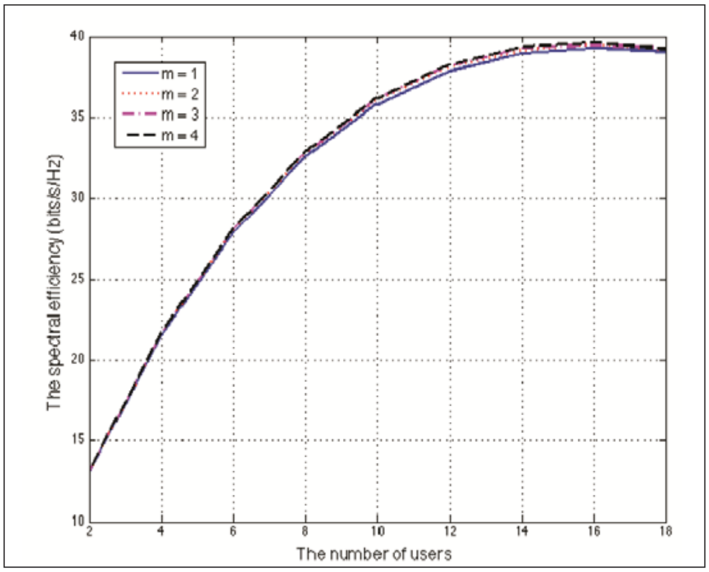

Figure 6. The spectral efficiency over Nakagami-*m* fading channel with different number of single antenna users with MMSE at *M* =20 and SNR = 0 dB

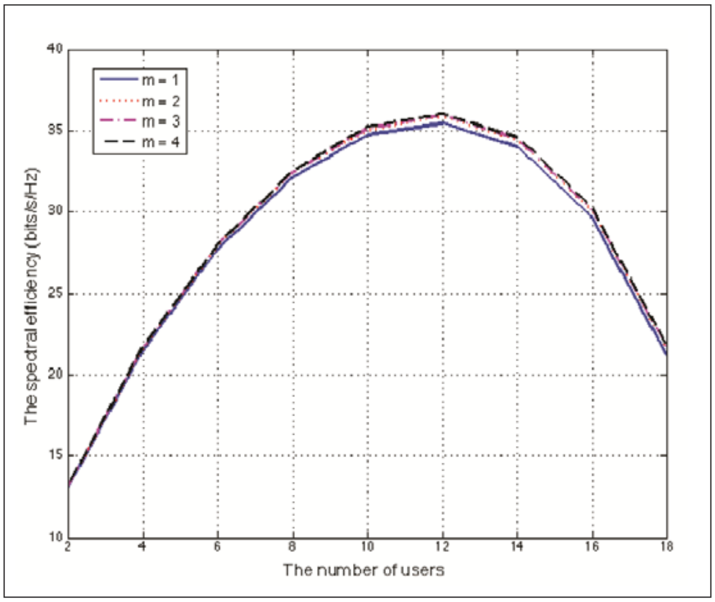

Figure 7. The spectral efficiency over Nakagami-*m* fading channel with different number of single antenna users with ZF at *M*=20 and SNR = 0 dB.

Figure 8 shows the spectral efficiency with MRT, where it increases slowly as the number of single antenna user increases. The spectral efficiency increases whereas the numbers of single antenna users are approaching the numbers of base station antennas.

All the results for scenario 2 are compared in Figure 9. The figure shows that massive MU-MIMO system with MMSE precoding scheme gives the best the spectral efficiency as the number of single antenna users increases. Morever , ZF precoding schemegives higher spectral efficiency than MRT precoding scheme. Nevertheless the spectral efficiency differences between ZF and MRT precoding schemes are very narrow as the number of single antenna user approaches the number of base station antennas.

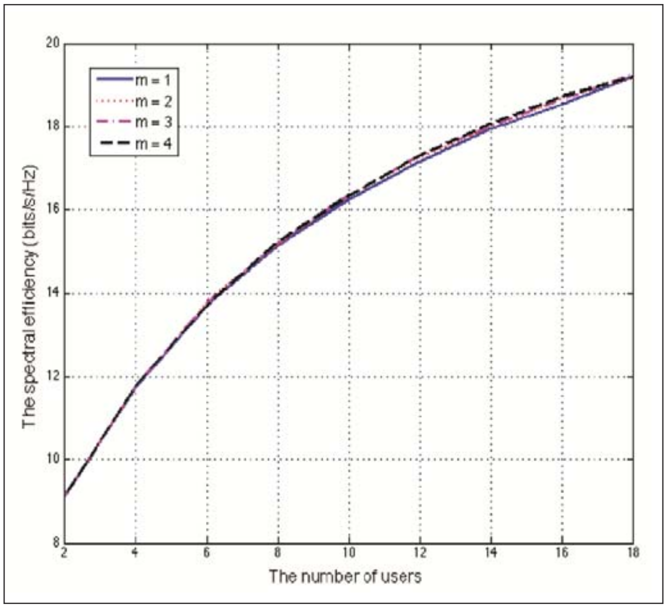

Figure 8. The spectral efficiency over Nakagami-*m* fading channel with different number of single antenna users with MRT at $M = 20$ and SNR = 0 dB.

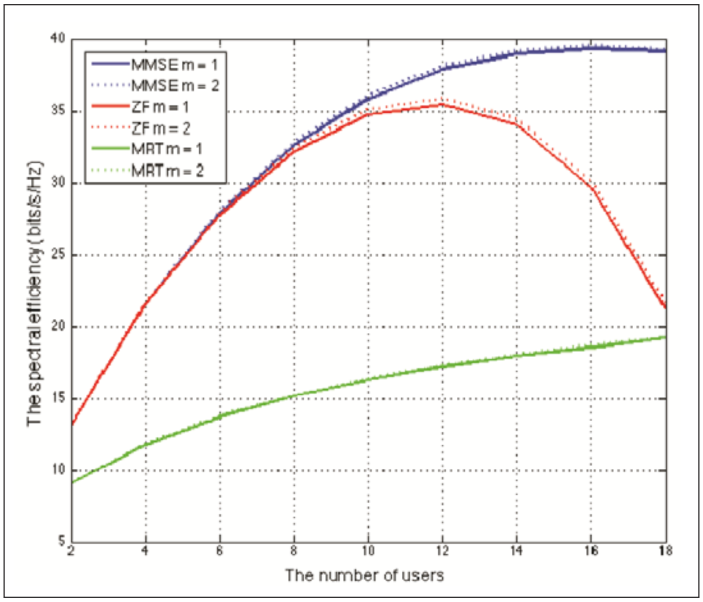

Figure 9. The spectral efficiency versus the number of single antenna users over N akagami-m fading channel with MMSE, ZF, and MRT at $M = 20$ and SNR = 0 dB.

5. Conclusion

A massive MIMO system introduces the opportunity of increasing the spectral efficiency in terms of bits/s/Hz. This system is able to utilise simple processing schemes such as MMSE, ZF, and MRT at the base station and using channel estimation from the uplink. Generally, ZF gives better performances at high transmission power while MRT gives better performance at the low transmission power. However, MMSE gives the best performance at low and high transmission power. Apart from massive MU-MIMO can use a simple processing, the propagation environment does not affect much on the system. With a practical channel model without interference such as the Nakagami-*m* fading channel, the massive MU-MIMO system with the linear precoding improves the performance with different *m* parameters when the number of antennas was increased. Therefore, massive MU-

MIMO systems are the key to the next wireless systems. Furthermore, this system offers advantages in term of achievable rate and spectral efficiency.

6. Future Work

Presently, wireless generation such as LTE, MU-MIMO system uses orthogonal Frequency Division Multiplexing (OFDM) to modulate the digital data to multiple sub-carrier frequencies. The advantages of OFDM on the MU-MIMO systems are to change the multipath environments from frequency selective to flat fading, and to eliminate inter-symbol interference (ISI).

In the simulations, we do not consider massive MU-MIMO system over a frequency selective fading channel. We assume that the multipath environment is flat. Our system model captures well the OFDM system with flat fading. Therefore we can apply this system in a practical way. The next step in our research is to investigate the performance of massive MU-MIMO downlink system in a practical system, which includes orthogonal frequency division multiplex (OFDM), amplifier, reliability, and phase noise.

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